

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P} = \epsilon_0 (1 + \chi_e) \underline{E} = \epsilon_0 \epsilon \underline{E}$$

$$\underline{B} = \mu_0 (\underline{H} + \underline{M}) = \mu_0 (1 + \chi_M) \underline{H} = \mu_0 \mu \underline{H}$$

$$\underline{M} = \frac{1}{\mu_0} \underbrace{\frac{\chi_M}{1 + \chi_M}}_{\substack{> 0 \\ < 0}} \underline{B}$$

paramagn. : $\chi_M > 0 \Rightarrow \mu > 1$
 diamagn. : $-1 < \chi_M < 0 \Rightarrow 0 < \mu < 1$

Bem. (i) $\underline{E} = 0 \Rightarrow \underline{P} = 0$ beschreibt kein Ferroelektrikum
 $\underline{B} = 0 \Rightarrow \underline{M} = 0$ kein Ferromagnet

(ii) $\chi_e > 0$, aber $\chi_M \geq 0$ { Para- } Magnetismus
 { Dia- }

(iii) $\sim \underline{B}$ in \underline{P}
 $\sim \underline{E}$ in \underline{M} } kann nicht auftreten
 wegen des falschen
 Raumspiegelungsverh.
 \underline{E} : polarer Vektor, \underline{B} axialer Vektor

$\rho_p = -\text{div } \underline{P}$ ist Skalar $\Rightarrow \underline{P}$ polarer Vektor

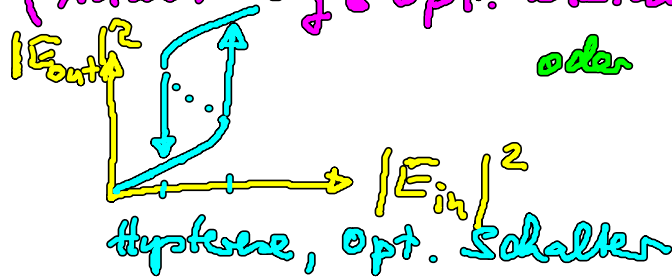
$\underline{j}_M = \text{rot } \underline{M}$ ist pol. Vektor $\Rightarrow \underline{M}$ axialer Vektor

(iv) Abweichungen von den einfachen Materialgls.

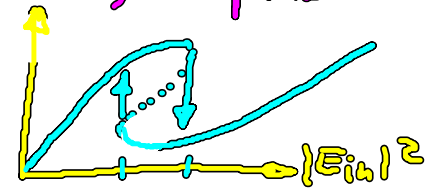
- für anisotrope : $\underline{P} = \epsilon_0 \underline{\chi}_e \underline{E}$ (Symm. Tensor)

- für starke Felder : $\underline{P} = \epsilon_0 (\chi_e^{(1)} \underline{E} + \chi_e^{(2)} |\underline{E}| \underline{E} + \dots)$

(Anwendung : opt. Bistabilität) nichtlineare Optik



oder



- für hochfrequente Felder :

$$\underline{P}(\underline{r}, t) = \epsilon_0 \int d^3 r' dt' \chi_e(\underline{r}, \underline{r}', t, t') \underline{E}(\underline{r}', t')$$

(räumliche bzw. zeitliche Dispersion :

$$\hat{P}(\underline{k}, \omega) = \epsilon_0 \hat{\chi}_e(\underline{k}, \omega) \hat{\underline{E}}(\underline{k}, \omega)$$

$$\hat{\underline{E}}(\underline{k}, \omega) = 1 + \hat{\chi}_e(\underline{k}, \omega) \text{ dielekt. Fkt.}$$

5.4 Grenzbed. für Felder

Frage : Wie verhalten sich $\underline{E}, \underline{D}, \underline{H}, \underline{B}$ an Grenzflächen, die verschiedene el. u. mag. Materialien (oder Vakuum / Materie) trennen?

