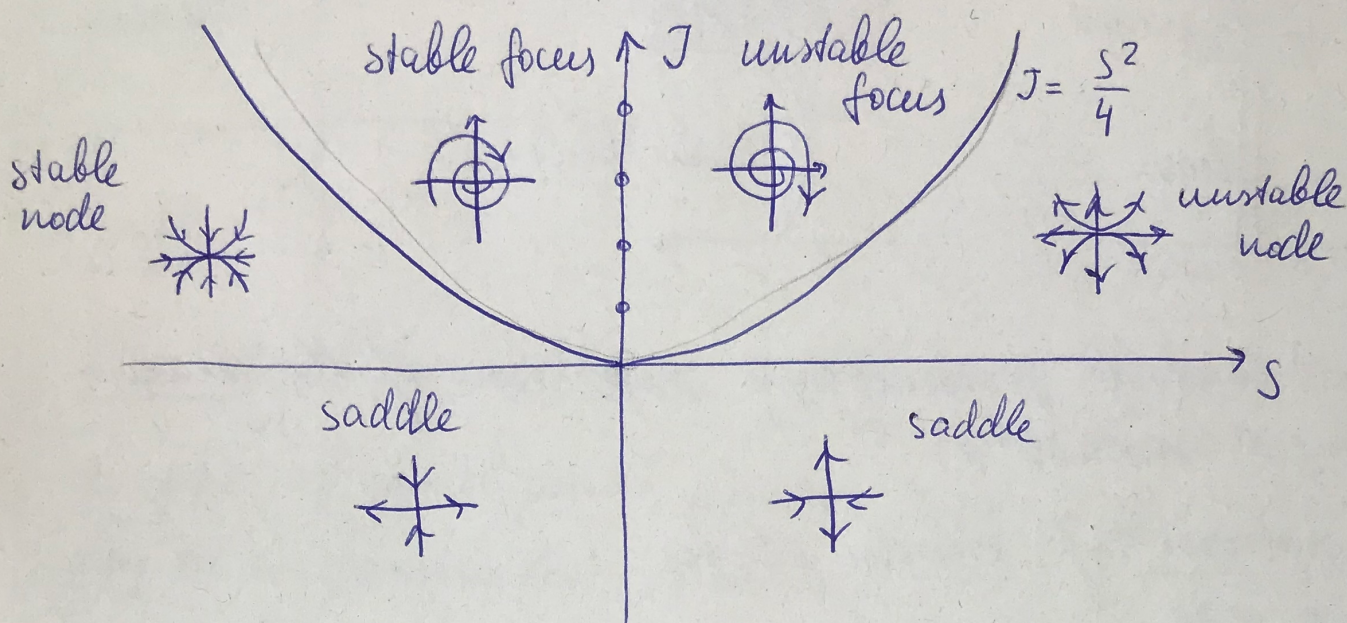


Types of steady states

L3 23.04.19



$S=0 \Rightarrow$ center

Conservative systems: they have two types of steady states

center

saddle

Dissipative: stable/unstable node, focus; saddle

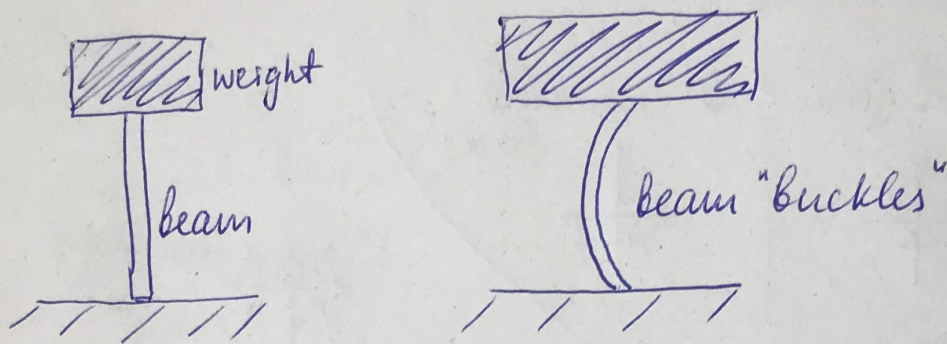
Attractors: stable node and focus

Bifurcations - qualitative change of the phase portrait (caused by the parameter tuning) which takes place when one or several control parameters are varied.

