English Summary:

2.3 Adaptive Control

minimize cost fct. $Q(x(t),t)$ $\Rightarrow$ find control fct. $u(t)$

speed gradient method: minimize $\dot{Q} = \frac{\partial Q}{\partial t} + \nabla_{\underline{x}} Q(\underline{x},t)\,\dot{\underline{x}} = f(\underline{x},\underline{u},t)$

$\dot{\underline{u}} = -\Gamma \nabla_u \dot{Q}$ $\qquad$ $\dot{Q}$ decreases $\Rightarrow \dot{Q} < 0 \Rightarrow Q$ decreases $\rightarrow 0$

e.g. find K, τ of time-delayed feedback control

3. Zeitverzögerte Rückkopplungsverfahren

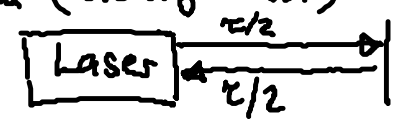
3.1. Retardierte komplexe Systeme

Delay-Differenzialgln.: $\dot{x}(t) = f(x(t)) + g(x(t-\tau))$

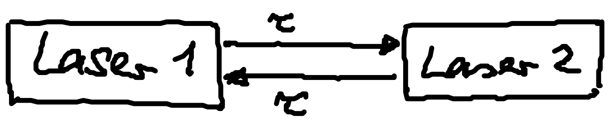
Verzögerungszeit τ

Delay (Retardierung) ist weit verbreitet in nichtlin. Systemen

- mechan. Systemen: Balancierung, Segway
- elektr. Stromkreis: Signalverarbeitungszeiten (Latenzzeit)
  kapazitive Effekte
- optische Systeme: Signallaufzeiten (Lichtgeschw.)
  - Laser mit opt. Rückkoppl.



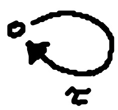
  - 2 gekoppelte Laser



- biolog. Systeme: Zell-Zyklus-Zeit τ
  biolog. Uhren
- neuronale Netzwerke: zeitverzögerte Kopplung

$\circ \overset{\tau}{\rightleftarrows} \circ$

zeitverzögerte Rückkopplung



z.B. biochem. Prozesse
(Neurotransmitter)
neuro-vaskuläre Kopplung

SCIENTIFIC PROGRAMS AND ACTIVITIES

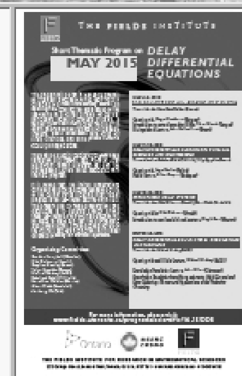
May 19, 2015

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

Short Thematic Program on Delay Differential Equations May 2015

Organizing Committee

Odo Diekmann (Utrecht)
Sue Ann Campbell (Waterloo)
Stephen Gourley (Surrey)
Yuliya Kyrychko (Sussex)
Eckehard Schöll (TU Berlin)
Michael Mackey (McGill)
Hans-Otto Walther (Giessen)
Glenn Webb (Vanderbilt)
Jianhong Wu (York)



May 11-15, 2015
Theme: Delay-Differential equations in physical sciences and engineering
Thematic leader: Eckehard Schöll, TU Berlin; Yuliya Kyrychko, Sussex

Schedule

Monday, 11 May 2015	
8:30-9:00	Coffee and Registration
9:00-9:30	**Opening Remarks**
9:30-10:30	**Ingo Fischer**, Institute for Cross-Disciplinary Physics and Complex Systems, Palma de Mallorca *Delays in Physical Systems: Nuisance, Challenges and Opportunities*
10:30-11:00	Coffee
11:00-11:45	**Anna Zakharova,** Technische Universität Berlin *Time delay control of symmetry-breaking patterns: oscillation death and chimera states*
11:45-12:30	**Jan Sieber,** University of Exeter *Extended time-delayed feedback and its odd-number limitation*
12:30-14:30	Lunch
14:30-15:15	**Laurent Larger**, Franche-Comté Électronique Mécanique Thermique et Optique - Sciences et Technologies, *Delay dynamics explored through signal and information photonic processing*
15:15-16:00	**Bernd Krauskopf**, The University of Auckland, New Zealand *Pulsed lasers with delay*
16:00-16:30	Tea

