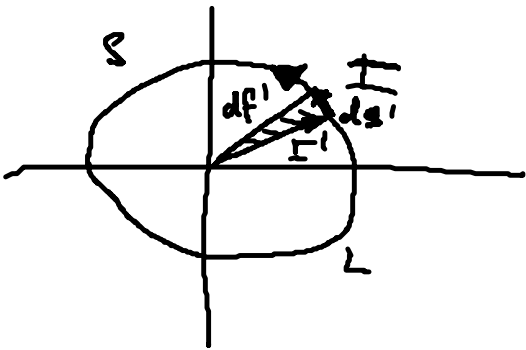


Beispiel für einen magnetischen Dipol:



$$d\mathbf{f}' = \frac{1}{2} \mathbf{r}' \times d\mathbf{s}'$$

$$d^3\mathbf{r}' \mathbf{j}(\mathbf{r}') = d\mathbf{s}' I$$



$$\mathbf{m} = \frac{1}{2} \oint_L d^3\mathbf{r}' \mathbf{r}' \times \mathbf{j}(\mathbf{r}') = \frac{I}{2} \oint_L \mathbf{r}' \times d\mathbf{s}' = I \int_F d\mathbf{f}' = I F \mathbf{n}$$

Ringstrom \rightarrow magn. Dipolmoment $\mathbf{m} = I F \mathbf{n}$
 (analog: 2 Punktlad. \rightarrow el. Dipolmoment $\mathbf{p} = q \mathbf{a}$)
 $q \oplus \xleftarrow{\mathbf{a}} \ominus -q$

2. Beispiel: Bewegte Ladungen

N Teilchen mit Massen m_i u. Ladungen q_i ,
 spezif. Ladung $\frac{q_i}{m_i} = \frac{q}{m} = \text{const.}$

$$\rho(\mathbf{r}) = \sum_i q_i \delta(\mathbf{r} - \mathbf{r}_i) \quad \mathbf{r}_i = \mathbf{r}_i(t)$$

$$\mathbf{j}(\mathbf{r}) = \sum_i q_i \mathbf{v}_i \delta(\mathbf{r} - \mathbf{r}_i) \quad \mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$$

magn. Dipolmoment:

$$\mathbf{m} = \frac{1}{2} \int d^3\mathbf{r}' \mathbf{r}' \times \mathbf{j}(\mathbf{r}') = \frac{1}{2} \sum_i q_i \int d^3\mathbf{r}' \mathbf{r}' \times \mathbf{v}_i \delta(\mathbf{r}' - \mathbf{r}_i)$$

$$= \frac{1}{2} \sum_i q_i \underline{r}_i \times \underline{v}_i = \frac{1}{2} \sum_i \frac{q_i}{m_i} m_i \underline{r}_i \times \underline{v}_i = \frac{q}{2m} \underline{L}$$

$$\boxed{\underline{m} = \frac{q}{2m} \underline{L}} \quad \text{gilt auch für starr Körper!}$$

(Spin eines Elektrons: \underline{S} , $\underline{m} = g \frac{e}{2m} \underline{S}$ mit $g \approx 2$)
quantenmech. Effekt

$$\underline{A} = \frac{\mu_0}{4\pi r^3} \underline{m} \times \underline{r}$$

Kraft auf Stromverteilung $\underline{j}(\underline{r}') = \rho(\underline{r}') \underline{v}(\underline{r}')$
im Feld einer magn. Induktion $\underline{B}(\underline{r}')$ (extern):

$$\underline{F} = \int d^3 r' \underline{j}(\underline{r}') \times \underline{B}(\underline{r}') \quad \text{Lorentz-Kraft}$$

Taylorentw.:

$$\underline{B}(\underline{r}') = \underline{B}(\underline{r}) + [(\underline{r}' - \underline{r}) \cdot \nabla_r] \underline{B}(\underline{r}) + \dots$$

$$\underline{F} = \underbrace{\left[\int d^3 r' \underline{j}(\underline{r}') \right]}_0 \times \underline{B}(\underline{r}) + \int d^3 r' \underline{j}(\underline{r}') \times [(\underline{r}' - \underline{r}) \cdot \nabla_r] \underline{B}(\underline{r}) + \dots$$

$$= \int d^3 r' \underline{j}(\underline{r}') \times \underbrace{[(\underline{r}' \cdot \nabla_r) \underline{B}(\underline{r})]}_0 \text{ (stat.)} - \int d^3 r' \underline{j}(\underline{r}') \times \underbrace{[(\underline{r} \cdot \nabla_r) \underline{B}(\underline{r})]}_0 \text{ (stat.)}$$

$$= \int d^3 r' \underline{j}(\underline{r}') \times \nabla_r (\underline{r}' \cdot \underline{B}(\underline{r})) - \underline{r} \times \underbrace{(\nabla_r \times \underline{B}(\underline{r}))}_0 \text{ externes Feld!}$$

$$= \int d^3 r' \underline{j}(\underline{r}') \times \nabla_r (\underline{r}' \cdot \underline{B}(\underline{r})) - \nabla_r \times [(\underline{r}' \cdot \underline{B}(\underline{r})) \underline{j}(\underline{r}')]]$$

$$= -\nabla_r \times (\underline{m} \times \underline{B}(\underline{r}))$$

$$= (\underline{m} \cdot \nabla_r) \underline{B}(\underline{r})$$

$$\underline{m} = \frac{1}{2} \int d^3 r' \underline{r}' \times \underline{j}$$

$$\nabla \times (\underline{m} \times \underline{B}) = \underline{b} (a \cdot \underline{c}) - \underline{c} (a \cdot \underline{b})$$

