

English Summary:

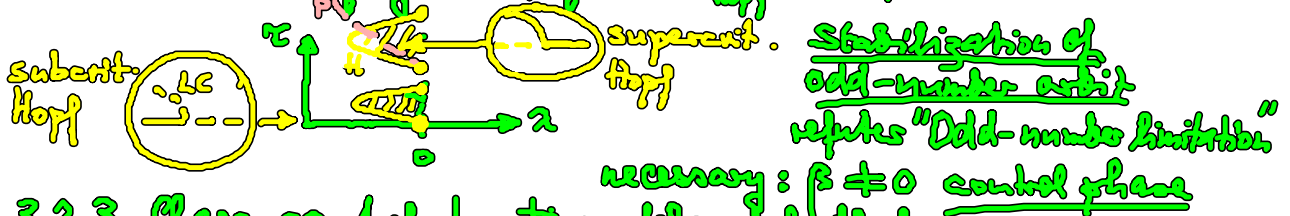
3.2.2 Stabilization of unstable periodic orbits (UPO)

Normal form of subcrit. Hopf bif. (creates UPO without torsion)

$$\dot{z} = (\alpha + i\omega + (1 + i\beta)|z|^2)z + b(z(t-\tau) - z(t)) \quad \begin{matrix} \alpha < 0, \omega = 1, \beta < 0 \\ b = b_0 e^{i\varphi} \in \mathbb{C} \end{matrix}$$

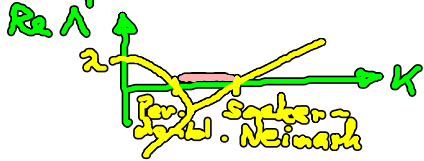
↓
odd no. of
real Floquet
exp. > 0

Pyragas curve $\tau_p(\alpha) = \frac{2\pi}{1-\beta} > \tau_{\text{Hopf}}(\alpha)$ Hopf curve



3.2.3 Chaos control by time-delayed feedback

$$\dot{x} = f(x) + K[x(t-\tau) - x(t)] \quad \text{Floquet exp.} \Rightarrow \lambda + K(1 - e^{-\lambda\tau}) = \alpha + i\omega$$



3.2.4 Kontrolle raum-zeitlicher Systeme

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = f(u, v) + \begin{pmatrix} \Delta_u \nabla_u^2 \\ \Delta_v \nabla_v^2 \end{pmatrix} + KA \begin{pmatrix} u(t-\tau) - u(t) \\ v(t-\tau) - v(t) \end{pmatrix}$$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ Aktivator-
 $= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Inhibitor-
Kontrolle

- je nach Kontrollschema (A), Delay-Zeit (τ) und Kontrollstärke (K) sowie Vorzeichen von K können verschiedene Raum-Zeit-Muster stabilisiert werden:
 - raum-zeitl. Chaos
 - hom. Fixpt. E_1
 - gemischte Turing-Hopf-Muster
 - Vergrößerung
 - Bistab. mit hom. Fixp.
 - Turing-Muster
 - laufende Wellen

