7. May 2019
Lecture 5

# What is synchronization? Classical example of a pendulum clock.

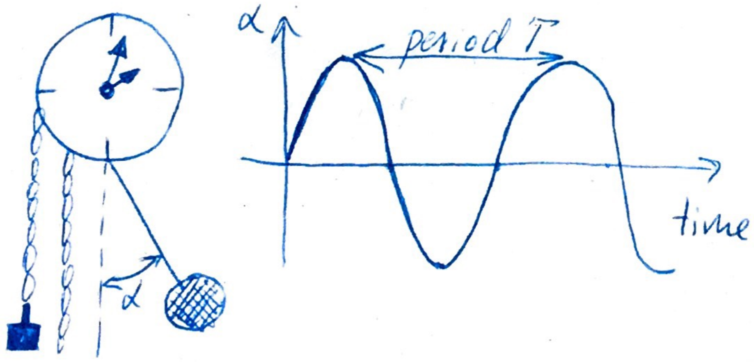
Sync is an adjustment of rhythms of oscillating objects due to their weak interaction.

* What is an oscillating object?
* What do we understand by the notion "rhythm"?
* What is an interaction of oscillating systems?
* What is an adjustment of rhythms?

We consider a classical example – a pendulum clock

## Self-sustained oscillator

The mechanism of the clock → transforms the potential energy of the lifted weight (or compressed spring, or electrical battery) into the oscillatory motion of the pendulum. In its turn, this oscillation is transferred into the rotation of the hands on the clock's face.




1) It takes energy from the source and maintains a steady oscillation of the pendulum, which continues without any change until the supply of energy expires.

2) The exact form of the oscillatory motion is entirely determined by the internal parameters of the clock and does not depend on how the pendulum was put into motion. After being slightly perturbed the pendulum restores its previous internal rhythm.

These two features are typical not only for clocks but also for many oscillating objects of diverse nature. In physics such oscillatory objects are called self-sustained oscillators.

might be

Properties of self-sustained oscillators:

1) contain an internal source of energy that is transformed into oscil-ry motion. Being isolated, the oscillator continues to generate the same rhythm, until the source of energy expires; mathematically an oscillator is described by an autonomous (without explicit time dependence) dynamical system.

2) the form of oscillation is determined by the parameters of the system and does not depend on how the system was "switched on" (initial conditions).

3) the oscillation is stable to (at least rather small) perturbations: being disturbed, the oscillation soon returns to its original shape

Examples: lasers, pacemakers of human hearts, electronic circuits.

$\Rightarrow$ they can be synchronized.

Period and frequency

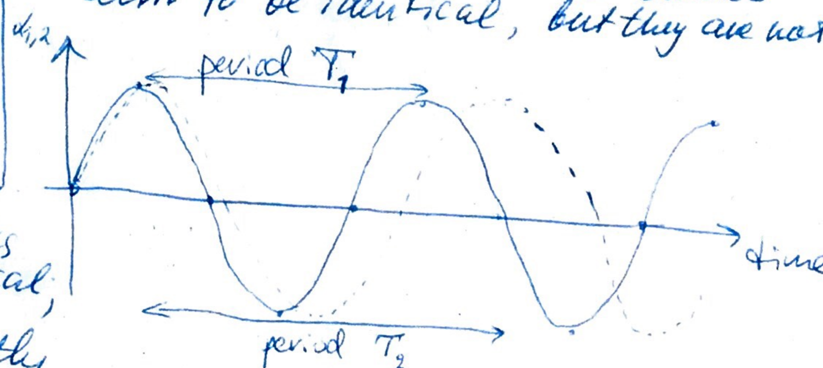
$$f = \frac{1}{T};$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Natural frequency is the frequency of the autonomous (isolated) system.

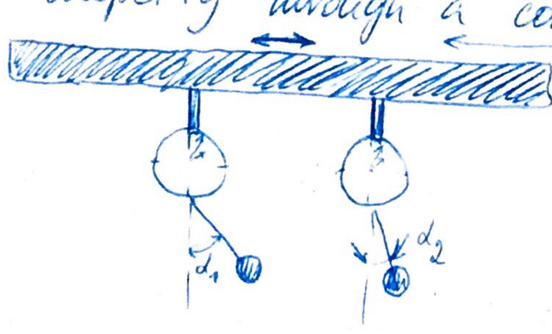
Coupling of oscillating objects

Pendulum clock: the clocks seem to be identical, but they are not.




Two similar pendulum clocks cannot be perfectly identical; due to a tiny parameter mismatch they have slightly different periods ($T_2 > T_1$).

Let us now consider two weakly interacting clocks: coupling through a common support. The beam on which the clocks are fixed is not rigid, but can vibrate slightly as indicated by the arrows at the top. This vibration is caused by the motions of both pendula $\Rightarrow$ they interact. The vibration



might be practically imperceptible. However in spite of its weakness, it may alter the rhythms of both clocks!

## Adjustment of rhythms

Experiments show that even a weak interaction can synchronize two clocks → they start to oscillate with a common period → frequency locking.

What are the factors which define whether they sync or not?

