Chapter 9 Superconductivity

so far our materials were „mix" of electrons and ions:

- Born-Oppenheimer: electrons and ions separable (elec-phonon coupling requires different treatment)
- independent electron approximation for many-electron ground state (always exists (DFT))
- independent quasiparticle picture as simple extension (e.g. photoemission, Fermi liquid theory)

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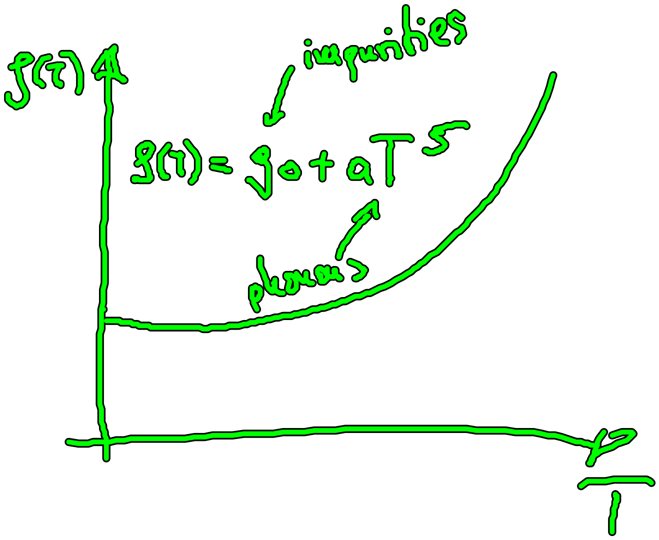
but: independent electron / quasiparticle picture can break down:

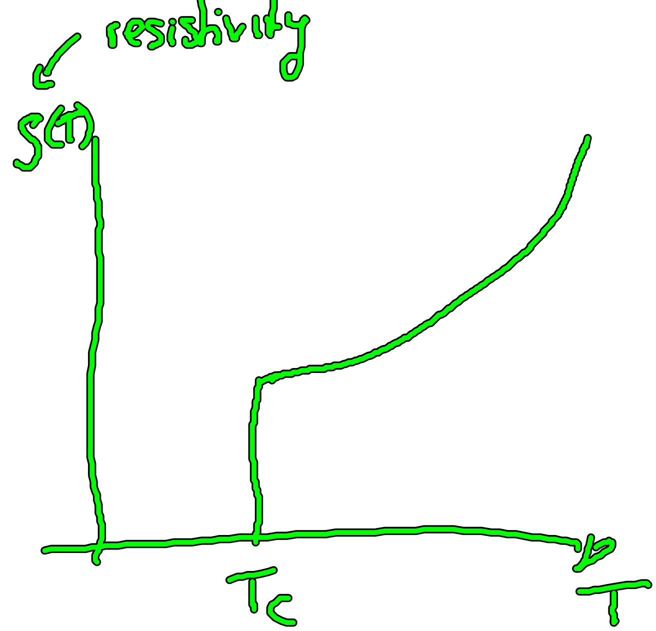
e.g. most magnetic ordering phenomena are

Many-body in nature

even worse:



normal conductor



superconductor

Kamerlingh Onnes 1911

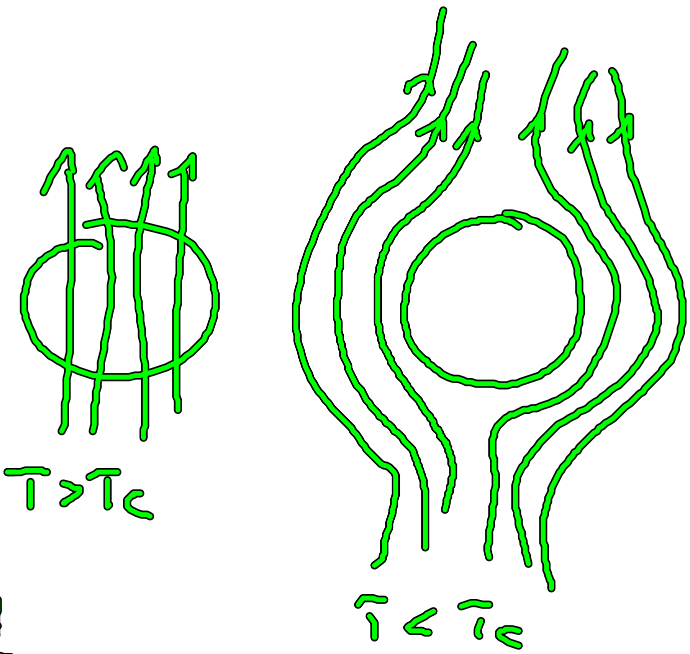
- But perfect conductivity "should" not exist at $T \approx 0$
- it's also not an isolated phenomenon
- typical transition temperatures:

