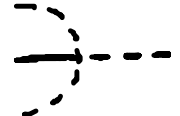
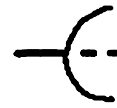
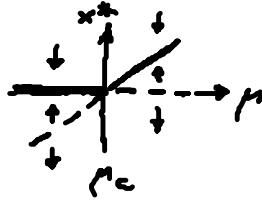
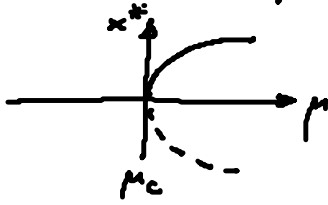


English Summary:

1.3 Bifurcations

(A) zero-eigenvalue bifurcation: $\lambda < 0 \rightarrow \lambda > 0$
 $\det A > 0$ $\det A < 0$

Saddle-node bifurcation transcrit. bif. pitchfork bif.



supercrit.

subcrit.

(B) Hopf bifurcation: $\lambda_{1,2} = \lambda_0 \pm i\omega$, $\lambda_0 < 0 \rightarrow \lambda_0 > 0$

Hopf normal form
 (Stuart Landau etc.) $z = r e^{i\varphi} \in \mathbb{C}$

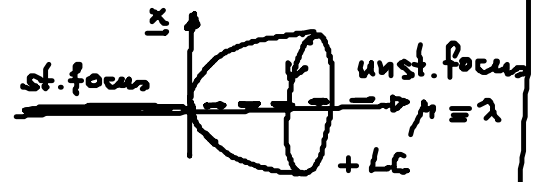
$$\dot{z} = (\lambda + i\omega \mp (1+i\gamma)|z|^2)z$$

$$\dot{r} = (\lambda \mp r^2)r \quad \text{super/subcrit.}$$

$$\dot{\varphi} = (\omega \mp \gamma r^2)$$

$$\begin{matrix} \text{amplitude } r \rightarrow 0 \\ \text{frequency } \omega \neq 0 \end{matrix}$$

Floquet multiplier $\mu = e^{\lambda T} u''(t+T)$



supercrit.

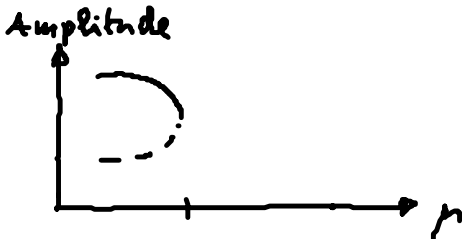
linear stab.:
 Floquet modes
 $\delta z = e^{\lambda t} u(t)$

(C) Lokale Bifurkation von Grenzzyklen

Startpt. jetzt: Grenzzyklus, nicht Fixpt.
 keine einfache lin. Stabilitätsanalyse

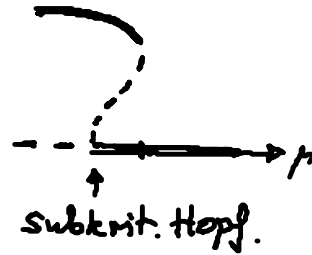
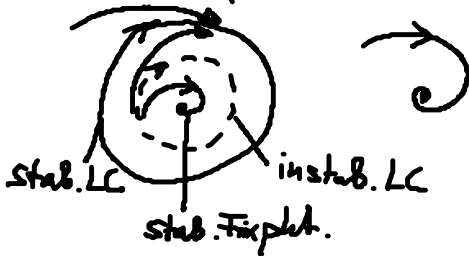
(C1) Sattel-Knoten-Bifurkation eines Grenzzyklus

(Kondensation von Pfaden, fold bifurcation of limit cycles LC)

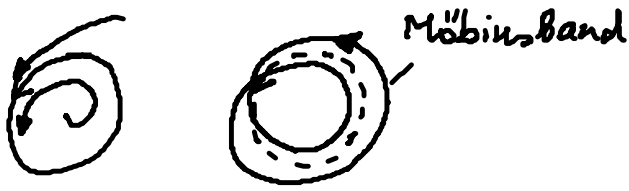


Amplitude $\neq 0$
Frequenz $\neq 0$

Bistabilität zwischen Fixpt. und LC, häufig:

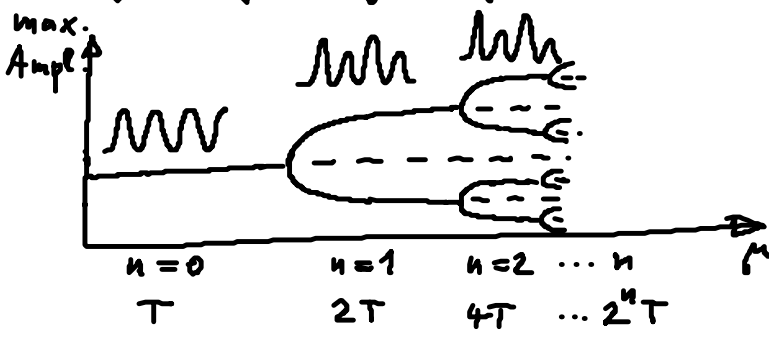
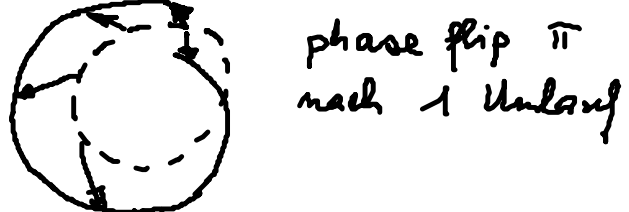


(C2) Perioden-Verdopplung (flip-Bifurkation, subharmon. Bif.)



mind. 3D Phasenraum

Periode-1 Grenzzykl. Periode-2 Grenzzykl.



Torsion benachbarter Traj. !

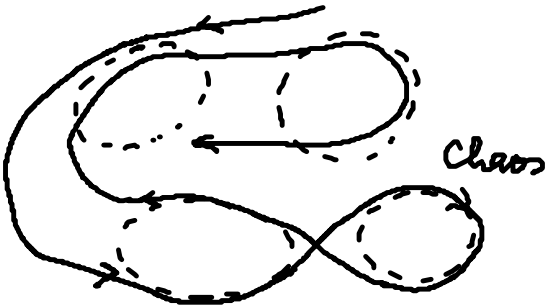
Floquet-Exp. $\Lambda = \lambda + i\omega$
Bif. : $\lambda = 0$, $\omega T = \pi$

\Rightarrow Floquet-Multiplikation

$\mu = e^{\Lambda T} = e^{i\pi} = -1$

häufig Periodenverdoppl. kaskade
 \rightarrow Chaos (Feigenbaum-Szenario)

\rightarrow unendlich viele instabile periodische Orbits der Periode $2^n T$
($n=0, 1, 2, \dots$)

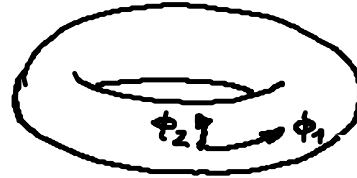


(C3) Sacker-Neimark-Bifurkation

(sekundäre Hopf-Bif. eines LC)



1. Hopf-Bif. des Fixpt. : $\phi_1 = \omega_1 t$
2. Hopf-Bif. des LC : $\phi_2 = \omega_2 t$



LC \longrightarrow 2-Torus

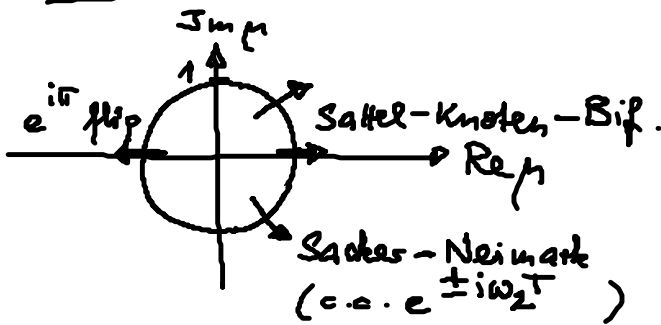
mild. 3D Phasenraum

inkommensurable Frequenzen $\frac{\omega_1}{\omega_2} \notin \mathbb{Q}$
 (quasiperiodisch) irrational

\rightarrow Trajektorie schließt sich nicht, liegt dicht auf dem Torus

falls $\frac{\omega_1}{\omega_2} \in \mathbb{Q} \Rightarrow$ geschlossener Orbit = LC
 (frequency locking)
 Modenkopplung

- Stabilität von Grenzzyklen : Floquet-Multiplikator



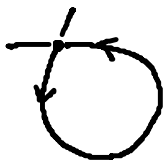
$\mu = e^{\lambda T}$
 Stab.grenze $|\mu| = 1$

oder transkrit. Bif. eines LC



(D) Globale Bifurkation von Grenzzyklen

globale qualitative Änderung des Phasenporträts,
 z.B.