16:30-17:15	**Cristina Masoller,** Universitat Politecnica de Catalunya *Optical spikes in the delayed Lang-Kobayashi equations: interplay of modulation and delay*

Tuesday, 12 May 2015	
9:00-9:45	**Thomas Erneux,** Universite Libre de Bruxelles *Time delay, excitability, and multi – rhythmicity*
9:45-10:30	**Kathy Lüdge,** Freie Universität Berlin *Optical delays for improving the dynamic behavior of passively mode-locked lasers*
10:30-11:00	Coffee break
11:00-11:45	**Xinzhi Liu,** University of Waterloo *Analysis of hybrid dynamical systems with time delays*
11:45-12:30	**Yuliya Kyrychko,** University of Sussex *Dynamics of neural and genetic networks with discrete and distributed delays*
12:30-14:30	Lunch break
14:30-17:30	Poster Session and Scientific Discussions

Wednesday, 13 May 2015	
9:00-9:45	**Jian-Qiao Sun,** University of California, Merced *Multi-objective optimal design of feedback controls for nonlinear dynamical systems with time delay*
9:45-10:30	**Thilo Gross,** University of Bristol Dynamical motifs in networks of delay-coupled delay oscillators
10:30-11:00	Coffee break
11:00-11:45	**Konstantin Blyuss,** University of Sussex *Dynamics of coupled oscillators with distributed-delay coupling*
11:45-12:30	**Oleksandr Popovych**, Juelich, Germany
12:30-	Lunch/Excursion

Thursday, 14 May 2015	
9:00-9:45	**Daniel J. Gauthier,** Duke University, Durham, NC *Reservoir computing using autonomous time-delay Boolean networks*
9:45-10:30	**Otti D'Huys,** Duke University, Durham, NC *Extreme transients in autonomous time-delay Boolean networks*
10:30-11:00	Coffee break
11:00-11:45	**Sabine Klapp,** Technische Universität Berlin *Delay-differential equations for driven soft-matter systems*
11:45-12:30	**Philipp Hövel,** Technische Universität Berlin *Control of cluster synchronization in delay-coupled oscillators by network adaptation*
12:30-14:30	Lunch
14:30-15:30	**Public Lecture** **Gabor Stepan,** Budapest University of Technology and Economics *Delay models for dynamic contact problems: machine tool vibrations*
15:30-16:00	Tea break
16:00-17:00	Panel Discussion of open problems
19:00-	Conference dinner

Friday, 15 May 2015	
9:30-10:15	**Andre Longtin,** University of Ottawa *Delay-induced linearization and paradoxical oscillations in feedforward nets*
10:15-11:00	**Rachel Kuske,** University of British Columbia *New pattern dynamics in stochastic PDEs with Pyragas control*
11:00-11:30	Coffee break
11:30-12:15	**Serhiy Yanchuk,** Humboldt University of Berlin *Multistable jittering in oscillators with pulsatile delayed feedback*

Retardierung generiert reichhaltiges komplexes Verhalten

- Retardierung erhöht die Phasenraumdim. einer ODE (ordinary diff. eq.) auf unendlich.
  Anfangsbed. auf ganzem Intervall $[-\tau, 0]$ notwendig :
  history fct. $x(t)$ auf $-\tau \le t \le 0$
- Einfache Dgl. produzieren komplexes nichtlin. Verhalten
  - delay-induzierte Bifurkation, Instabilität
  - delay-induzierte Multistabilität
  - Stabilisierung von instab. period. oder stat. Zuständen
  - Chaoskontrolle (Unterdrückung von Chaos)