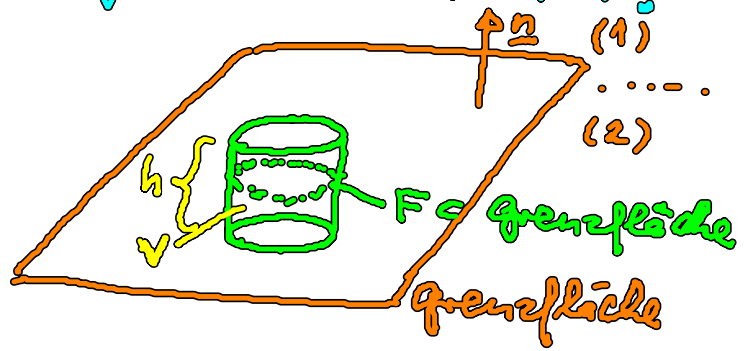
Integration der Maxwell-Gln. über Volumen V :

$$(1) \int_V d^3r \nabla \times \underline{E} = - \int_V d^3r \dot{\underline{B}}$$

$$(2) \int_V d^3r \nabla \times \underline{H} = \int_V d^3r (\underline{j} + \dot{\underline{D}})$$

$$(3) \int_V d^3r \nabla \cdot \underline{B} = 0 \stackrel{\text{Gauß}}{=} \oint_{\partial V} d\mathbf{f} \cdot \underline{B}$$

$$(4) \int_V d^3r \nabla \cdot \underline{D} = \int_V d^3r \rho \stackrel{\text{Gauß}}{=} \oint_{\partial V} d\mathbf{f} \cdot \underline{D}$$



Normalkomponenten

$$h \rightarrow 0 : (3) \Rightarrow \lim_{h \rightarrow 0} \oint_{\partial V} d\mathbf{f} \cdot \underline{B} = \int_F d\mathbf{f} (\underline{B}^{(1)} - \underline{B}^{(2)}) = \int_F d\mathbf{f}_n (\underline{B}^{(1)} - \underline{B}^{(2)})$$

$$(4) \Rightarrow \lim_{h \rightarrow 0} \oint_{\partial V} d\mathbf{f} \cdot \underline{D} = \int_F d\mathbf{f} (\underline{D}^{(1)} - \underline{D}^{(2)}) = \int_F d\mathbf{f}_n (\underline{D}^{(1)} - \underline{D}^{(2)})$$

Annahme: Grenzfläche trägt freie Flächenladungsdichte σ

$$\rho(\underline{r}, t) = \sigma(x, y, t) \delta(z) \quad \underline{e}_z \equiv \underline{n}$$

$$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \rho = \int_F d\mathbf{f} \sigma$$

Also für bel. Fläche F :

$$\int_F d\mathbf{f}_n (\underline{B}^{(1)} - \underline{B}^{(2)}) = 0$$

$$\Rightarrow \underline{n} (\underline{B}^{(1)} - \underline{B}^{(2)}) = 0$$

$$\int_F d\mathbf{f}_n (\underline{D}^{(1)} - \underline{D}^{(2)}) = \int_F d\mathbf{f} \sigma$$

$$\Rightarrow \underline{n} (\underline{D}^{(1)} - \underline{D}^{(2)}) = \sigma$$

Tangentiale Komp.

Verallg. Gauß'scher Satz:

$$(1) \Rightarrow \int_{\partial V} d\underline{f} \times \underline{E} = - \int_V d\underline{f} \cdot \underline{D}$$

$$(2) \Rightarrow \int_{\partial V} d\underline{f} \times \underline{H} = \int_V d\underline{f} \cdot (\underline{j} + \underline{\dot{D}})$$

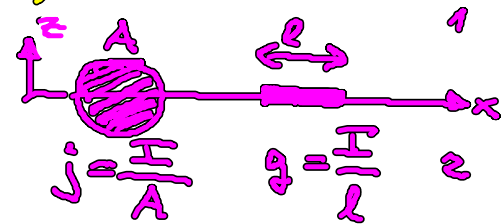
h $\rightarrow 0$: (1) $\Rightarrow \lim_{h \rightarrow 0} \int_{\partial V} d\underline{f} \times \underline{E} = \int_F d\underline{f} \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)})$
Tangentiale Komp.

$$(2) \Rightarrow \lim_{h \rightarrow 0} \int_{\partial V} d\underline{f} \times \underline{H} = \int_F d\underline{f} \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)})$$

