Wellengl. für $\rho(\underline{r},t) = \rho(\underline{r})\, e^{-i\omega t}$

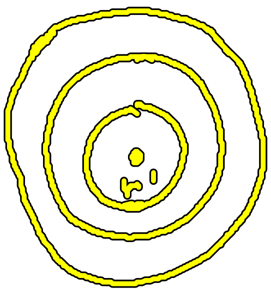
$$\boxed{\begin{aligned} (\Delta + k^2)\, \tilde{G}(\underline{r}-\underline{r}') &= -\delta(\underline{r}-\underline{r}') \\ \phi(\underline{r}) &= \int d^3r'\, \tilde{G}(\underline{r}-\underline{r}')\, \rho(\underline{r}')/\epsilon_0 \end{aligned}}$$

(a) <u>Greenfkt. des unendl. Raumes:</u>

Randbed. $\tilde{G}(\underline{r}-\underline{r}') \to 0$ für $r \to \infty$

$\Rightarrow$ retard. Potenzial

$$G(\underline{r}-\underline{r}', \underbrace{t-t'}_{\tau}) = \begin{cases} \frac{1}{4\pi|\underline{r}-\underline{r}'|}\, \delta\left(\tau - \frac{|\underline{r}-\underline{r}'|}{c}\right) & \tau > 0 \\ 0 & \tau < 0 \end{cases}$$



$$\boxed{\tilde{G}(\underline{r}-\underline{r}') := \int_0^\infty d\tau\, G(\underline{r}-\underline{r}',\tau)\, e^{i\omega\tau} = \frac{e^{ik|\underline{r}-\underline{r}'|}}{4\pi|\underline{r}-\underline{r}'|}}$$

$$k := \frac{\omega}{c}$$

$$\Rightarrow \phi(\underline{r},t) = \int_{\mathbb{R}^3} d^3r'\, \tilde{G}(\underline{r}-\underline{r}')\, e^{-i\omega t}\, \rho(\underline{r}')/\epsilon_0$$

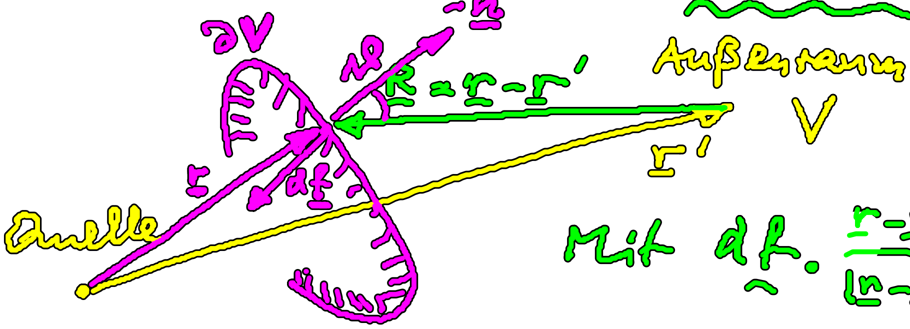
$$= \int_{\mathbb{R}^3} d^3r'\, \frac{e^{i(k|\underline{r}-\underline{r}'| - \omega t)}}{4\pi|\underline{r}-\underline{r}'|}\, \rho(\underline{r}')/\epsilon_0$$

beschreibt Überlagerung <u>auslaufender Kugelwellen</u>

(Ausstrahlbed., Konsequenz der Kausalität

Mit $\underline{R} := \underline{r} - \underline{r}'$ lauten die Kirchhoff-Identität

$$\phi(\underline{r}') = \frac{1}{4\pi} \oint_{\partial V} d\underline{f}_R \left\{ \frac{e^{ikR}}{R} \nabla_r \phi(\underline{r}) - \phi(\underline{r}) \underbrace{\nabla_r \frac{e^{ikR}}{R}}_{\frac{e^{ikR}}{R}\left(ik - \cancel{\frac{1}{R}}\right)\frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|}} \right\}$$



