6.2 Feldoperatoren

$$\sum_{\lambda=1}^{\infty} a_\lambda^+ a_\lambda = \sum_{\lambda=1}^{\infty} \underbrace{\int \psi_\lambda(\underline{x}) \hat{\psi}^+(\underline{x})\, d\tau \int \psi_\lambda^*(\underline{x}')}_{=\delta(\underline{x}-\underline{x}')} \hat{\psi}(\underline{x}')\, d\tau'$$

$$= \int \delta(\underline{x}-\underline{x}') \hat{\psi}^+(\underline{x}) \hat{\psi}(\underline{x}')\, d\tau\, d\tau' = \int \hat{\psi}^+(\underline{x}) \hat{\psi}(\underline{x})\, d\tau$$

$$\hat{H} = \sum_{\lambda,\mu}^{1\ldots\infty} A_{\lambda\mu}\, a_\lambda^+ a_\mu = \sum_{\lambda\mu}^{1\ldots\infty} a_\lambda^+ \int \psi_\lambda^*(\underline{x}) A(\underline{x}) \psi_\mu(\underline{x})\, d\tau\; a_\mu$$

$$= \sum_{\lambda,\mu}^{1\ldots\infty} \int \underbrace{\psi_\lambda(\underline{x}') \hat{\psi}^+(\underline{x}') \cdot \psi_\lambda^*(\underline{x})}_{\delta(\underline{x}'-\underline{x})} A(x) \underbrace{\psi_\mu(x)\, \psi_\mu^*(\underline{x}'')}_{\delta(\underline{x}-x'')} \hat{\psi}(\underline{x}'')\, d\tau\, d\tau'\, d\tau''$$

$$= \int \delta(\underline{x}'-x)\, \delta(\underline{x}-\underline{x}'')\, \hat{\psi}^+(\underline{x}')\, A(\underline{x})\, \hat{\psi}(\underline{x}'')\, d\tau\, d\tau'\, d\tau''$$

$$= \int \hat{\psi}^+(\underline{x}) A(\underline{x}) \hat{\psi}(\underline{x})\, d\tau$$

$$U^+ = \exp\left\{\frac{i}{\hbar} H(t-t_0)\right. \quad ; \quad \dot{U} = -\frac{i}{\hbar} H U$$

$$\varrho = U \varrho_0 U^+ \qquad\qquad \dot{U}^+ = \frac{i}{\hbar} H U^+$$

$$\frac{\partial \varrho}{\partial t} = \dot{U} \varrho_0 U^+ + U \varrho_0 \dot{U}^+ = -\frac{i}{\hbar} H \underbrace{U \varrho_0 U^+}_{\varrho} + \frac{i}{\hbar} \underbrace{U \varrho_0 H U^+}_{= U\varrho_0 U^+ H = \varrho H}$$

$$= -\frac{i}{\hbar} [H, \varrho]$$

$$[\hat{\psi}(\underline{x},t), \hat{A}(t)] = \int [\hat{\psi}, \hat{\psi}^{+\prime} A' \hat{\psi}'] \, d\tau'$$

$$= \int \left(\hat{\psi} \hat{\psi}^{+\prime} A' \hat{\psi}' - \hat{\psi}^{+\prime} A' \hat{\psi}' \hat{\psi} \right) d\tau'$$

$$= \int \left(\hat{\psi}^{+\prime} \hat{\psi} A' \hat{\psi}' + \delta(\underline{x}-\underline{x}') A' \hat{\psi}' - \hat{\psi}^{+\prime} \hat{\psi} A' \hat{\psi}' \right) d\tau' = A \hat{\psi}$$