homoklin. Orbit \rightarrow

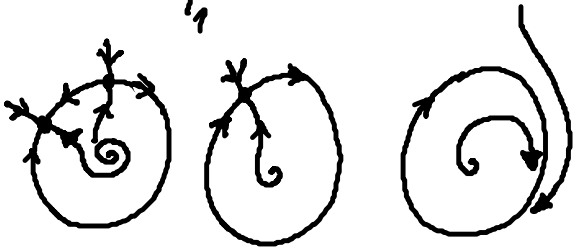
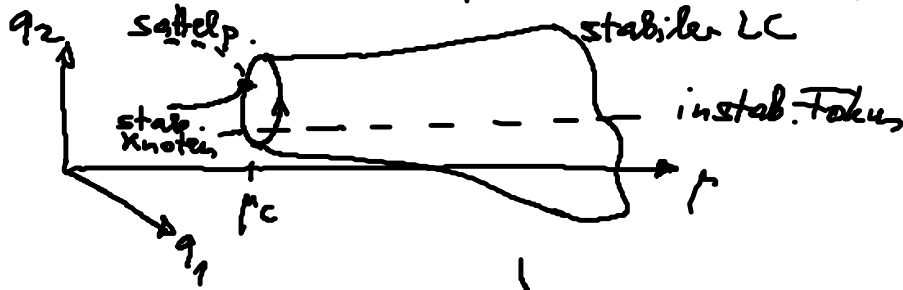


invar. Mannigfaltigk.
 des Sattelpnts.

(D1) Sattel-Knoten-Bif. auf einem Grenzzyklus (Omega explosion)

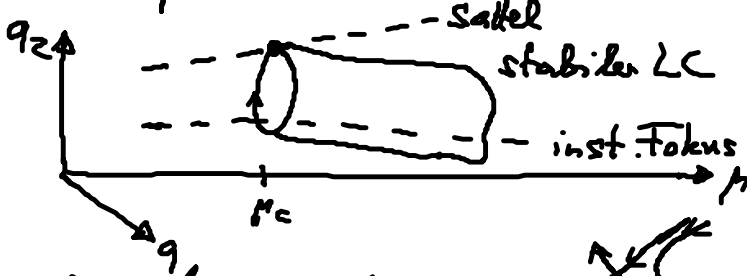
saddle-node on invariant cycle (SNIC)
 saddle-node infinite period (SNIPER)

Amplitude $\neq 0$ Frequ. $\rightarrow 0$
--

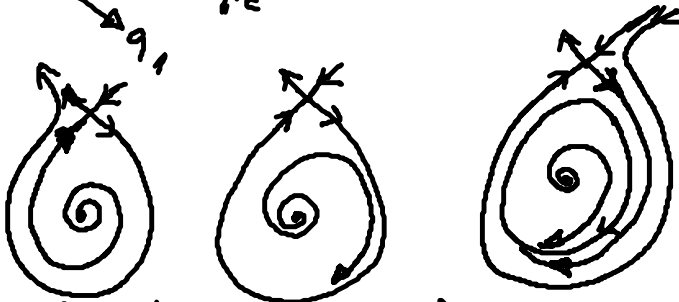


(D2) Homokline Bif. (blue-sky catastrophe)

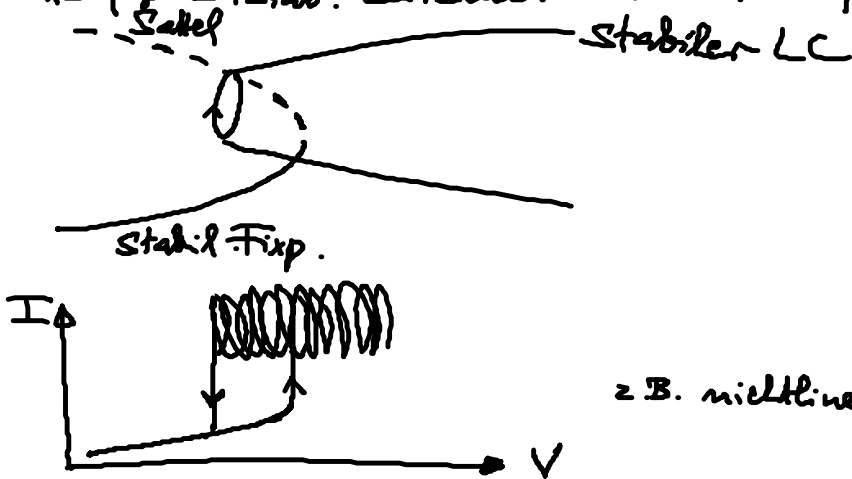
Sattelpkt. kollidiert mit LC: homokliner Orbit (saddle-to-saddle loop)



Ampl. $\neq 0$ Frequ. $\rightarrow 0$
--



häufig Bistab. zwischen Osz. u. Fixpkt., z.B.:



z.B. nichtlinearer Halbleiter: Hypothese von Osz. u. stationärem Strom