Annahme : Grenzfläche trägt freie Flächenstromdichte \underline{g} :

$$\underline{j}(\underline{r}, t) = \underline{g}(x, y, t) \delta(z) \quad \text{z.B. Metall}$$

$$\Rightarrow \lim_{h \rightarrow 0} \int_V d\underline{f} \cdot \underline{j} = \int_F d\underline{f} \cdot \underline{g}$$



$\underline{E}, \underline{D}$ und $\underline{E}, \underline{D}$ sind beschränkt:

$$\lim_{h \rightarrow 0} \int_V d\underline{f} \cdot \underline{D} = \lim_{h \rightarrow 0} \int_V d\underline{f} \cdot \underline{\dot{D}} = 0$$

Also für bel. Fläche F :

$$\int_F d\underline{f} \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)}) = 0$$

$$\int_F d\underline{f} \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)}) = \int_F d\underline{f} \underline{g}$$

$$\underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)}) = 0$$
$$\underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)}) = \underline{g}$$

Zusammenfassung : $\delta \underline{E} = \underline{E}^{(1)} - \underline{E}^{(2)}$ usw.

Maxwell-Ges.	Grenzbed.
$\nabla \times \underline{E} = -\dot{\underline{B}}$	$\underline{n} \times \Delta \underline{E} = 0$
$\nabla \cdot \underline{D} = \rho$	$\underline{n} \cdot \Delta \underline{D} = \sigma$
$\nabla \times \underline{H} = \underline{j} + \dot{\underline{D}}$	$\underline{n} \times \Delta \underline{H} = \underline{g}$
$\nabla \cdot \underline{B} = 0$	$\underline{n} \cdot \Delta \underline{B} = 0$

Tangentialkomp. v. \underline{E} stetig
 Normalkomp. v. \underline{D} springt
 („Flächendivergenz“)
 Tang. komp. v. \underline{H} springt
 („Flächenrotation“)
 Normalkomp. v. \underline{B} stetig

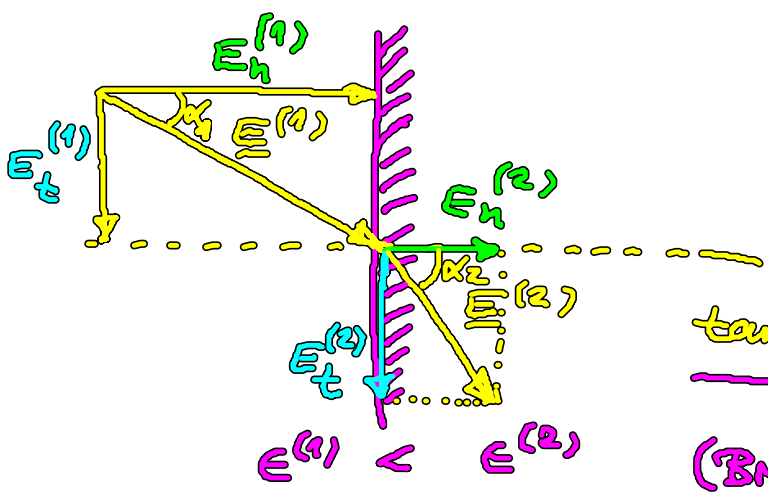
Beispiele

(1) Grenzfläche zwischen 2 dielektr. Materialien
 mit $\epsilon^{(1)} < \epsilon^{(2)}$ und $\sigma = 0$

$$\underline{E}^{(i)} = \underline{E}_t^{(i)} + \underline{E}_n^{(i)} \quad \text{mit} \quad \underline{E}_t^{(1)} = \underline{E}_t^{(2)}$$

$$\underline{D}_n^{(1)} = \underline{D}_n^{(2)} \Rightarrow \epsilon_1 \underline{E}_n^{(1)} = \epsilon_2 \underline{E}_n^{(2)}$$

$$\Rightarrow \underline{E}_n^{(2)} = \frac{\epsilon_1}{\epsilon_2} \underline{E}_n^{(1)}$$



$$\tan \alpha_1 = \frac{|E_t^{(1)}|}{|E_n^{(1)}|} = \frac{\epsilon_1 |E_t^{(2)}|}{\epsilon_2 |E_n^{(2)}|} = \frac{\epsilon_1}{\epsilon_2} \tan \alpha_2$$

(Brechungsgesetz der Feldlinien)