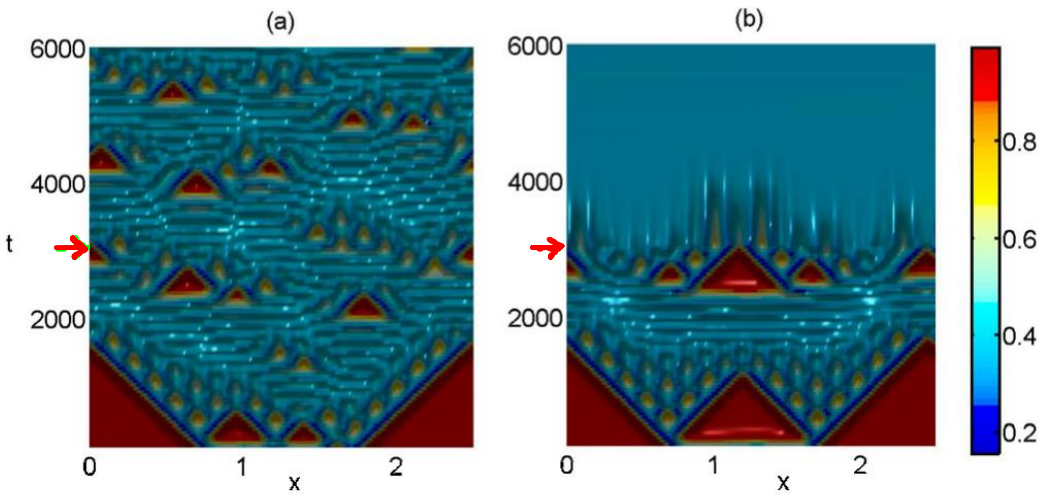


FIG. 4. (Color online) Space-time plot in the case of activator control. (a) Spatiotemporal chaos. (b) Nontrivial steady state E_1 . The color code corresponds to the values of $u(x,t)$. Parameter values are $a=0.028$, $b=0.053$, $D_u=2 \times 10^{-5}$, $D_v=10^{-5}$, (a) $K=-0.4$, $\tau=0.1$ and (b) $K=-0.8$, $\tau=0.6$. Control is switched on at $t=3000$.

Kopylov,
Slyus,
Togan,
Schöll:
Chaos 19, 045126
(2009)

Regime des
Raum-Zeit-
Chaos

period Rundheit.

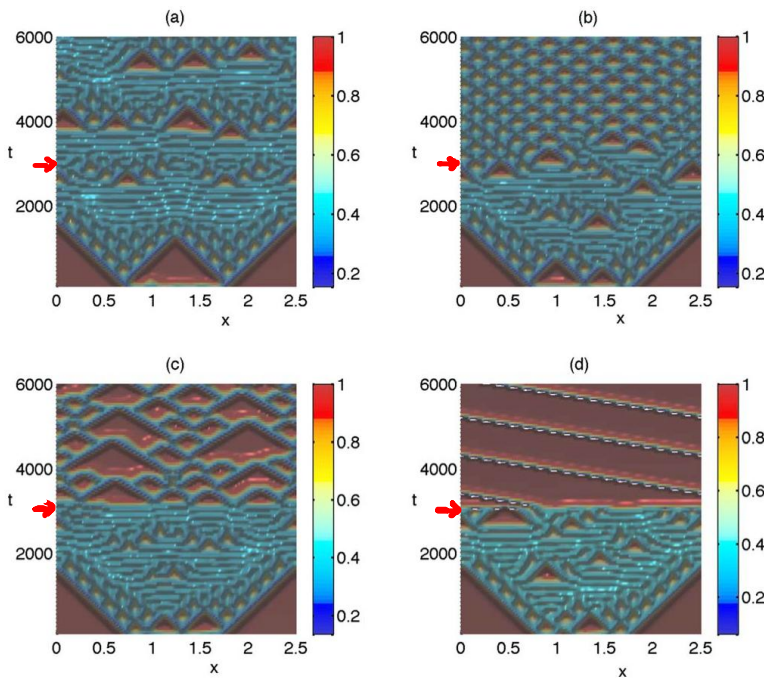


FIG. 5. (Color online) Space-time plot in the case of inhibitor control. (a) Spatiotemporal chaos. (b) Mixed (Turing-Hopf) mode. (c) Coarsening. (d) Transition to traveling waves. Parameter values are $a=0.028$, $b=0.053$, $D_u=2 \times 10^{-5}$, $D_v=10^{-5}$, (a) $K=-0.05$, $\tau=0.3$, (b) $K=-0.3$, $\tau=0.35$, (c) $K=-0.4$, $\tau=0.4$, and (d) $K=-0.6$, $\tau=0.75$. Control is switched on at $t=3000$.

