Vollständigkeit der Basis

1) $\mathcal{H}_1 : \psi_\nu(\underline{x})$

2) $\mathcal{H}^{(N)} : \psi_{\nu_1\nu_2\cdots\nu_N}(1,2,\dots N)$ $\quad (\underline{x}_\nu \longleftrightarrow \nu)$

3) $\mathcal{H}^{sym}, \mathcal{H}^{antisym} \subset \mathcal{H}^{(N)}, \quad \mathcal{H}^{s,a} : |n_1 n_2 \dots\rangle$

Orthogonalität: wenn ein $n_\nu \neq n'_\nu$

$$\langle \psi_{\nu_1}(1)\psi_{\nu_2}(2)\dots | \psi_{\nu_1'}(1)\dots\rangle = \langle\psi_{\nu_1}(1)|\psi_{\nu_1'}(1)\rangle\langle\psi_{\nu_2}(2)|\psi_{\nu_2'}(2)\rangle\cdot\dots$$

Normierung: Bsp. Fermionen $N=2$

$$|n_1 n_2 \dots\rangle = |1\,1\,0\dots\rangle = \frac{1}{\sqrt{2}}\left[\psi_1(1)\psi_2(2) - \psi_2(1)\psi_1(2)\right]$$

$$A(i)\,\psi_{\nu_i}(i) = \sum_{\lambda=1}^{\infty}\psi_\lambda(i)\,A_{\lambda\nu_i} \quad ; A_{\lambda\nu_i} = \langle\psi_\lambda|A|\psi_{\nu_i}\rangle$$

$$\sum_{i=1}^{N} A(i)|n_1 n_2 \dots\rangle = \sum_{i=1}^{N}\frac{1}{\sqrt{N!\prod_j n_j!}}\sum_{P\in S}(\pm1)^P T_P\{\psi_{\nu_1}(1)\dots\underline{A(i)\psi_{\nu_i}(i)}\dots\psi_{\nu_N}(N)\}$$

$$= \sum_{\lambda=1}^{\infty}\sum_{i=1}^{N}\underbrace{\frac{1}{\sqrt{N!\prod_j n_j!}}\sum_{P\in S}(\pm1)^P T_P\{\psi_{\nu_1}(1)\dots A_{\lambda\nu_i}\psi_\lambda(i)\dots\psi_{\nu_N}(N)\}}_{X_{\lambda\nu_i}}$$

wenn $\lambda = \nu_i$, $\quad X_{\lambda\nu_i} = A_{\lambda\lambda}|n_1 n_2 \dots\rangle$

sei $\lambda \neq \nu_i$: $X_{\lambda\nu_i} = \sqrt{\frac{n_\lambda+1}{n_{\nu_i}}} A_{\lambda\nu_i} |n_1 \dots n_{\nu_i}-1 \dots n_\lambda+1 \dots\rangle$

$$\sum_{i=1}^{N} A(i) |n_1 n_2 \dots\rangle = \sum_{\lambda=1}^{\infty} n_\lambda A_{\lambda\lambda} |n_1 n_2 \dots\rangle +$$

$$+ \sum_{\lambda,\mu}^{1\dots\infty} \sqrt{n_\mu (n_\lambda+1)} A_{\lambda\mu} |n_1 \dots n_\mu-1 \dots n_\lambda+1 \dots\rangle$$

$$\boxed{\nu_i \longleftrightarrow n}$$

$$\sum_{i=1}^{N} \longrightarrow \sum_{\mu=1}^{\infty} n_\mu$$

$a_\lambda = (a_\lambda^+)^+$: $\langle n_1 \dots n_\lambda-1 \dots | a_\lambda | n_1 \dots n_\lambda \dots \rangle = \sqrt{n_\lambda} \langle n_1 \dots | n_1 \dots \rangle = \sqrt{n_\lambda}$

$$= \langle n_1 \dots n_\lambda | a_\lambda^+ | n_1 \dots n_\lambda - 1 \rangle = \sqrt{n_\lambda}$$

$$= \langle a_\lambda^+ |n_1 \dots n_\lambda-1 \dots | n_1 n_2 \dots n_\lambda \rangle$$

$$a_\lambda a_\lambda^+ |n_1 n_2 \dots\rangle = \sqrt{n_\lambda+1} \sqrt{n_\lambda+1} |n_1 \dots\rangle = (n_\lambda+1) |n_1 n_2 \dots\rangle$$

$$[a_\lambda, a_\lambda^+] = 1$$