$Hg(\alpha)$	4.15 K
Sn	3.72 K
Pb	7.19 K

Nb 9.26 K

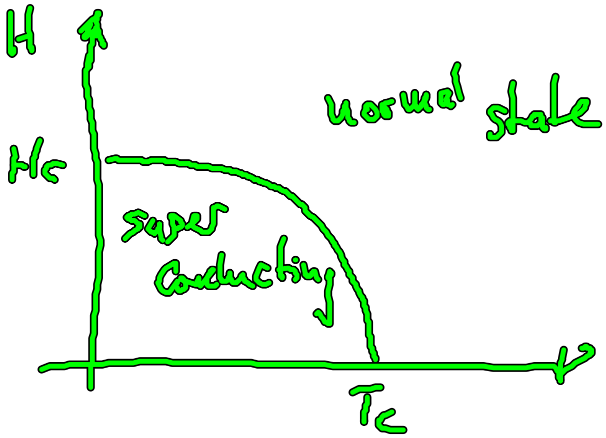
1933 Meissner and Ochsenfeld

- external field can't enter SC
- any original field in SC material is expelled in SC state



$$0 = \underline{B} = \mu_0 \mu \underline{H} = \mu_0 (1+\chi) \underline{H}$$

$\Rightarrow \chi = -1$ very large diamagnetism



- this odd, so let's consider a perfect conductor

n_s is fraction of superconducting electrons
(rest scatters normally)
$\vec{E}$ applied elec field, accelerates electrons:

$$m \frac{d v_s}{dt} = -e \underline{E}$$

current density: $\underline{j} = -e \underline{v_s} n_s$

$$\curvearrowright \quad \frac{d\underline{j}}{dt} = \frac{n_s e}{m} \underline{E}$$

Substituting this into Faraday's law of induction:

$$\underline{\nabla} \times \underline{E} = -\frac{1}{c} \frac{\partial \underline{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t} \left(\nabla \times \underline{j} + \frac{n_s e^2}{mc} \underline{B} \right) = 0$$

together with Maxwell's equation

$$\underline{\nabla} \times \underline{B} = \frac{4\pi}{c} \underline{j}$$

this determines $\underline{B}$ and $\underline{j}$ in superconductor.

- in particular: any change in $\underline{B}$ field is immediately screened ✓
- however: there are static $\underline{j}$ and $\underline{B}$ that satisfy equations

↯ inconsistent with expulsion of magnetic fields

$\Rightarrow$ perfect conductivity not sufficient to explain Meissner effect.

F. and H. London proposed to restrict $\underline{j}$ and $\underline{B}$ of SC to those that satisfy:

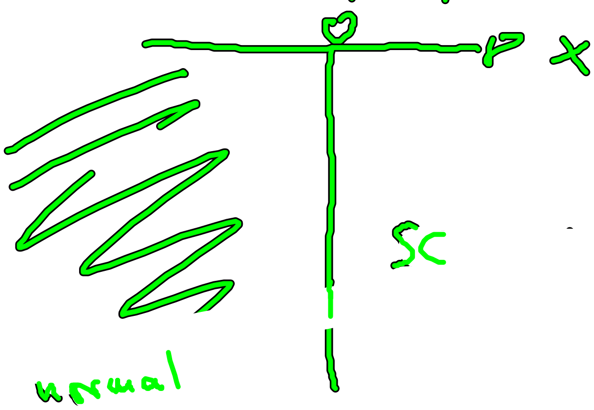
$$\underline{\nabla} \times \underline{j} + \frac{n_s e^2}{mc} \underline{B} = 0 \qquad \text{(London equation)}$$

$\curvearrowright$ Maxwell's equation:

$$\underbrace{\underline{\nabla} \times (\underline{\nabla} \times \underline{B})} + \frac{4\pi n_s e^2}{mc^2} \underline{B} = 0$$

$$= \nabla(\underbrace{\nabla\cdot B}_{\text{Gauss law} = 0}) - \nabla^2 B \Rightarrow \nabla^2 B = \frac{4\pi n_s e^2}{mc^2} B$$

consider half infinite SC

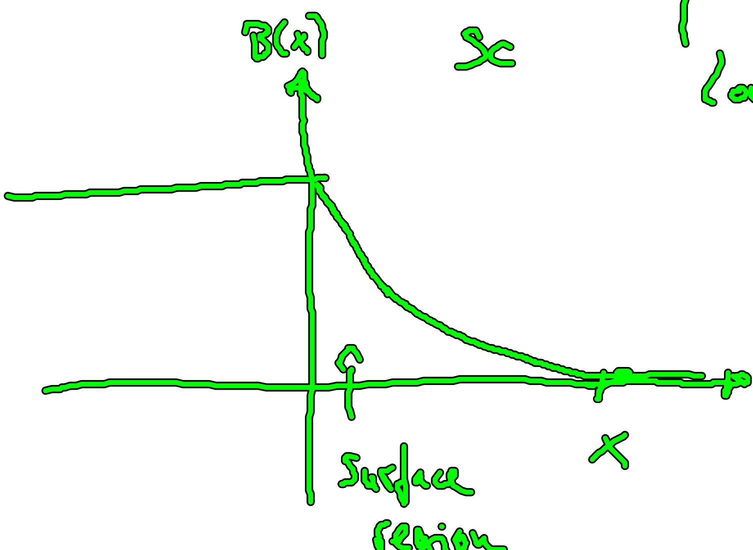


$$B(x) = B(0)\, e^{-\frac{x}{\lambda_L}}$$

$$\lambda_L = \sqrt{\frac{mc^2}{4\pi n_s e^2}} = 41.9 \left(\frac{r_s}{a_0}\right)^{3/2} \left(\frac{n}{n_s}\right)^{1/2} Å$$

$\approx 10^2 - 10^3$ Å typically

London penetration depth



- magnetic field only penetrates SC surface
- eddy currents at surface expel the field

Questions:

what creates phase transition of electronic structure to superconducting state?

