Introduction

L1 09.04.19

Nonlinear dynamics in complex networks.

Dynamics is an approach to analyze complex systems. Viewed from the perspective of dynamics the subjects can be placed in a common framework.

Nonlinear – linear system = sum of its parts
– nonlinear system ≠ sum of its parts, the parts interfere, cooperate or compete.

Most of everyday life is nonlinear => the principle of superposition fails.

Example: if you listen to two your favourite songs at the same time you do not get double pleasure.

- operation of laser
- formation of turbulence in a fluid
- superconductivity of materials

Nonlinear dynamics is an approach to analyze nonlinear systems. What is a dynamical system?

(physics): any system that evolves in time

(math): an object for which: 1) the state is uniquely defined at a given time

2) there is a law (operator) which defines the evolution of this state in time

if we know the state at $t = t_0$ =>

=> we can uniquely define the state of the system at $t > t_0$ (in the future)

$$\boxed{\dot{\vec{x}} = \vec{F}(\vec{x}(t), t)} \quad (*)$$

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_N, t), \quad i = 1 \ldots N$$

$$\dot{x}_i = f_i(x_1, x_2, \ldots, x_N, t), \quad i = 1 \ldots N$$

$x_1, \ldots x_N$ → populations of different species in an ecosystem

→ concentrations of different chemicals in a reactor

→ positions and velocities of the planets in a solar system

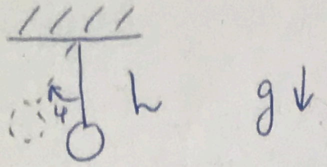
$\vec{F}$ is a vector function of dimension N

(*) defines a velocity vector field

$\vec{F}$ is the law (operator) which defines the evolution of the system

$$\begin{cases} \dot{x}_1 = \dfrac{dx_1}{dt} = f_1(x_1, x_2, \ldots, x_N, t) \\ \dot{x}_2 = \dfrac{dx_2}{dt} = f_2(x_1, x_2, \ldots x_N, t) \\ \vdots \quad - - - - - - - \\ \dot{x}_N = \dfrac{dx_N}{dt} = f_N(x_1, x_2, \ldots, x_N, t) \end{cases} \quad \Longrightarrow \quad \dot{\vec{x}} = \vec{F}(\vec{x}(t), t)$$

Example Swinging of a pendulum



φ - the angle

L - length of the pendulum

g - acceleration due to gravity

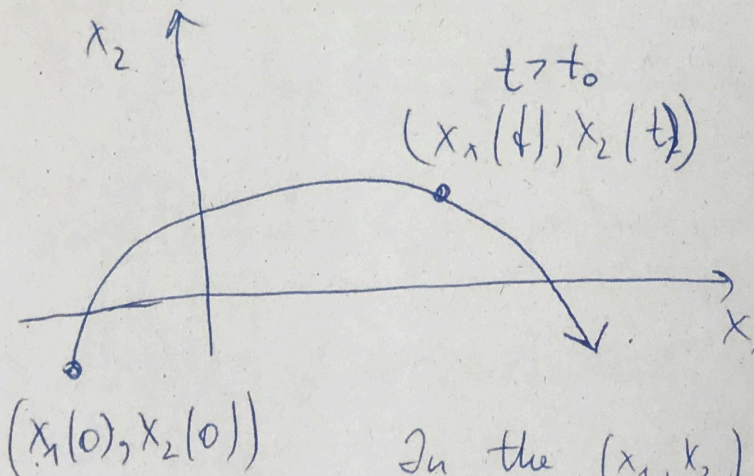
$$\boxed{\ddot{\varphi} + \frac{g}{L}\sin\varphi = 0} \Rightarrow \boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\underbrace{\sin x_1}_{\text{nonlinearity}} \end{aligned}} \quad (**)$$

$N = 2$

x_1 is the position

x_2 is the velocity

We construct an abstract space with coordinates (x_1, x_2)



Suppose we know a solution for $(**)$ for a particular initial condition.
The solution would be a pair of functions: $x_1(t)$ and $x_2(t)$.

In the (x_1, x_2)-plane the solution corresponds to a point (phase point) $(x_1(t), x_2(t))$ moving along a curve in this space.

The curve is called trajectory and the space is called phase space for the system.

We need the initial values of both x_1 and x_2 to define the solution uniquely.

Complex networks

Undirected graph $G = (\mathcal{N}, \mathcal{L})$

consists of two sets

$\mathcal{N}$ $\quad$ $\mathcal{L}$

$\mathcal{N} \neq \emptyset$

$\mathcal{L}$ is a set of unordered (ordered) pairs of elements of $\mathcal{N}$

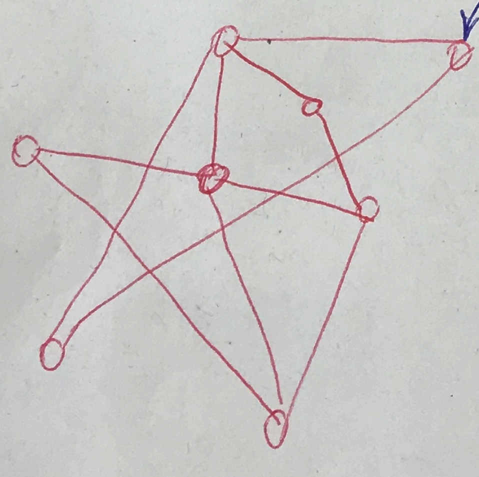
$\mathcal{N} \equiv \{n_1, n_2, \dots, n_N\}$ - nodes / elements / units

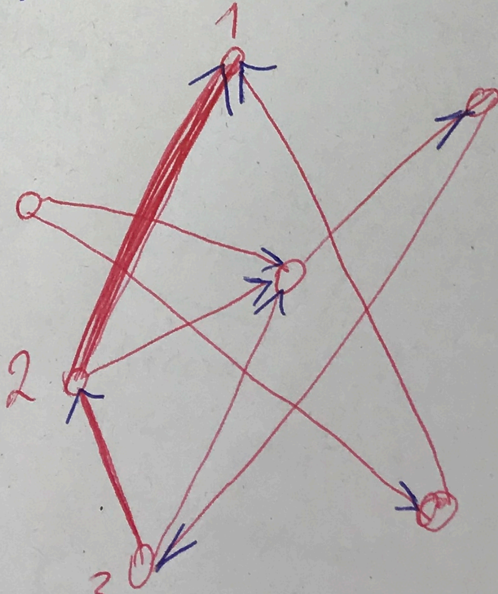
$\mathcal{L} \equiv \{l_1, l_2, \dots l_K\}$ - links / edges / connections

$G(N, K) = (\mathcal{N}, \mathcal{L}) = G_{NK}$ $\rightarrow$ notation

- interplay of dynamics and topology

dynamical system





- undirected
- directed
- weighted

$w_{12} > w_{23}$