Mit $d\underline{f} \cdot \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} = df \cos\vartheta$

und Beschränkung auf Fernzone von ∂V

(d.h. $R \gg \frac{1}{k}$)

$$\boxed{\phi(\underline{r}') \approx \frac{1}{4\pi} \oint_{\partial V} df_R \underbrace{\left\{ \frac{\partial}{\partial n} \phi(\underline{r}) - ik\phi(\underline{r}) \cos\vartheta \right\}}_{\text{richtungsabh. Amplitude}} \underbrace{\frac{e^{ikR}}{R}}_{\text{Kugelwelle}}}$$

„Sekundärwelle"

Exakte Formulierung des Huygens'schen Prinzips

(Jeder Pkt. der Oberfläche eines Hindernisses ist Ausgangspkt. einer Kugelwelle, deren phasengerechte Überlagerung ergibt das Wellenfeld in $\underline{r}'$.)

(b) Greenfkt. zu Randbed. $\tilde{G}(\underline{r}-\underline{r}')\Big|_{\substack{\underline{r}\in\partial V\\ \underline{r}'\in V}} = 0$

$$\Longrightarrow \boxed{\phi(\underline{r}') = -\int_{\partial V} d\underline{f}\, \phi(\underline{r})\, \underline{\nabla}_r \tilde{G}(\underline{r}-\underline{r}')}$$

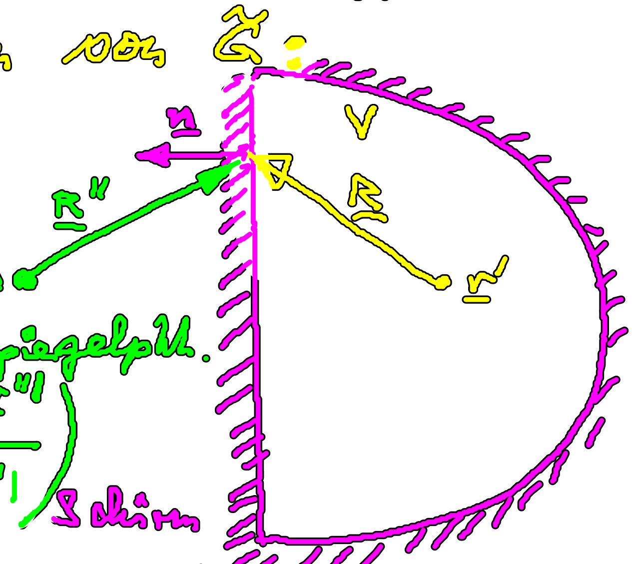
neue Greenfkt. unterscheidet sich von der alten nur um additive Lösung g der homog. Wellengl.

$$\tilde{G}(\underline{R}) = g(\underline{R}) + \frac{1}{4\pi}\frac{e^{ikR}}{R}$$

$$(\Delta + k^2)\, g = 0 \quad \text{mit Randbed. } g\Big|_{\partial V} = -\frac{1}{4\pi}\frac{e^{ikR}}{R}\Big|_{\partial V}$$

Beispiel für Konstruktion von $\tilde{G}$:

Ebener Schirm



Spiegelladungsmethode:

$$\tilde{G}(\underline{r}-\underline{r}') = \frac{1}{4\pi}\left(\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} - \frac{e^{ik|\underline{r}-\underline{r}''|}}{|\underline{r}-\underline{r}''|}\right)$$

$$\underline{\nabla}_r \tilde{G}(\underline{r}-\underline{r}') = \frac{1}{4\pi}\left[\underline{\nabla}_r \frac{e^{ikR}}{R} - \underline{\nabla}_r \frac{e^{ikR''}}{R''}\right]$$

$$= \frac{1}{4\pi}\left[\frac{e^{ikR}}{R}\left(ik-\frac{1}{R}\right)\frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} - \frac{e^{ikR''}}{R''}\left(ik-\frac{1}{R''}\right)\frac{\underline{r}-\underline{r}''}{|\underline{r}-\underline{r}''|}\right]$$