How can we create a microscopic theory (e.g. of London's equation) beyond standard electrodynamics?

9.2. London theory

- still phenomenological
- but quantum mechanical analog of London equation

Current density

$$\underline{j}(\underline{r}) = \frac{ie}{2m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{e^2}{m}\underline{A}\,\psi^*\psi$$

sum over all electron states below Fermi energy

with vector potential $\underline{A}(\underline{r})$ such that

$$\underline{\nabla}\times\underline{A} = \underline{B}$$

- for electrons this yields weak Landau diamagnetism (see Chapter on magnetism)

  $\Rightarrow$ cannot explain Meissner effect

So let's assume bosons with charge e^* and mass m^*

- they satisfy the Schrödinger equation:

$$-\frac{1}{2m^*}\left(\underline{\nabla} + ie^*\underline{A}\right)^2\phi(\underline{r}) = E\,\phi(\underline{r})$$

- assume Maxwell gauge $\underline{\nabla}\,\underline{A} = 0$ and expand $(\ )^2$

$$\Rightarrow \left(-\frac{1}{2m^*}\nabla^2 - \frac{ie^*}{m^*}\underline{A}\,\underline{\nabla} + \frac{e^{*2}\underline{A}^2}{2m^*}\right)\phi(\underline{r}) = E\,\phi(\underline{r})$$

now we assume $\underline{A}$ to be small: apply perturbation theory

$\phi(\underline{r}) = \phi_0(\underline{r}) + \phi_e(\underline{r})$ for ground state

ground state without field (from ϕ_0)

comes from weak perturbation $\underline{A}$ (from ϕ_e)

Bose condensate at low temperature

$$\phi_0(\underline{r}) = \frac{1}{\sqrt{V}} \quad , \underline{k} = 0 , \ E_0 = 0$$

⤳ insert ϕ into SE and keep terms to first order in $\underline{A}$ and ϕ_e

$$\underbrace{-\frac{1}{2m^*}\nabla^2\phi_0}_{=0} - \frac{1}{2m^*}\nabla^2\phi_e - \underbrace{\frac{ie^*}{m^*}\underline{A}\,\underline{\nabla}\phi_0}_{=0} = \underbrace{E_0\phi_e(\underline{r})}_{=0} + E_1\phi_0(\underline{r})$$

$$E_1 \sim \underline{\nabla}\,\underline{A}$$ (because it has to be first order in $\underline{A}$ and a scalar)

$\Rightarrow$ $E_1 = 0$ vanishes in Maxwell gauge

$\Rightarrow$ $\phi_e(\underline{r}) = 0$ first order change vanishes (to first order in $\underline{A}$)

$$j(\underline{r}) = \frac{ie}{2m^*}\underbrace{\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right)}_{=0} - \frac{e^*}{2m^*}\underline{A}\,\psi\,\psi^*$$

$\psi = \phi_0$

$$j(\underline{r}) = -\frac{e^{*2}}{2m^*}\underbrace{\left(\frac{N_s}{V}\right)}_{n_s}\cdot\underline{A}$$

N_s: # of sc carriers (from sum over states)

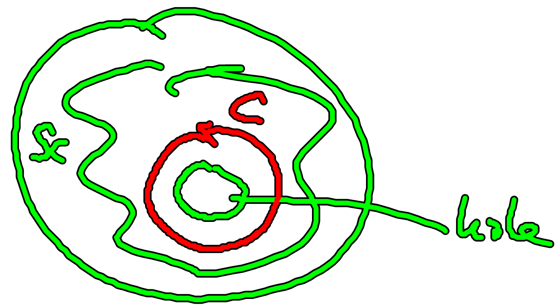
with $\underline{\nabla}\times\underline{A} = \underline{B}$ and Maxwell's eq. : $\underline{\nabla}\times\underline{B} = \frac{4\pi}{c}\,j$

$\Rightarrow \quad \underline{B} + \lambda_L^2 \, \underline{\nabla} \times \underline{\nabla} \times \underline{B} = 0$ London equation

but this time derived quantum mechanically

and we know: Bosons are the carriers in superconductivity ??

Flux quantization



$$\phi(\underline{r}) = \sqrt{n_s}\, e^{i\varphi(\underline{r})}$$

← phase factor

Can we understand phase?

magnetic flux through hole:

$$\Phi_B = \int_{surf} d\underline{S}\, \underline{B} = \oint_C d\underline{\ell}\, \underline{A}(\underline{r})$$

$\underline{B} = \underline{\nabla} \times \underline{A}$ + Stoke theorem