Bifurcation value of the parameter.

- profound structural changes of the phase portrait
- appearance / disappearance of limit sets
- changes in the stability of the trajectories - 1-

A simple example: buckling of a beam



weight - control parameter; deflection of the beam - dynamical variable x .

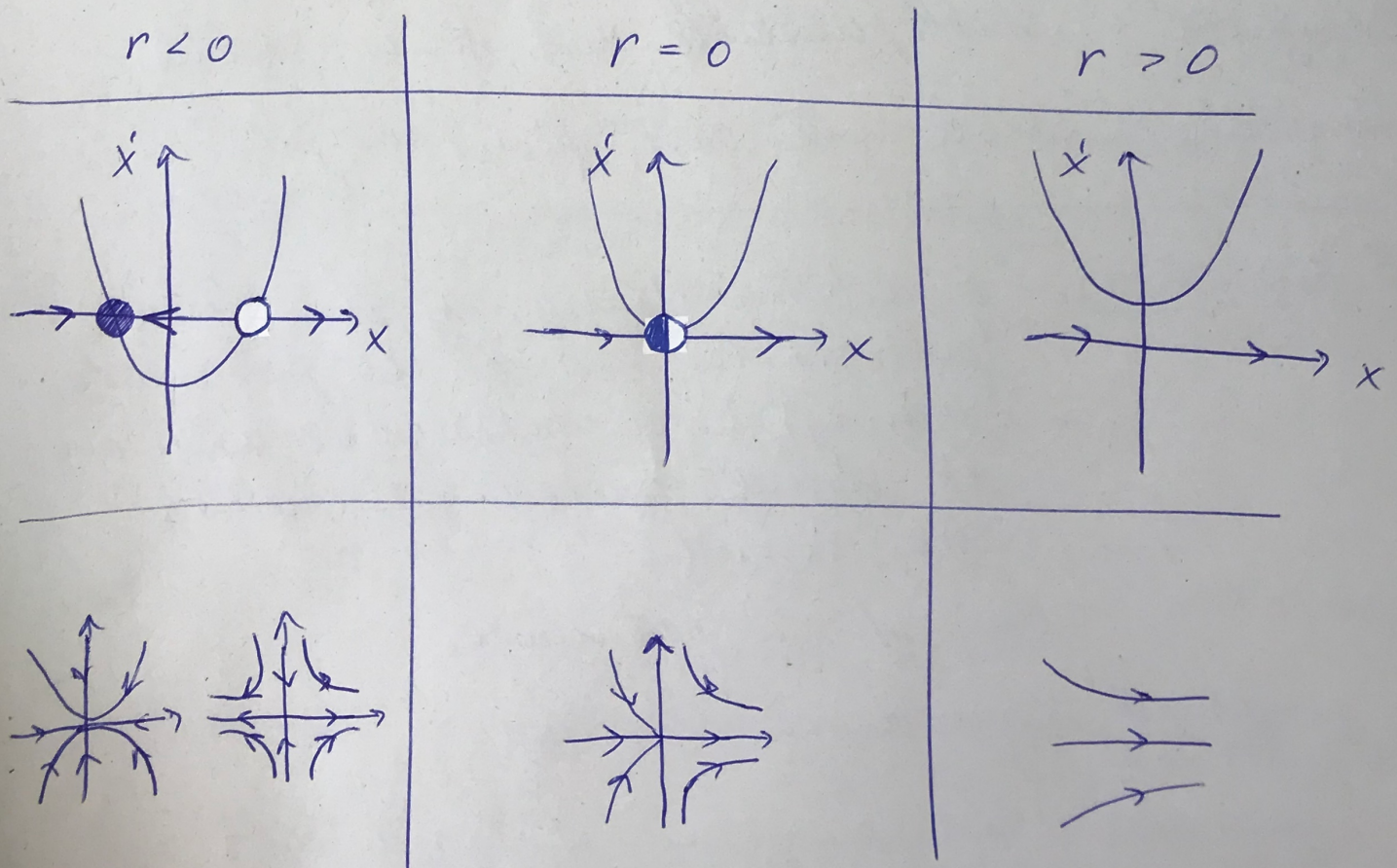
Bifurcations of steady states

1) Saddle node bifurcation -

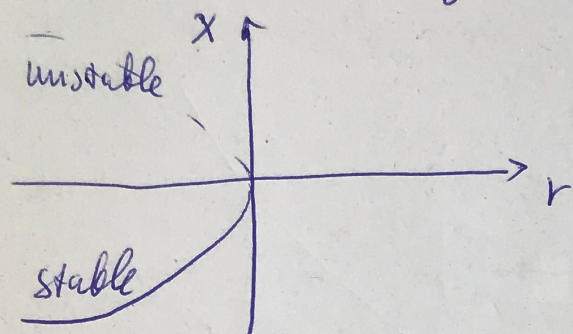
- basic mechanism by which fixed points are created and destroyed. As a parameter is varied, two fixed points move toward each other, collide, and mutually annihilate.