$\Rightarrow V = -\underline{m} \cdot \underline{B}(\underline{r})$ pot. Energie eines magn. Dipols
im Magnetfeld

3. Maxwell-Gleichungen

Ziel: Dynamik der Felder

Methode: Erweiterung der elektostat. u. magnetostat. Feldgl., so dass allg. Invarianz-Prinzipien erfüllt sind.

3.1 TCP-Invarianz

Zeitumkehr $T: t \rightarrow t' = -t$

Ladungsumkehr $C: Q \rightarrow Q' = -Q$

Paritätsumkehr $P: \underline{r} \rightarrow \underline{r}' = -\underline{r}$

(i) Zeitumkehr-Trafo

$$\begin{aligned} T_g &:= \{ T\text{-invar. Obs. } A : TA = A \} \text{ gerade unter } T \\ &= \{ \underline{r}, d\underline{r}, \underline{a} = \frac{d^2 \underline{r}}{dt^2}, m, q, \rho, \underline{F} = m\underline{a}, \underline{E} = \frac{\underline{F}}{q}, \\ &\quad \phi, \dots \} \end{aligned}$$

$$\begin{aligned} T_u &:= \{ A : TA = -A \} \text{ ungerade unter } T \\ &= \{ \underline{v} = \frac{d\underline{r}}{dt}, \underline{j} = q\underline{v}, \underline{B}, \underline{A}, \dots \} \\ &\quad \begin{array}{ccc} \underline{F} = q\underline{v} \times \underline{B} & \underline{B} = \nabla \times \underline{A} \\ q & q & u & u & u & q & u \end{array} \end{aligned}$$

\Rightarrow T -Invarianz der stat. Grundgl.:

$$T: \{ \nabla_{\underline{r}} \times \underline{E} = 0 \} \rightarrow \{ \nabla \times \underline{E} = 0 \}$$

$$T: \{ \epsilon_0 \nabla_{\underline{r}} \cdot \underline{E} = \rho \} \rightarrow \{ \epsilon_0 \nabla_{\underline{r}} \cdot \underline{E} = \rho \}$$

$$T: \{ \nabla_{\underline{r}} \cdot \underline{B} = 0 \} \rightarrow \{ -\nabla \cdot \underline{B} = 0 \}$$

$$T: \{ \underline{\nabla} \times \underline{B} = \underline{\mu_0 j} \} \rightarrow \{ -\underline{\nabla} \times \underline{B} = -\underline{\mu_0 j} \}$$

Kontinuitätsgl.:

$$T: \{ \frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{j} = 0 \} \rightarrow \{ -\frac{\partial \rho}{\partial t} - \underline{\nabla} \cdot \underline{j} = 0 \}$$

Diese Gln. sind forminvariant

(ii) Ladungsumkehr (-konjugation)

$$C_g = \{ \underline{E}, \underline{u}, \underline{v}, \underline{a}, \dots \} \text{ gerade}$$

$$C_u = \{ \underline{E} = \frac{1}{q} \underline{E}, \underline{B}, \rho, \underline{j}, \dots \} \text{ ungerade}$$

$$\begin{array}{ccccc} \underline{E} & = & q & \underline{v} & \times & \underline{B} \\ q & & u & q & & u \end{array}$$

\Rightarrow C-Invarianz der stat. Gln.:

$$C: \{ \underline{\nabla} \times \underline{E} = 0 \} \rightarrow \{ -\underline{\nabla} \times \underline{E} = 0 \}$$

$$C: \{ \epsilon_0 \underline{\nabla} \cdot \underline{E} = \rho \} \rightarrow \{ -\epsilon_0 \underline{\nabla} \cdot \underline{E} = -\rho \}$$

$$C: \{ \underline{\nabla} \cdot \underline{B} = 0 \} \rightarrow \{ -\underline{\nabla} \cdot \underline{B} = 0 \}$$

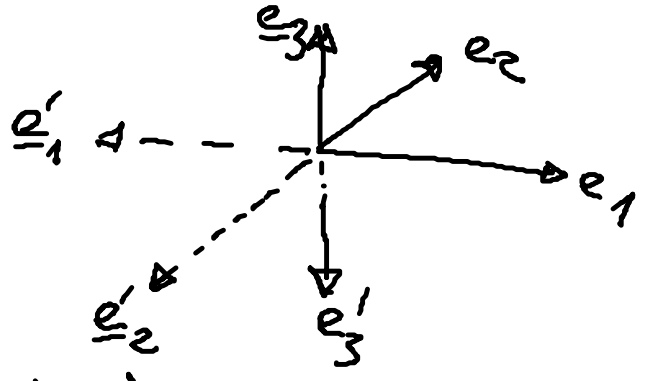
$$C: \{ \underline{\nabla} \times \underline{B} = \underline{\mu_0 j} \} \rightarrow \{ -\underline{\nabla} \times \underline{B} = -\underline{\mu_0 j} \}$$

$$C: \{ \frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{j} = 0 \} \rightarrow \{ -\frac{\partial \rho}{\partial t} - \underline{\nabla} \cdot \underline{j} = 0 \}$$

