

6.2 Feldoperatoren

$$\begin{aligned} \sum_{\lambda=1}^{\infty} a_{\lambda}^{\dagger} a_{\lambda} &= \sum_{\lambda=1}^{\infty} \int \psi_{\lambda}(\underline{x}) \hat{\psi}^{\dagger}(\underline{x}) d\tau \underbrace{\int \psi_{\lambda}^*(\underline{x}') \hat{\psi}(\underline{x}') d\tau'}_{=\delta(\underline{x}-\underline{x}')} \\ &= \int \delta(\underline{x}-\underline{x}') \hat{\psi}^{\dagger}(\underline{x}) \hat{\psi}(\underline{x}') d\tau d\tau' = \int \hat{\psi}^{\dagger}(\underline{x}) \hat{\psi}(\underline{x}) d\tau \end{aligned}$$

$$\begin{aligned} \hat{H} &= \sum_{\lambda, \mu}^{1, \dots, \infty} A_{\lambda\mu} a_{\lambda}^{\dagger} a_{\mu} = \sum_{\lambda, \mu}^{1, \dots, \infty} a_{\lambda}^{\dagger} \int \psi_{\lambda}^*(\underline{x}) A(\underline{x}) \psi_{\mu}(\underline{x}) d\tau a_{\mu} \\ &= \sum_{\lambda, \mu}^{1, \dots, \infty} \int \underbrace{\psi_{\lambda}(\underline{x}') \hat{\psi}^{\dagger}(\underline{x}') \psi_{\lambda}^*(\underline{x}) A(\underline{x}) \psi_{\mu}(\underline{x})}_{\delta(\underline{x}'-\underline{x})} \underbrace{\psi_{\mu}^*(\underline{x}'') \hat{\psi}(\underline{x}'')}_{\delta(\underline{x}-\underline{x}'')} d\tau d\tau' d\tau'' \\ &= \int \delta(\underline{x}'-\underline{x}) \delta(\underline{x}-\underline{x}'') \hat{\psi}^{\dagger}(\underline{x}') A(\underline{x}) \hat{\psi}(\underline{x}'') d\tau d\tau' d\tau'' \\ &= \int \hat{\psi}^{\dagger}(\underline{x}) A(\underline{x}) \hat{\psi}(\underline{x}) d\tau \end{aligned}$$

$$U^{\dagger} = \exp\left\{ \frac{\varepsilon}{\hbar} H(t-t_0) \right\} ; \quad \dot{U} = -\frac{\varepsilon}{\hbar} H U$$

$$\vartheta = U \vartheta_0 U^{\dagger}$$

$$\dot{U}^{\dagger} = \frac{\varepsilon}{\hbar} H U^{\dagger}$$

$$\begin{aligned} \frac{\partial \vartheta}{\partial t} &= \dot{U} \vartheta_0 U^{\dagger} + U \vartheta_0 \dot{U}^{\dagger} = -\frac{\varepsilon}{\hbar} H \underbrace{U \vartheta_0 U^{\dagger}}_{\vartheta} + \frac{\varepsilon}{\hbar} \underbrace{U \vartheta_0 H U^{\dagger}}_{= U \vartheta_0 U^{\dagger} H = \vartheta H} \\ &= -\frac{\varepsilon}{\hbar} [H, \vartheta] \end{aligned}$$

$$[\hat{\psi}(x, t), \hat{A}(t)] = \int [\hat{\psi}, \hat{\psi}^{\dagger'} A' \hat{\psi}'] d\tau'$$

$$= \int (\hat{\psi} \hat{\psi}^{\dagger'} A' \hat{\psi}' - \hat{\psi}^{\dagger'} A' \hat{\psi}' \hat{\psi}) d\tau'$$

$$= \int (\underbrace{\hat{\psi}^{\dagger'} \hat{\psi} A' \hat{\psi}' + \delta(x-x')}_{A'} A' \hat{\psi}' - \hat{\psi}^{\dagger'} \hat{\psi} A' \hat{\psi}') d\tau' = A \hat{\psi}$$