

# Dispersionrelation für Welle in einem Medium

$$k^2 = \epsilon(\omega) \frac{\omega^2}{c^2} \left(1 + i \frac{1}{\omega\tau}\right)$$

$$\tau = \frac{\epsilon\epsilon_0}{\sigma}$$

$$i(kx - \omega t)$$

$$E(x,t) = E_0 e$$

Isolator (disp. Dielektrikum):

$$k^2 \approx \epsilon(\omega) \frac{\omega^2}{c^2}$$

$$\tilde{n}(\omega) = n(\omega) + i\gamma(\omega)$$

$$\tilde{n}(\omega)^2 = \epsilon(\omega) \equiv \epsilon' + i\epsilon''$$

$$\gamma = \frac{1}{2} \sqrt{\epsilon'^2 + \epsilon''^2 - \epsilon'} \\ n = \frac{1}{2} \sqrt{\epsilon'^2 + \epsilon''^2 + \epsilon'}$$

Abs. koeff.  
reeller Brech. index

(i) Absorption

a)  $\epsilon'' = 0 \Rightarrow \gamma = 0$   
 $n = \sqrt{\epsilon'}$   $\Rightarrow$  ungedämpfte Welle

b)  $\epsilon'' > 0 \Rightarrow \gamma > 0$  (gedämpfte Welle)

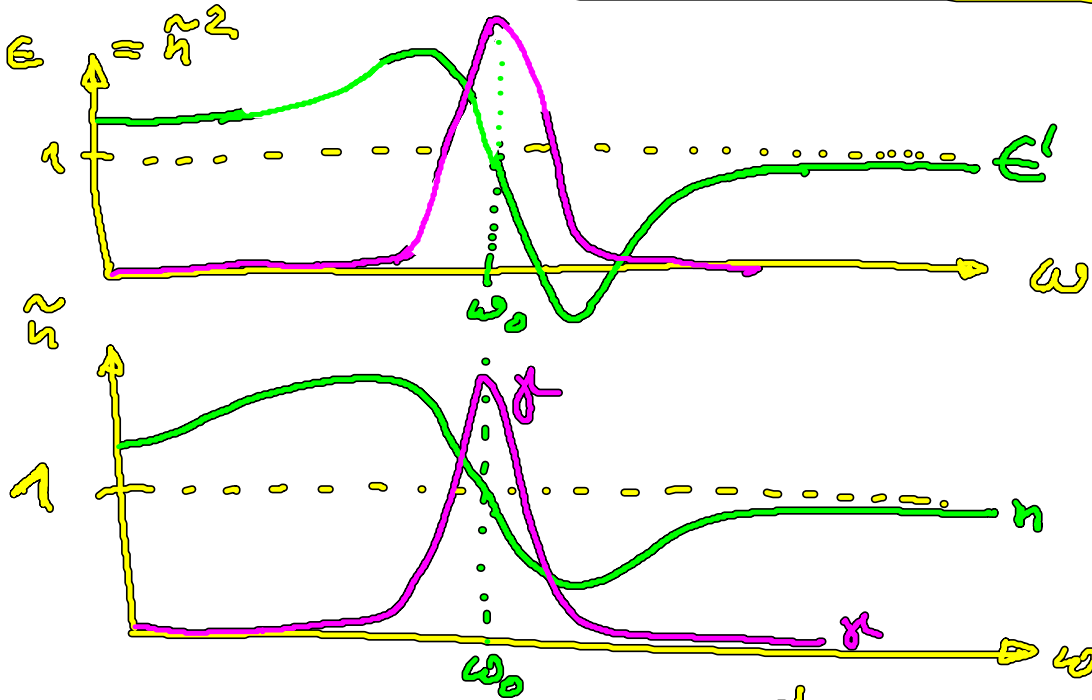
Transparenzgebiet der Substanz:  $\epsilon'' < \epsilon'$

(ii) Dispersion

$$\text{Re } k \equiv k' = \frac{\omega}{c} n(\omega) \Rightarrow \text{richtig. Dispersion}$$

Gruppengeschw.  $v_g := \frac{d\omega}{dk} = \frac{1}{\frac{dk}{d\omega}} = \frac{c}{\frac{d}{d\omega}[\omega n(\omega)]}$   
 Phasengeschw.  $v_{ph} := \frac{\omega}{k}$