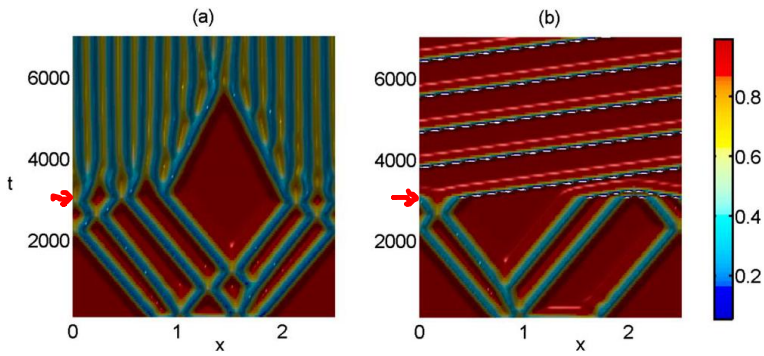


FIG. 7. (Color online) Space-time plot of control of traveling waves. (a) Activator control: Development of a stationary Turing pattern. (b) Inhibitor control: Transition to traveling wave. Parameter values are $a=0.022$, $b=0.053$, $D_u=2 \times 10^{-5}$, $D_v=10^{-5}$, (a) $K=-0.5$, $\tau=0.6$ and (b) $K=-0.8$, $\tau=0.4$. Control is switched on at $t=3000$.

Regime der
laufenden Wellen

3.2.5 zeitverzögerte Rückkopplungskontrolle mit verteilter Verzögerungzeit (distributed delay)

Kopylov, Blyuss, Schöll : *Eur. Phys. J. B* 84, 307 (2011)

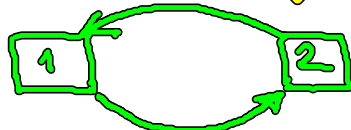
" " " : *Phil. Trans. R. Soc.* (2013)

" " " : *Chaos* 24, 043117 (2014)

statt $\delta(t-\tau)$: kontinuierlich verteilte Verzögerungzeiten

Beispiel : Stabilisierung eines instabilen Fokus :

Betrachte 2 gekoppelte Stuart-Landau-Osc.
mit zeitverzögerter Kopplung



$g(t)$ verteilte delay mit $g(t) \geq 0$, $\int_0^\infty g(t) dt = 1$

$$\dot{z}_1 = (1 + i\omega_0)z_1 - |z_1|^2 z_1 + K e^{i\theta} \left[\int_0^\infty g(t') z_2(t-t') dt' - z_2(t) \right]$$

$$\dot{z}_2 = (1 + i\omega_0)z_2 - |z_2|^2 z_2 + K e^{i\theta} \left[\int_0^\infty g(t') z_1(t-t') dt' - z_1(t) \right]$$

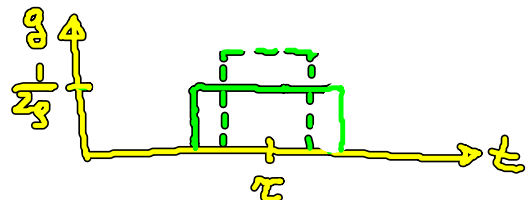
Linearisierung um Fixpt. $z_i^* = z_2^* = 0$: $z_i = e^{\lambda t}$, $i=1,2$

$$(1 + i\omega_0 - K e^{i\theta} - \lambda)^2 - K^2 e^{2i\theta} [\mathcal{L}g(\lambda)]^2 = 0$$

mit Laplace-Transformierte $\mathcal{L}g(s) = \int_0^\infty e^{-st'} g(t') dt'$

speziell : a) gleichförmige Verteilung

$$\Rightarrow \langle \tau \rangle := \int_0^\infty t' g(t') dt' = \tau$$



$$\sigma^2 := \int_0^{\infty} (t' - \langle t \rangle)^2 g(t') dt' = \frac{\sigma^2}{2}$$