### 1) Coupling strength

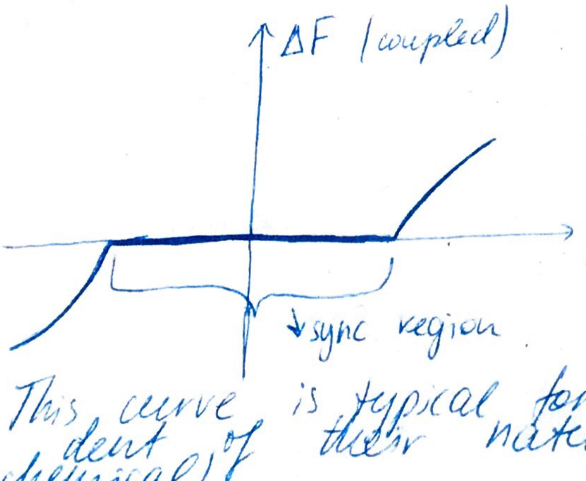
$\boxed{\text{Experimentally it is not always clear how to measure this quantity}}$ For the example of pendulum clocks it depends on the ability of the supporting beam to move: if the beam is absolutely rigid, then the motions of pendula do not influence the support, and therefore, there is no way for one clock to act on the other ⇒ if the clocks do not interact the coupling strength is zero. If the beam is not rigid, but can vibrate longitudinally or bend, then an interaction takes place.

### 2) Frequency detuning

Frequency detuning or mismatch $\Delta f = f_1 - f_2$ quantifies how different the uncoupled oscillators are. It can be easily measured in experiment.

$f_1, f_2$ → characterize free (uncoupled) oscillators

$F_1, F_2$ → obtained in the presence of coupling.




For a certain range of detuning the frequencies of coupled oscillators are identical:

$\Delta F = 0$, indicating sync.

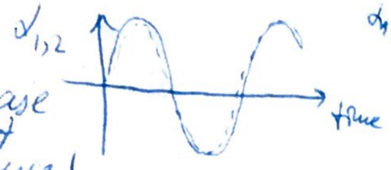
$\Delta f$ (uncoupled)

↓ detuning = frequency mismatch

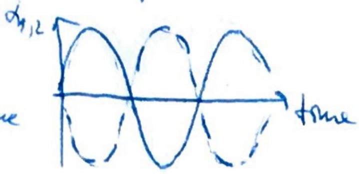
This curve is typical for interacting oscillators, independent of their nature (mechanical, electronic, chemical)!

## Synchronization

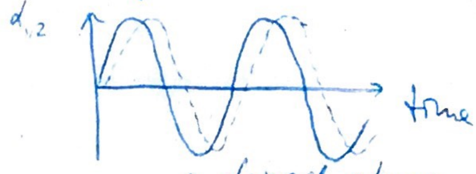
in-phase · anti-phase · phase-lag




no phase shift (difference)




phase shift $= \pi$

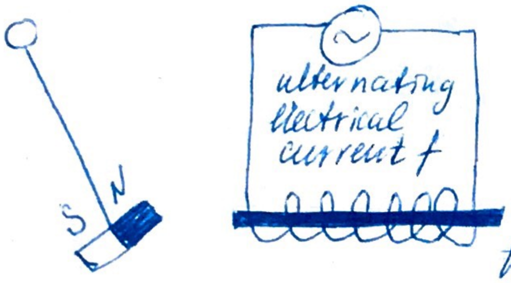



a fixed phase shift

## What is not sync?

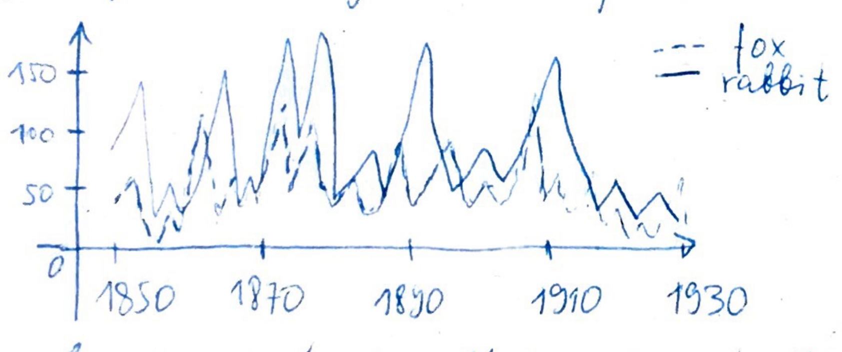
Counter-examples.

1) Resonance is not sync!



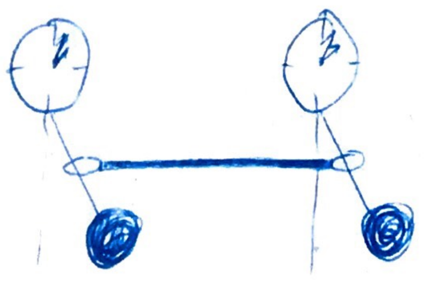

The magnetic pendulum oscillates in the electromagnetic field with the frequency of the electric current. This is an example of a forced system that has no rhythm on its own: if the pendulum is shielded from the electromagnet then its oscillation decays.

2) Population dynamics: foxes and rabbits




This ecological system can not be decomposed into two oscillators. If foxes and rabbits were separated, there would be no oscillations at all. We can not consider them as two subsystems having their own rhythms => we can not speak about sync.

The case of too strong coupling => not sync!



To call a phenomenon sync, we must be sure that:

* we analyze the behavior of two self-sustained oscillators, i.e. systems capable of generating their own rhythms (the form, period and freq of oscillations depend only on internal parameters of the system).

* the systems adjust their rhythms due to <u>weak</u> interaction.

* the adjustment of rhythms occurs in a certain range of system's mismatch, in particular, if the frequency of one oscillator is slowly varied, the second system follows this variation.

To be asked on 14 May

Questions to the lecturer:

1) What is a self-sustained oscillator? Give examples.
2) What are the properties of self-sustained oscillators?
3) What are the factors which define whether the systems can sync or not? Use the example of a pendulum clock to explain the factors.
4) Which types of sync do you know?

5) 5) What are the counter-examples? What is not sync?
6) To call a phenomenon sync we must be sure that certain conditions are fulfilled. What are these conditions?