Lit.: T. Erneux : Applied delay differential equations (Springer 2009)
P. Hövel : Control of Complex Nonlin. Systems with Delay (Springer 2010)
V. Flunkert : Delay-Coupled Complex Systems (Springer 2011)
C. Otto : Laser Dynamics (Springer 2014)
D. Rosin : Autonomous Boolean Networks (Springer 2015)
Just, Pelster, Schanz, Schöll (eds): Delayed Complex Systems (Theme Issue of Phil. Trans. Roy. Soc. A 368 (2010))
Flunkert, Fischer, Schöll (eds.) : Dyn., Control, and Information in Delay-coupled Systems (Theme Issue of Phil. Trans. Roy. Soc. A371 (2013))

3.2 Lineare Stabilitätsanalyse retardierter Differenzialgln.

einfachste lin. Delay-Dgl. $\dot{x} = -a x(t) + b x(t-\tau) \qquad a, b, x \in \mathbb{R}$

Anf. bed. $x(t) = \phi(t) \qquad -\tau \le t \le 0$

Fixpkt. $x^* = 0$

kleine Störung : $x(t) \sim e^{\Lambda t}$

$$\Rightarrow \Lambda e^{\Lambda t} = -a e^{\Lambda t} + b e^{\Lambda t} e^{-\Lambda \tau}$$

$$\boxed{\Lambda = -a + b e^{-\Lambda \tau}}$$ transzendente char. Gl. für Eigenwert $\Lambda \in \mathbb{C}$

Lösung für Λ : $\underbrace{(\Lambda + a)\tau}_{z} = b\tau e^{-\Lambda \tau}$

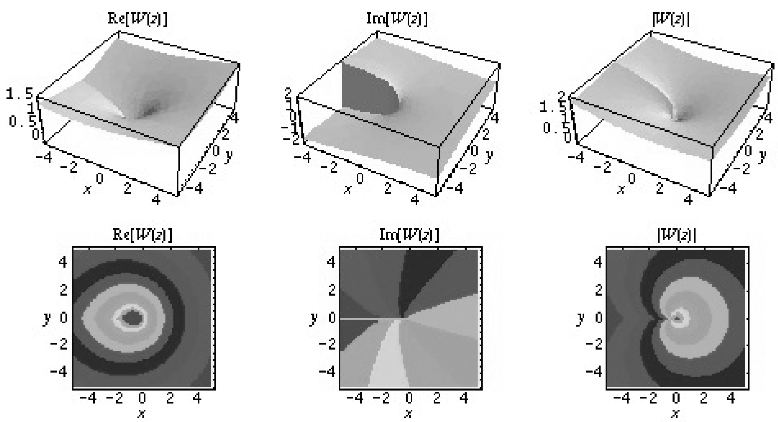
$$z e^{z} = b\tau e^{a\tau}$$

inverse Fkt. von $z e^{z} = y$: $z = W_l(y)$ Lambert-Fkt.
(vielblättrig, $l = \mathbb{Z}$)

(cf. $e^{z} = y \Leftrightarrow z = \ln y$)

$$\Rightarrow \Lambda_\ell = -a + \frac{1}{\tau} W_\ell\left(b\tau e^{a\tau}\right) \qquad \begin{pmatrix} a>0: \text{stab. Fixpkt. ohne Delay} \\ a<0: \text{instab. " \quad " \quad "} \end{pmatrix}$$

allg. Lös. $x(t) = \sum_{\ell=-\infty}^{\infty} c_\ell e^{\Lambda_\ell t}$



Hauptzweig

$$W_0(z) = \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} z^n$$

$\left(|z| < \frac{1}{e}\right)$

asymptot. Entwicklung für $z \to 0$ u. $z \to \infty$ $(\ell \neq 0)$

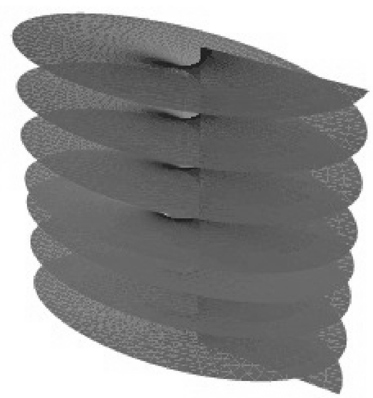
$$W_\ell(z) \approx \ln z + 2\pi i \ell - \ln(\ln z + 2\pi i \ell)$$