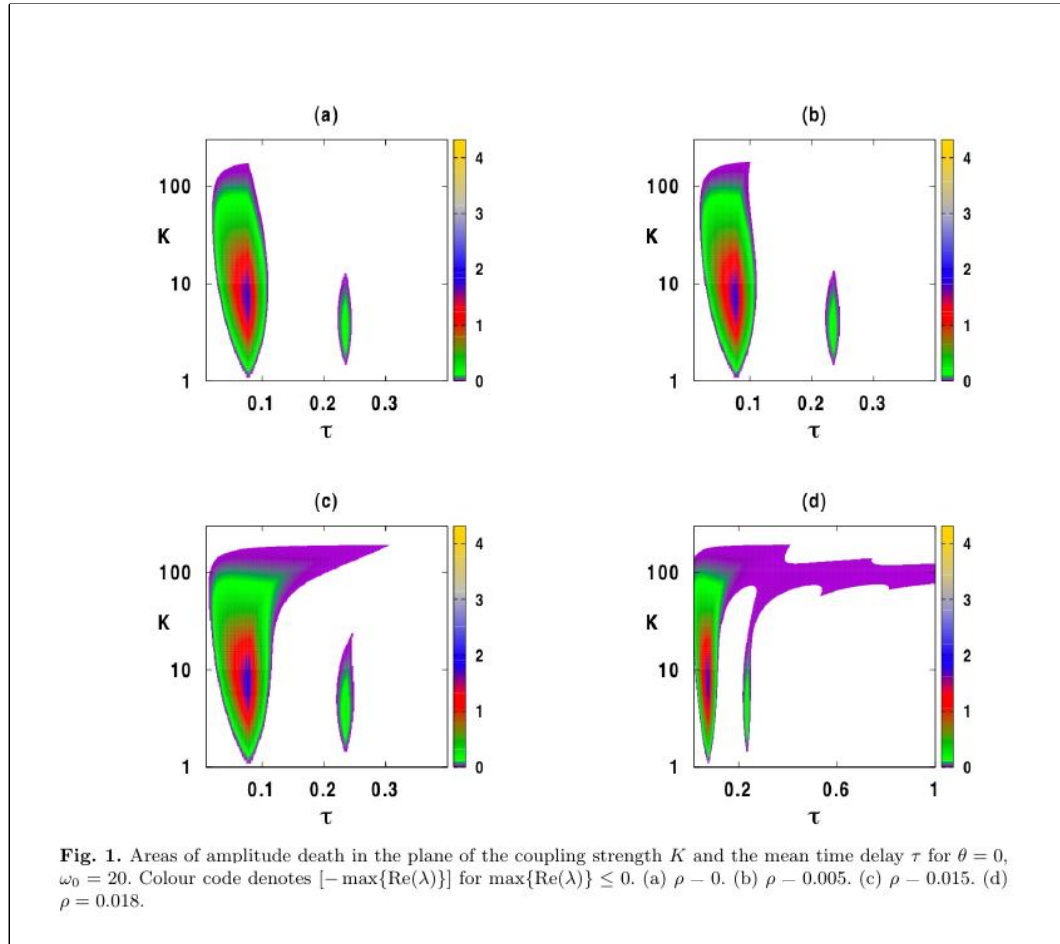
← σ_g

$$\Rightarrow \chi_g(\lambda) = e^{-\lambda \tau} \frac{\sinh(\lambda \sigma_g)}{\lambda \sigma_g}$$

$$\Rightarrow 1 + i\omega_0 - Ke^{i\theta} - \lambda = \pm Ke^{i\theta} e^{-\lambda \tau} \frac{\sinh(\lambda \sigma_g)}{\lambda \sigma_g}$$