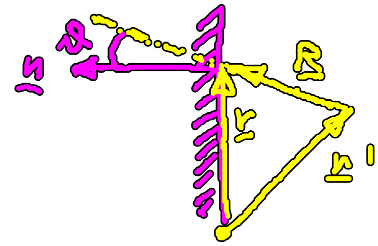
$$R = R'', \quad d\underline{f}\cdot\frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} = -d\underline{f}\,\frac{\underline{r}-\underline{r}''}{|\underline{r}-\underline{r}''|} = df\cos\alpha$$

$$\Rightarrow d\underline{f}\cdot\underline{\nabla}_r\tilde{G} = df\,\frac{1}{2\pi}\frac{e^{ikR}}{R}\left(ik-\cancel{\frac{1}{R}}\right)\cos\alpha$$

Für $\lambda \ll R$ (Fernzone):

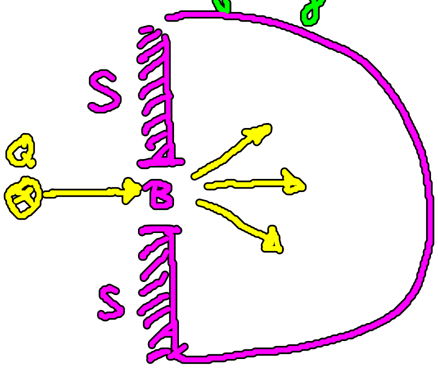
$$\boxed{\phi(\underline{r}') = -\int_{\partial V} \left[d\underline{f}\cdot\nabla_{r}\tilde{G}(\underline{r}-\underline{r}')\right]\phi(\underline{r}) \approx -\frac{i}{\lambda}\int_F df\;\phi(\underline{r})\,\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}\cos\vartheta}$$

Randwerte $\phi(\underline{r})\big|_F$ festlegen!



Kirchhoff'sche Näherung:

Beugung am Blenden B in einem ebenen Schirm S



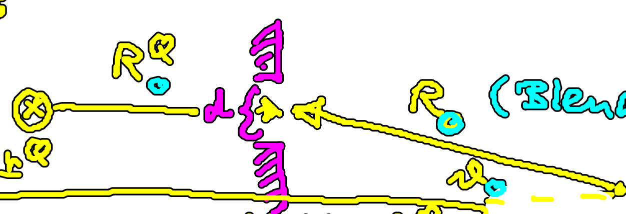
Annahme: $\phi(\underline{r})\big|_S = 0$ (Leiter)

$$\phi(\underline{r})\big|_B = \frac{e^{ikR^Q}}{R^Q} \quad \text{(freie einfall. Welle)}$$

$$\Rightarrow \phi(\underline{r}') = -\frac{i}{\lambda}\int_B df\,\frac{e^{ik(R+R^Q)}}{R\,R^Q}\cos\vartheta$$

$\underline{R} = \underline{r} - \underline{r}'$
$\underline{R}^Q = \underline{r} - \underline{r}^Q$
$df = d^2r$

Kleine Blende: (aber $\lambda \ll d$)

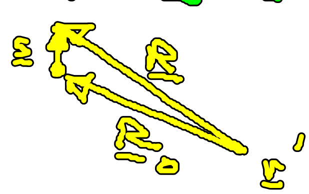


$$\boxed{\phi(\underline{r}') \approx -\frac{i}{\lambda}\frac{\cos\vartheta_0}{R_0 R_0^Q}\int_B df\; e^{ik(R+R^Q)}}$$

im schnell osz. Exp. darf nicht $R \approx R_0$, $R^Q \approx R_0^Q$ gesetzt werden!