$\underline{\tau \to 0}$ $(z \to 0)$

$$W_\ell(z) \approx \underbrace{\ln z}_{-\infty} + 2\pi i \ell + \text{höh. Ordn.}$$

Re $W(z)$

Im $W(z)$



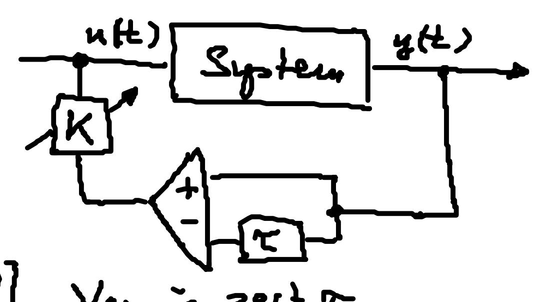
$$\boxed{\Lambda_\ell \to -\infty \quad \forall \ell \neq 0}$$

$\underline{\tau \to \infty}$ $(z \to \infty)$: $\Lambda_\ell \approx \underline{-a} + \frac{1}{\tau}\left[\ln(b\tau) + \underline{a\tau} + 2\pi i \ell - \ln(\ln z + 2\pi i \ell)\right]$

Lit.: Amann, Schöll, Just: Physica A **373**, 191 (2007)

3.2.1 Stabilisierung instabiler Fixpunkte durch zeitverzögerte Rückkopplungskontrolle

Pyragas (1992)
Phys. Lett. A **170**, 421



closed-loop control
(Rückkoppl. kontrolle)

$y(t)$: Ausgangsvar.

$u(t) = K[y(t) - y(t-\tau)]$
Kontrollvar.

Verzög. zeit τ
Rückkoppl. stärke K

nichtinvasiv
(Kontrollkraft verschwindet auf dem Zielzustand $y(t)=y(t-\tau)$)

Allg. Form eines 2-Var.-Systems (ohne Kontrolle)

Fixpkt. $\underline{x}^*$: $\quad 0 \overset{!}{=} \dot{\underline{x}} = f(\underline{x}^*) \qquad \underline{x} \in \mathbb{R}^2$

Linearisierung um $\underline{x}^*$ für kleine Störungen:

$$\underline{x}(t) = \underline{x}^* + \delta\underline{x}(t): \qquad \delta\dot{\underline{x}} = (Df)_* \, \delta\underline{x}$$

Jacobi-Matrix $(Df)_* \equiv A$

Lös. $\delta\underline{x} \sim e^{\Lambda t}$: $\quad 0 = \det(A - \Lambda 1) = \Lambda^2 - \Lambda \operatorname{tr} A + \det A$

$$\Rightarrow \Lambda = \frac{\operatorname{tr} A \pm \sqrt{(\operatorname{tr} A)^2 - 4 \det A}}{2}$$

Normalform eines instab. Fokus $\quad \Lambda = \lambda \pm i\omega \quad (\lambda > 0)$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \lambda & \omega \\ -\omega & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

komplexe Schreibweise $\quad \dot{z} = (\lambda \pm i\omega) z, \quad z = x \mp iy \in \mathbb{C}$

mit zeitverzögerter Rückkopplung

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \lambda & \omega \\ -\omega & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - K \begin{pmatrix} x(t) - x(t-\tau) \\ y(t) - y(t-\tau) \end{pmatrix}$$