→ für $\sigma_g \rightarrow 0$

Stabilisierungsbereiche in der (K, τ) -Ebene: amplitude death



mit zunehmen.
Breite σ_g
werden die
Stab. bereiche
größer
u. wandern
zusammen

Phasenabhängigkeit

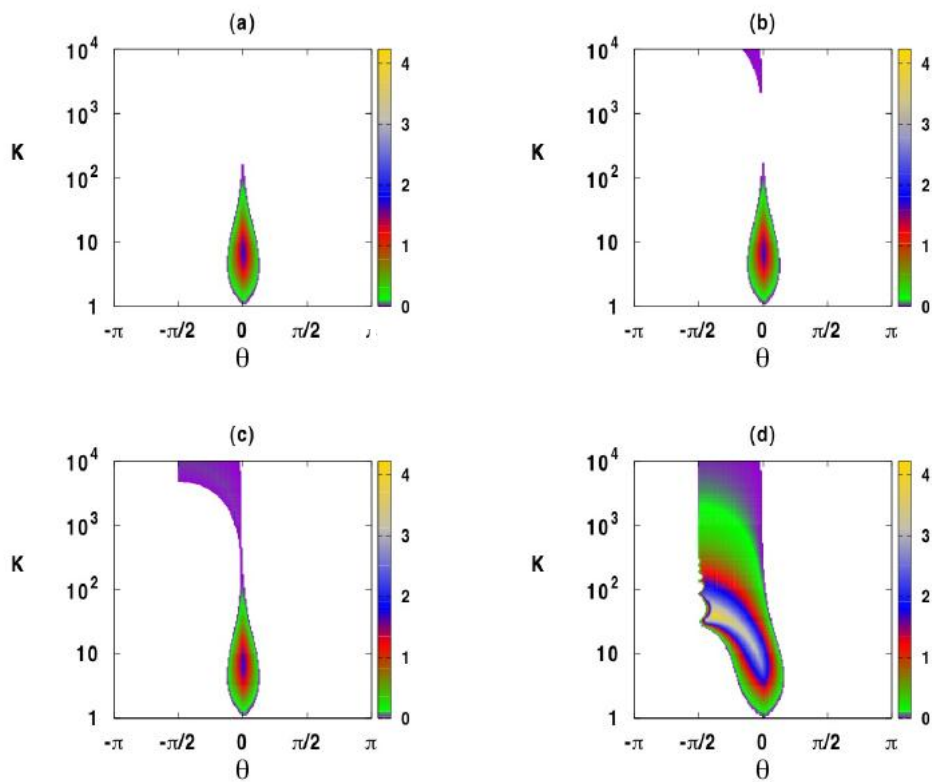


Fig. 2. Areas of amplitude death depending on the coupling strength K and the phase θ for $\tau = 0.08$ and $\omega_0 = 20$. Colour code denotes $[-\max\{\text{Re}(\lambda)\}]$. (a) $\rho = 0$. (b) $\rho = 0.002$. (c) $\rho = 0.006$; (d) $\rho = 0.026$.

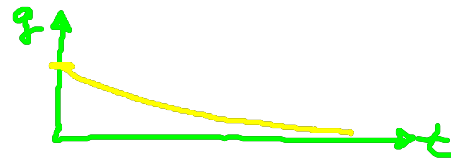
b) Exponentialverteilung

$$g(t) = \begin{cases} \alpha e^{-\alpha t} & (t \geq 0) \\ 0 & (t < 0) \end{cases}$$

$$\langle \tau \rangle := \int_0^{\infty} t' g(t') dt' = \frac{1}{\alpha}$$

$$\sigma^2 = \frac{1}{\alpha^2}$$

$$\mathcal{L}g(\lambda) = \frac{\alpha}{\lambda + \alpha}$$



$$\Rightarrow [\lambda^2 + \lambda(Ke^{i\theta} - 1 + \alpha - i\omega_0) - \alpha(1 + i\omega_0)] [\dots] = 0$$