(iii) Paritätsumkehr (Räuml. Spiegelung)

Vertauschung „rechts \leftrightarrow links“

Rechtssystem

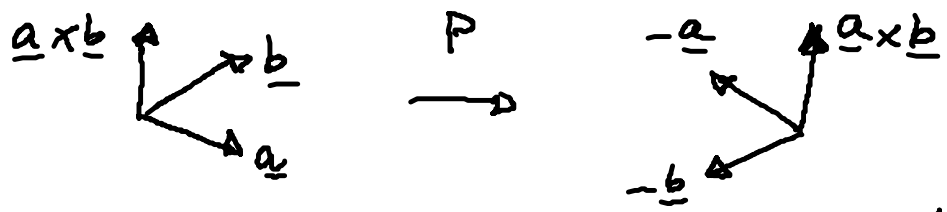


Links-system

$P \underline{r} = - \underline{r}$ polares Vektor

aber

$P(\underline{a} \times \underline{b}) = (-\underline{a}) \times (-\underline{b}) = \underline{a} \times \underline{b}$



P-invariant (axialer Vektor, Pseudovektor)

- Seien $\underline{a}, \underline{b}$ polar $\Rightarrow \underline{a} \times \underline{w}$ polar
- $\underline{w}, \underline{v}$ axial $\underline{a} \times \underline{b}, \underline{w} \times \underline{v}$ axial
- $\underline{a} \cdot \underline{b}$ skalar ($P(\underline{a} \cdot \underline{b}) = \underline{a} \cdot \underline{b}$)
- $\underline{a} \cdot \underline{w}$ pseudoskalar

$P_q = \{ \text{Skalare } m, q; \text{ axiale Vektoren } \underline{B} \}$ gerade

$\underline{F} = q \underline{v} \times \underline{B}$

($P(\underline{a} \cdot \underline{w}) = - \underline{a} \cdot \underline{w}$)

$$P_u = \left\{ \text{polare Vektoren } \underline{r}, \underline{d}_u, \underline{v}, \underline{a}, \underline{E}, \underline{E} = \frac{1}{q} \underline{F}, \underline{j} = \rho \underline{v}, \underline{A}, \right. \\ \left. \text{Pseudoskalar } \underline{\nabla} \cdot \underline{B} \right\} \quad \underline{B} = \underline{\nabla} \times \underline{A}$$

$$\begin{matrix} \underline{B} \\ q \\ u \\ u \end{matrix}$$

P-Invarianz der El./Magn. statik

$$P: \{ \underline{\nabla} \times \underline{E} = 0 \} \rightarrow \{ \underline{\nabla} \times \underline{E} = 0 \}$$

$$P: \{ \epsilon_0 \underline{\nabla} \cdot \underline{E} = \rho \} \rightarrow \{ \epsilon_0 \underline{\nabla} \cdot \underline{E} = \rho \}$$

$$P: \{ \underline{\nabla} \cdot \underline{B} = 0 \} \rightarrow \{ -\underline{\nabla} \cdot \underline{B} = 0 \}$$

$$P: \{ \underline{\nabla} \times \underline{B} = \mu_0 \underline{j} \} \rightarrow \{ -\underline{\nabla} \times \underline{B} = -\mu_0 \underline{j} \}$$

$$P: \left\{ \frac{\partial}{\partial t} \rho + \underline{\nabla} \cdot \underline{j} \right\} \rightarrow \left\{ \frac{\partial}{\partial t} \rho + \underline{\nabla} \cdot \underline{j} = 0 \right\}$$

NB: Gäbe es magn. Ladungen, dann wären sie Pseudoskalar!

NB: Schwache WW verletzt Paritätsabh.

3.2 Maxwell-gln. im Vakuum

Forderungen an dynam. gln. für zeitabh. Felder $\underline{E}(\underline{r}, t)$ u. $\underline{B}(\underline{r}, t)$:

$$(1) \text{ Stat. Grenzfall } \quad \begin{aligned} \underline{\nabla} \times \underline{E} &= 0 \\ \epsilon_0 \underline{\nabla} \cdot \underline{E} &= \rho \\ \underline{\nabla} \cdot \underline{B} &= 0 \end{aligned}$$

$$\nabla \times \underline{B} = \mu_0 \underline{j}$$

- (2) linear in $\underline{E}, \underline{B}$ (Superpositionsprinzip)
1. Ordnung in t (Kausalitätsprinzip :
zur Zeit $t=0$ soll
Zustand für $t > 0$ vollst.
festgelegt)
- (3) TCP - Invarianz
- (4) Ladungserhaltung
- (5) Lorentzhaft