Normal form

$$\dot{x} = r + x^2$$

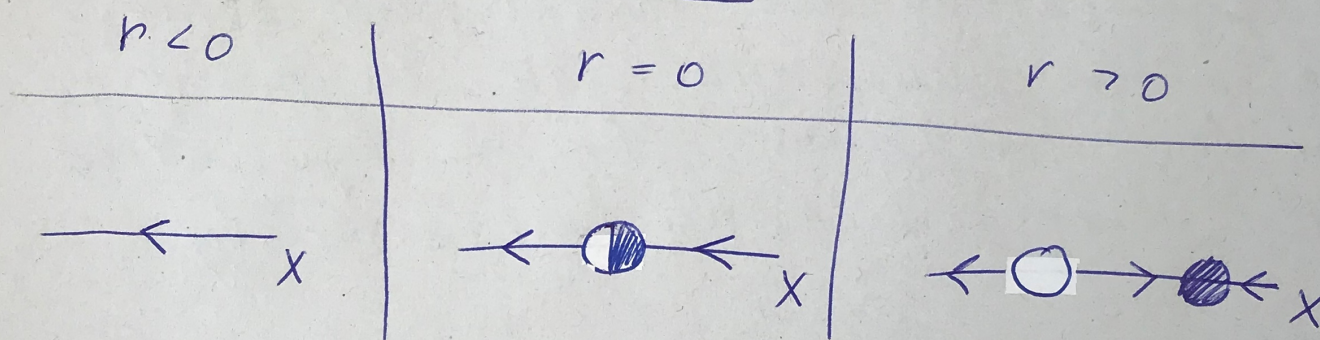


Bifurcation diagram



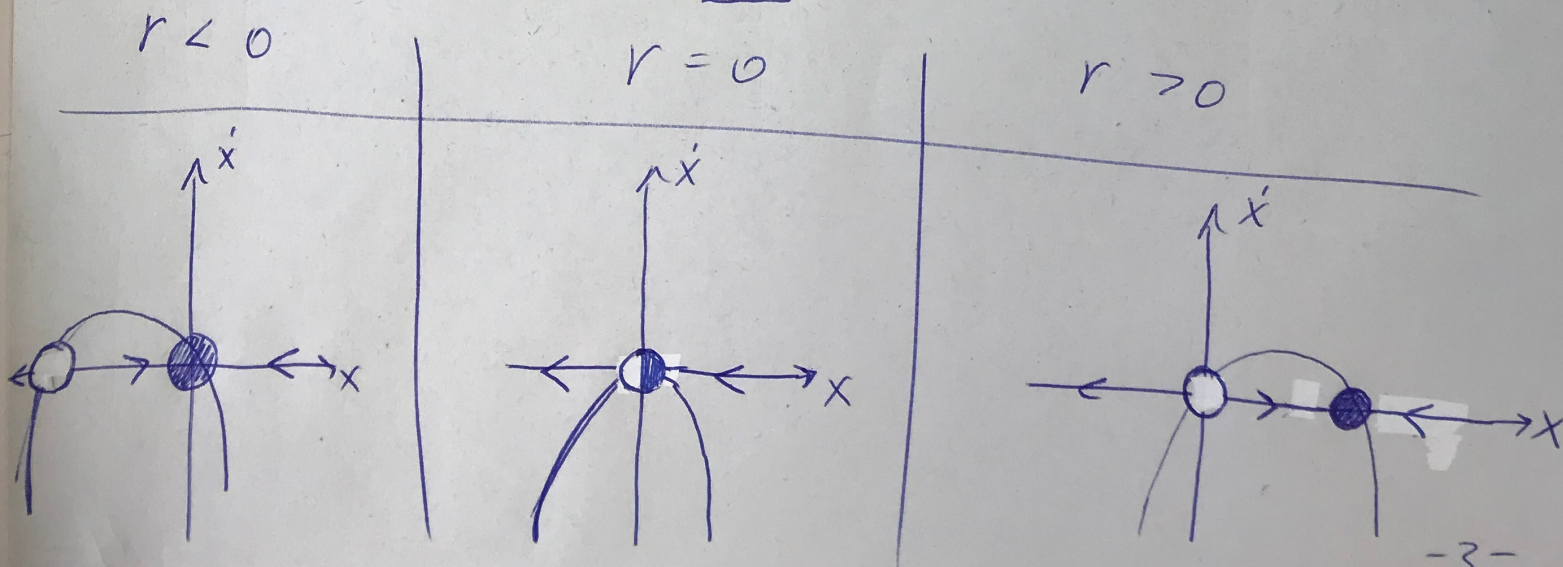
Blue sky bifurcation - saddle-node bifurcation in the other direction;
a pair of fixed points appears "out of the clear blue sky" as a parameter is varied.

Normal form $\boxed{\dot{x} = r - x^2}$

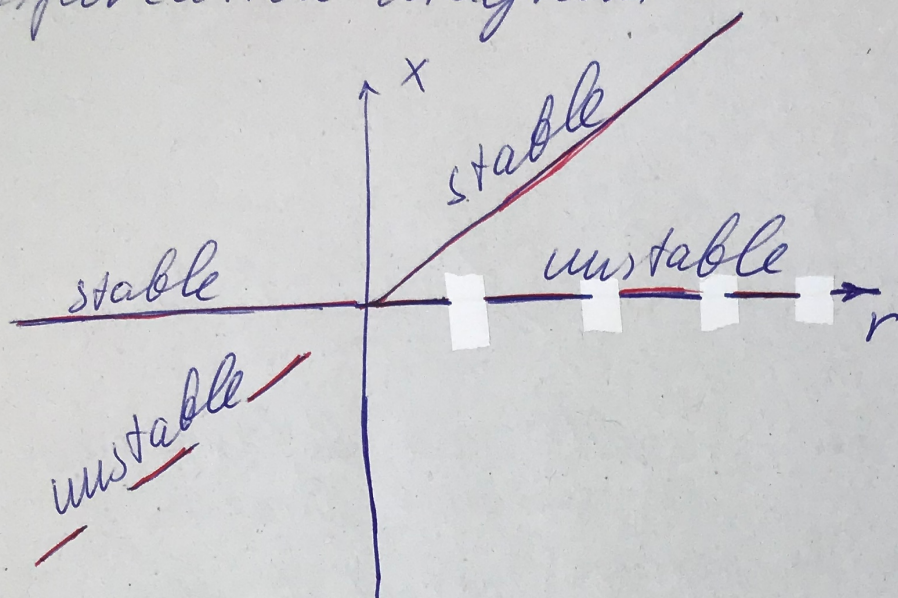


2) Transcritical bifurcation - standard mechanism for changes in stability

Normal form $\boxed{\dot{x} = rx - x^2}$



bifurcation diagram



The important difference between the saddle-node bifurcation and transcritical bifurcation: in the transcritical case, the two fixed points don't disappear after the bifurcation - instead they just switch their stability.