Grenzfälle:

(i) Fraunhofer'sche Beugung (Fernzone. $\lambda \ll d \ll R$)



Setze $\underline{R} = \underline{R}_0 + \underline{s}$

$$R^2 \approx R_0^2 + 2\underline{R}_0\cdot\underline{s}$$

$$\Rightarrow R \approx R_0 + \underline{\alpha}\cdot\underline{s} \quad \text{mit } \underline{\alpha} := \frac{\underline{R}_0}{R_0}$$

analog $R^Q \approx R_0^Q + \underline{\alpha}_0 \cdot \underline{s}$ mit $\underline{\alpha}_0 := \frac{\underline{R}_0^Q}{R^Q}$

$$\Rightarrow \boxed{\phi(\underline{r}') \approx -\frac{i}{\lambda}\,\frac{e^{ik(R_0+R_0^Q)}}{R_0 R_0^Q}\cos\vartheta_0 \int_B d^2s\, e^{ik(\underline{\alpha}+\underline{\alpha}_0)\cdot\underline{s}}}$$

(ii) <u>Fresnel'sche Beugung</u> (<u>Mittelzone</u> $\lambda \ll R \approx d$)

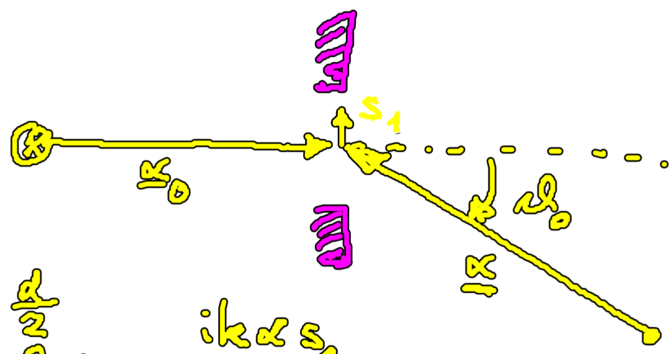
$$R^2 = R_0^2 + 2\underline{R}_0\underline{s} + s^2$$

<u>Beispiel</u>: Fraunhofer'sche Beugung am Spalt

(1-dim. Integr. ds_1)

Senkrechter Einfall

$\underline{\alpha}_0 \cdot \underline{s} = 0$



$$\phi(\underline{r}') = c\int_{-\frac{d}{2}}^{\frac{d}{2}} ds_1\, e^{ik\alpha s_1}$$

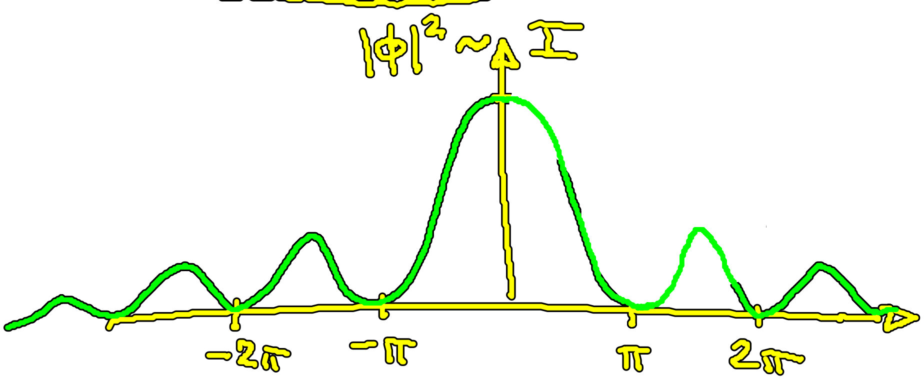
$\alpha := \sin\vartheta_0$ $\quad (\underline{\alpha}\cdot\underline{s} = s_1 \sin\vartheta_0)$

$$= \frac{c}{ik\alpha}\left(e^{ik\alpha\frac{d}{2}} - e^{-ik\alpha\frac{d}{2}}\right)$$

$$\boxed{\phi(\underline{r}') = Cd\,\frac{\sin\left(k\alpha\frac{d}{2}\right)}{k\alpha\frac{d}{2}}}$$

<u>Spaltfunktion</u>

(Fouriertransformierte der Rechteckfkt. $\hat{=}$ Blende)



$$k\alpha\frac{d}{2} = k\frac{d}{2}\sin\vartheta_0$$

(Beob.richtung)

<u>Beugungsminima</u> bei $\sin\vartheta_0 = n\frac{\lambda}{d}$