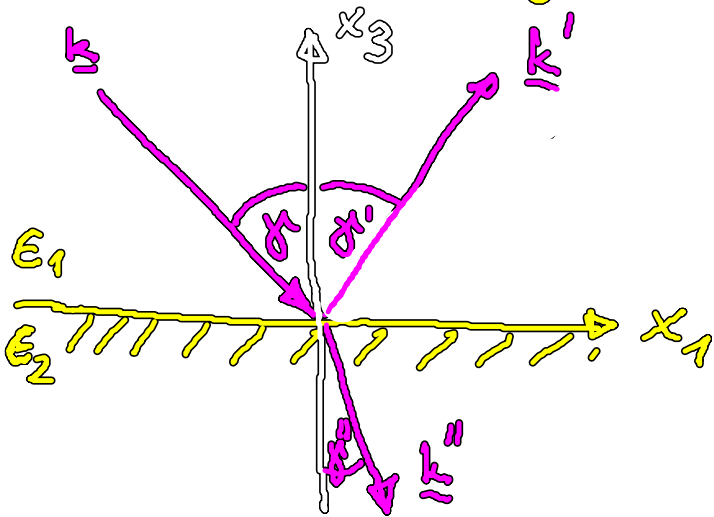
$$v_g = \frac{c}{n + \omega \frac{dn}{d\omega}} \neq \frac{c}{n(\omega)} = v_{ph}$$



normale Dispersion  $\frac{dn}{d\omega} > 0$  (Transparenzgebiet)

anomale Dispersion  $\frac{dn}{d\omega} < 0$  (Abs.)

# 5.7 Brechung und Reflexion



$$\epsilon_{1,2} \in \mathbb{R}$$

$$\frac{\omega}{c_1} = |k| \stackrel{\text{s.u.}}{=} |k'| = \frac{\omega}{c_1}$$

$$|k''| = \frac{\omega}{c_2}$$

$$c_i = \frac{c}{n_i} = \frac{c}{\sqrt{\epsilon_i}}$$

Einfallende Welle  
reflekt.  
transmitt. "

$$\underline{E} = \underline{E}_0 e^{i(k_n - \omega t)}$$

$$\underline{E}' = \underline{E}'_0 e^{i(k'_n - \omega t)}$$

$$\underline{E}'' = \underline{E}''_0 e^{i(k''_n - \omega t)}$$

Grenzbed. für  $\underline{E}$ :

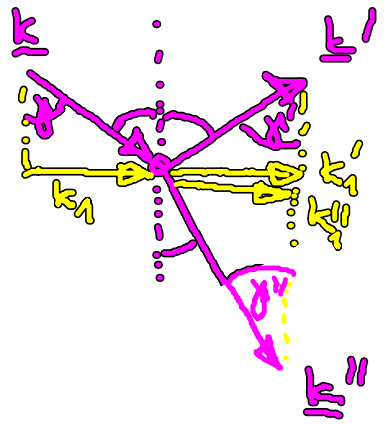
$$\left. \underline{E}_1 + \underline{E}'_1 \right|_{x_3=0} = \left. \underline{E}''_1 \right|_{x_3=0}$$

$$\Gamma = 0 : \underline{E}_{01} e^{-i\omega t} + \underline{E}'_{01} e^{-i\omega' t} = \underline{E}''_{01} e^{-i\omega'' t} \quad \forall t$$

$$\omega = \omega' = \omega''$$

$$E_{01} + E'_{01} = E''_{01}$$

$$t=0: E_{01} e^{ik_1 x_1} + E'_{01} e^{ik'_1 x_1} = E''_{01} e^{ik''_1 x_1} \quad \forall x_1$$



$$k_1 = k'_1 = k''_1$$

$$k \sin \gamma = k' \sin \gamma' = k'' \sin \gamma''$$

$$\frac{\omega}{c_1} \sin \gamma = \frac{\omega}{c_1} \sin \gamma' = \frac{\omega}{c_2} \sin \gamma''$$

Reflexionsgesetz

$$\sin \gamma = \sin \gamma'$$

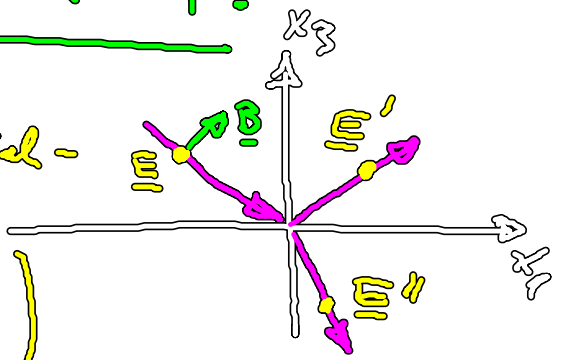
Brechungsgesetz  
(Snellius)

$$\frac{\sin \gamma''}{\sin \gamma} = \frac{c_2}{c_1} = \frac{n_1}{n_2}$$

Bestimmung der Amplituden:

$$(1) \quad E_{02} + E'_{02} = E''_{02}$$

Tangentenbed-komp.