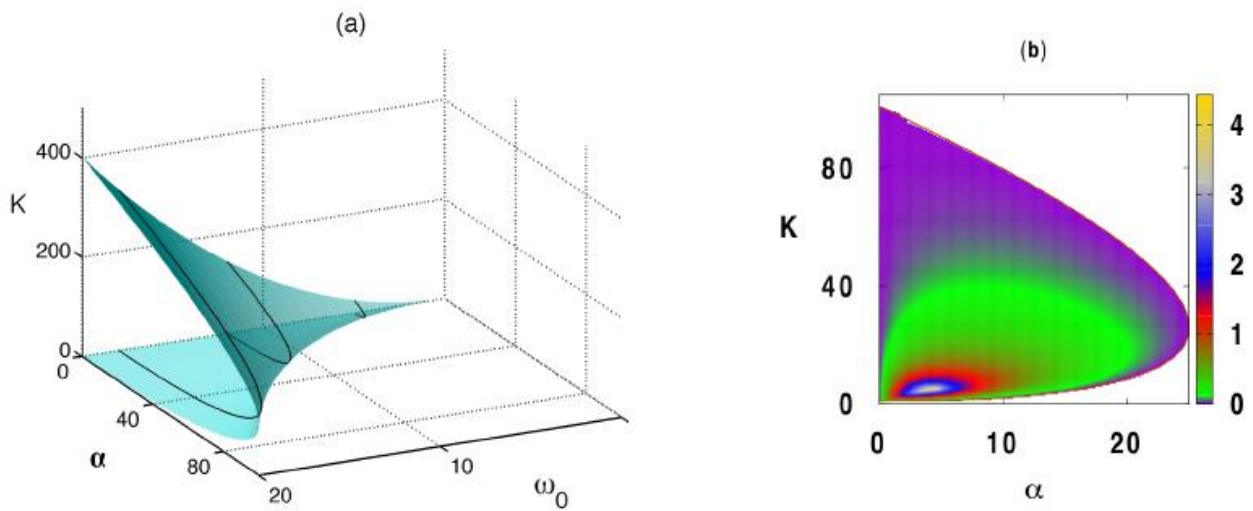


Fig. 4. (a) Stability boundary for the system (1) with a weak delay distribution kernel (13) for $\theta = 0$ ($p = 1$). The trivial steady state is unstable outside the boundary surface and stable inside the boundary surface. (b) Stability boundary for $\omega_0 = 10$. Colour code denotes $[-\max\{\text{Re}(\lambda)\}]$.

Exp. kern $\hat{=}$ Tiefpassfilter

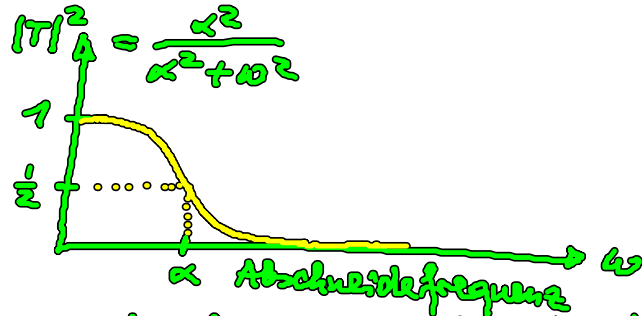
$$Y_1(t) = \int_0^{\infty} \alpha e^{-\alpha t'} e_1(t-t') dt' \quad \text{Gedächtnis}$$

$$Y_2(t) = \int_0^{\infty} \alpha e^{-\alpha t'} e_2(t-t') dt'$$

Def. Transferfkt. $T(\omega)$ im Fourier-Raum:

$$\hat{Y}(\omega) = \frac{\alpha}{\alpha + i\omega} \hat{z}(\omega)$$

$\underbrace{\hspace{2cm}}_{T(\omega)}$



Äquivalente Darstellung durch zusätzl. dyn. Var. Y_i :

$$\dot{e}_1 = (1 + i\omega_0) e_1 - |e_1|^2 e_1 + K e^{i\theta} (Y_2 - e_1) \quad (1)$$

$$\dot{e}_2 = (1 + i\omega_0) e_2 - |e_2|^2 e_2 + K e^{i\theta} (Y_1 - e_2) \quad (2)$$

$$\dot{Y}_1 = -\alpha Y_1 + \alpha e_1 \quad (3)$$

$$\dot{Y}_2 = -\alpha Y_2 + \alpha e_2 \quad (4)$$

Lösung von (3) bzw. (4) mittels Green'scher Fkt. und Substitution in (2) bzw. (1):

$$Y_1(t) = \int_{-\infty}^t e^{-\alpha(t-s)} \alpha e_1(s) ds \quad t-s = t'$$

$$Y_2(t) = \int_{-\infty}^t e^{-\alpha(t-s)} \alpha z_2(s) ds$$

c) allg. Gamma-Verteilung

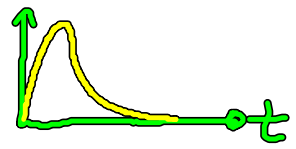
$$g(t) = \frac{t^{p-1} \alpha^p e^{-\alpha t}}{(p-1)!}$$

$$\alpha > 0, p \in \mathbb{N}$$

$$\Gamma(p+1) = p \Gamma(p) = p!$$

$$\Gamma(0) = 1$$

$p=1$: Exponentialverteilung (weak delay kernel)

