Vollständigkeit der Basis

1) $\mathcal{H}_1 : \psi_\nu(\underline{x})$

2) $\mathcal{H}^{(N)} : \psi_{\nu_1 \nu_2 \cdots \nu_N}(1, 2, \ldots N)$ $\qquad (\underline{x}_\nu \longleftrightarrow \nu)$

3) $\mathcal{H}^{sym}, \mathcal{H}^{antisym} \subset \mathcal{H}^{(N)}$, $\mathcal{H}^{s,a} : |n_1 n_2 \cdots\rangle$

<u>Orthogonalität</u>: wenn ein $n_\nu \neq n_\nu'$

$$\langle \psi_{\nu_1}(1) \psi_{\nu_2}(2) .. | \psi_{\nu_1'}(1) \ldots \rangle = \langle \psi_{\nu_1}(1) | \psi_{\nu_1'}(1) \rangle \langle \psi_{\nu_2}(2) | \psi_{\nu_2'}(2) \rangle \cdot \ldots\ldots$$

<u>Normierung</u>: Bsp. Fermionen $N=2$

$$|n_1 n_2 \ldots\rangle = |1 1 0 \ldots\rangle = \frac{1}{\sqrt{2}} \left[\psi_1(1) \psi_2(2) - \psi_2(1) \psi_1(2) \right]$$

$$A(i) \psi_{\nu_i}(i) = \sum_{\lambda=1}^{\infty} \psi_\lambda(i) A_{\lambda \nu_i} \quad ; A_{\lambda \nu_i} = \langle \psi_\lambda | A | \psi_{\nu_i} \rangle$$

$$\sum_{i=1}^{N} A(i) |n_1 n_2 \ldots\rangle = \sum_{i=1}^{N} \frac{1}{\sqrt{N! \prod_s n_s!}} \sum_{P \in S} (\pm 1)^P T_P \left\{ \psi_{\nu_1}(1) \ldots \underline{A(i) \psi_{\nu_i}(i)} \ldots \psi_{\nu_N}(N) \right\}$$

$$= \sum_{\lambda=1}^{\infty} \sum_{i=1}^{N} \underbrace{\frac{1}{\sqrt{N! \prod_s n_s!}} \sum_{P \in S} (\pm 1)^P T_P \left\{ \psi_{\nu_1}(1) \ldots A_{\lambda \nu_i} \psi_\lambda(i) \ldots \psi_{\nu_N}(N) \right\}}_{\chi_{\lambda \nu_i}}$$

wenn $\lambda = \nu_i$, $\chi_{\lambda \nu_i} = A_{\lambda \lambda} |n_1 n_2 \ldots\rangle$

sei $\lambda \neq \nu_i$: $\chi_{\lambda \nu_i} = \sqrt{\frac{n_\lambda+1}{n_{\nu_i}}} A_{\lambda \nu_i} | n_1 \dots n_{\nu_i}-1 \dots n_\lambda+1 \dots \rangle$

$$\sum_{i=1}^{N} A(i) | n_1 n_2 \dots \rangle = \sum_{\lambda=1}^{\infty} n_\lambda A_{\lambda\lambda} | n_1 n_2 \dots \rangle +$$

$$+ \sum_{\lambda, \mu}^{1 \dots \infty} \sqrt{n_\mu (n_\lambda+1)} A_{\lambda \mu} | n_1 \dots n_\mu-1 \dots n_\lambda+1 \dots \rangle$$

$$\boxed{\nu_\mu \longleftrightarrow \mu}$$

$$\sum_{i=1}^{N} \longrightarrow \sum_{\mu=1}^{\infty} n_\mu$$

$a_\lambda = (a_\lambda^+)^+$: $\langle n_1 \dots n_\lambda-1 \dots | a_\lambda | n_1 \dots n_\lambda \dots \rangle = \sqrt{n_\lambda} \langle n_1 \dots | n_1 \dots \rangle = \sqrt{n_\lambda}$

$$= \langle n_1 \dots n_\lambda | a_\lambda^+ | n_1 \dots n_\lambda - 1 \rangle = \sqrt{n_\lambda}$$

$$= \langle a_\lambda^+ | n_1 \dots n_\lambda-1 \dots | n_1 n_2 \dots n_\lambda \rangle$$

$$a_\lambda a_\lambda^+ | n_1 n_2 \dots \rangle = \sqrt{n_\lambda+1} \sqrt{n_\lambda+1} | n_1 \dots \rangle = (n_\lambda+1) | n_1 n_2 \dots \rangle$$

$$[a_\lambda, a_\lambda^+] = 1$$