$$\underline{B}_0 = \frac{c}{\omega} (\underline{k} \times \underline{E}_0) = \frac{c}{\omega} E_{02} \begin{pmatrix} -k_3 \\ 0 \\ k_1 \end{pmatrix}$$

$$B_{01} + B'_{01} = B''_{01} \Rightarrow k_3 E_{02} + k'_3 E'_{02} = k''_3 E''_{02}$$

Reflex.  $k_3 = -k'_3 \Rightarrow k_3 (E_{02} - E'_{02}) = k''_3 E''_{02} \quad (2)$

$$\Rightarrow \frac{E_{02}'}{E_{02}} = \frac{k_3 - k_3''}{k_3 + k_3''}$$

$$\frac{E_{02}''}{E_{02}} = \frac{2k_3}{k_3 + k_3''}$$

$$k_3'' = |k''| \cos \gamma'' = |k| \underbrace{\frac{n_2}{n_1}}_{\frac{\sin \gamma}{\sin \gamma''}} \cos \gamma''$$

$$k_3 = |k| \cos \gamma$$

$$\frac{\sin \gamma}{\sin \gamma''}$$

$$\frac{E_{02}'}{E_{02}} = \frac{\sin(\gamma'' - \gamma)}{\sin(\gamma'' + \gamma)}$$

Fresnel'sche  
Formeln

$$\frac{E_{02}''}{E_{02}} = \frac{2 \sin \gamma'' \cos \gamma}{\sin(\gamma'' + \gamma)}$$

Intensitätsverhältnisse

$$\langle \underline{S} \rangle = \frac{1}{T} \int_0^T dt \underline{E} \times \underline{H} \sim |\underline{E}_0|^2$$

Reflexionsvermögen

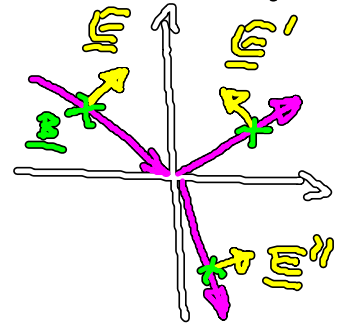
$$R_{\perp} = \left| \frac{E_{02}'}{E_{02}} \right|^2 = \frac{\sin^2(\gamma'' - \gamma)}{\sin^2(\gamma'' + \gamma)}$$

Transmissionsvermögen

$$T_{\perp} = \left| \frac{E_{02}''}{E_{02}} \right|^2 = \frac{4 \sin^2 \gamma'' \cos^2 \gamma}{\sin^2(\gamma'' + \gamma)} = 1 - R_{\perp}$$

## (b) Polarisation von $E \parallel$ Einfallsebene

$$R_{\parallel} = 1 - T_{\parallel} = \frac{\tan^2(\gamma'' - \gamma)}{\tan^2(\gamma'' + \gamma)}$$



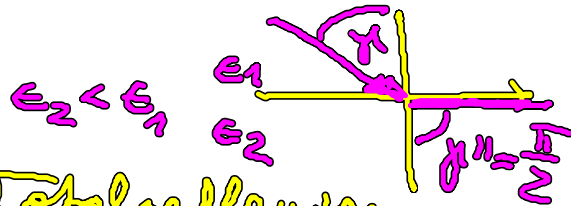
(i) Speziell  $R_{\parallel} = 0$  :  
 $\gamma'' + \gamma = \frac{\pi}{2} : \tan(\gamma'' + \gamma) = \infty \Rightarrow R_{\parallel} = 0$

Brewster-Winkel  $\gamma_B$  so, daß  $\tan \gamma_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$$\left( \tan \gamma = \frac{\sin \gamma}{\cos \gamma} = \frac{\sin \gamma}{\sin(\frac{\pi}{2} - \gamma)} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \right)$$



## (ii) Totalreflexion



Grenzwinkel der Totalreflexion

$$\gamma'' = \frac{\pi}{2}, \quad R_{\perp} = R_{\parallel} = 1$$

$$T_{\perp} = T_{\parallel} = 0$$

$$\frac{\sin \gamma}{\sin \gamma''} = \frac{\sin \gamma}{\sin \frac{\pi}{2}} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \Rightarrow \boxed{\sin \gamma_C = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

Für  $\gamma > \gamma_C : k_3'' = i/d$

$$\underline{E}'' = \underline{E}_0'' e^{-|x_3|/d} e^{i(k_1 x_1 - \omega t)}$$