$p=2$:  $g(t) = \alpha^2 t e^{-\alpha t}$ (strong delay kernel)

$$\langle \tau \rangle = \frac{2}{\alpha}, \quad \sigma^2 = \frac{2}{\alpha^2}$$

$$\hat{g}(\lambda) = \frac{\alpha^2}{(\lambda + \alpha)^2}$$

$$\dot{z}_1 = (1 + i\omega_0) z_1 - |z_1|^2 z_1 + K e^{i\theta} (Y_{12} - z_1)$$

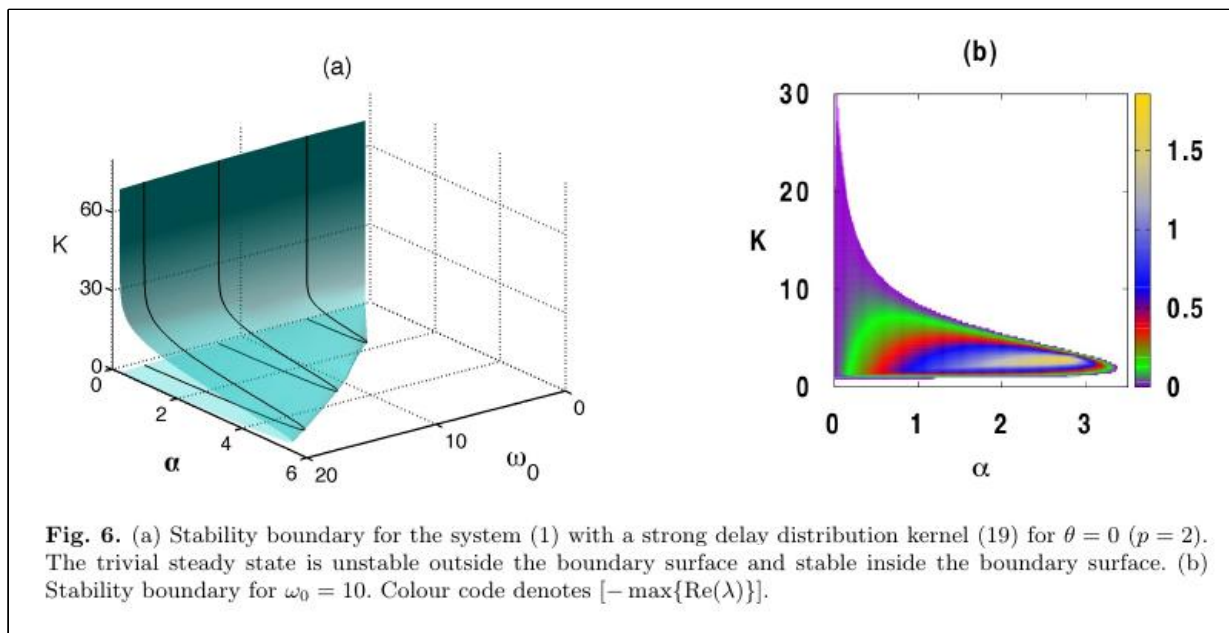
$$\dot{z}_2 = (1 + i\omega_0) z_2 - |z_2|^2 z_2 + K e^{i\theta} (Y_{22} - z_2)$$

$$\dot{Y}_{11} = -\alpha Y_{11} + \alpha z_1$$

$$\dot{Y}_{12} = \alpha Y_{11} - \alpha Y_{12} + \alpha^2 z_1$$

$$\dot{Y}_{21} = -\alpha Y_{21} + \alpha z_2$$

$$\dot{Y}_{22} = \alpha Y_{21} - \alpha Y_{22} + \alpha^2 z_2$$



• Zustandsabh. Verzögerungszeit (state-dep. delay)

zeitlich verändernde Verzögerungszeit (periodisch, stochast.);

A. Gijbels, F. J. Steutel, H. Van den Broek, Scholl: PRE 88, 052912 (2015)
" " , Zubizarreta, " : : 89, 052915 (2014)

für schnell modulierte $\tau \stackrel{\approx}{=} \text{verteilt}$ mit $q(t) = \text{Wahrscheinl. verteilung}$.