"diagonale Rückkopplung"

$$\dot{z} = (\lambda \pm i\omega) z - K (z(t) - z(t-\tau))$$

Ansatz $\begin{pmatrix} x \\ y \end{pmatrix} \sim e^{\Lambda t}$

char. Gl. $\quad 0 = \det \left[\begin{pmatrix} \lambda - \Lambda & \omega \\ -\omega & \lambda - \Lambda \end{pmatrix} - K \begin{pmatrix} 1 - e^{-\Lambda\tau} & 0 \\ 0 & 1 - e^{-\Lambda\tau} \end{pmatrix} \right]$

$$= \left[\Lambda + K(1 - e^{-\Lambda\tau}) - \lambda \right]^2 + \omega^2$$

$$\Leftrightarrow \quad [\quad \dots \quad]^2 = -\omega^2$$

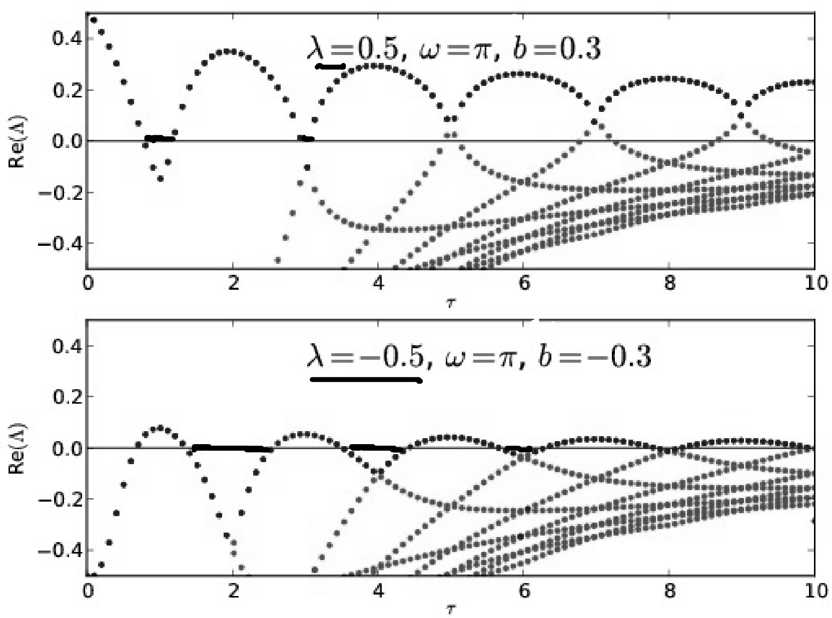
$$\Rightarrow \boxed{\Lambda + K(1 - e^{-\Lambda\tau}) = \lambda \pm i\omega}$$

Lösung durch Lambertfkt. $\underbrace{(\Lambda + K - (\lambda \pm i\omega))\tau}_{\tilde{z}} = K\tau e^{-\Lambda\tau}$

$$\tilde{z} e^{\tilde{z}} = K\tau e^{-(\lambda \pm i\omega)\tau + K\tau}$$

$$\Lambda\tau = W_l\left(K\tau e^{-(\lambda \pm i\omega)\tau + K\tau}\right) + (\lambda \pm i\omega)\tau - K\tau$$

<u>natürliche Zeitskala</u> $T_0 = \frac{2\pi}{\omega}$ (Osz. periode ohne Delay)



unkontrolliert: Fixpkt. <u>instabil</u> ($\lambda > 0$)

unkontrolliert: Fixpkt. stabil ($\lambda < 0$)

nichtmonotones Verhalten der Eigenmoden Re Λ als Fkt. von τ führt abwechselnd zu Stabilisierung / Destabilisierung

Stabilitätsumkehr bei $\tau \approx \frac{2n+1}{2} T_0$ $(n = 0, 1, 2, \ldots)$

$T_0 = \frac{2\pi}{\omega} = 2$