7.1 Quantisierung freier elektromagnetischer Felder

$$\hat{A} = \frac{1}{\sqrt{2}} \sqrt{\frac{\hbar}{\varepsilon\omega}} \frac{1}{\sqrt{V}} \vec{u} \left[e^{i\vec{q}\cdot\vec{r}} c(t) + e^{-i\vec{q}\cdot\vec{r}} c^+ \right]$$

$$\hat{\pi} = \frac{1}{\sqrt{2}} \sqrt{\frac{\hbar}{\varepsilon\omega}} \frac{1}{\sqrt{V}} \vec{u} \left[-i\omega e^{i\vec{q}\cdot\vec{r}} c + i\omega e^{-iq\vec{r}} c^+ \right]$$

$$[\hat{A}_k, \hat{\pi}_l] = \frac{1}{2} \frac{\hbar}{\varepsilon\omega} \frac{1}{V} \underbrace{u_k u_l}_{\delta_{kl}} i\omega \left[\underbrace{-e^{iq(\vec{r}+\vec{r}')} c^2}_{0} + \underbrace{e^{iq(\vec{r}-\vec{r}')} c c^+ - e^{-iq(r-\vec{r}')} c^+ c}_{\delta(\vec{r}-\vec{r}')} - \underbrace{\ldots c^+ c^+}_{0} \right]$$

$$[c, c^+] = 1 \quad , \Rightarrow \quad c^+ c = c c^+ - 1$$

$$[\hat{A}_k, \hat{\pi}_l] = \frac{\hbar}{2} \delta_{kl} \delta(\vec{r}-\vec{r}')$$

$$\hat{A} = \frac{1}{\sqrt{2}} \sqrt{\frac{\hbar}{\varepsilon\omega}} \frac{\vec{u}}{\sqrt{V}} \left[a c + a^* c^+ \right] \; ; \; a = \exp\{i(\vec{q}\cdot\vec{r} - \omega t)\} \; , \vec{u}^2 = 1$$

$$\hat{H} = \varepsilon \dot{\hat{A}}^2 = \frac{1}{2} \frac{\hbar}{\cancel{\varepsilon}\omega} \frac{1}{V} (i\omega)^2 \left[(-ac + a^* c^+)(-ac + a^* c^+) \right] \; ; \quad a a^* = 1$$

$$= \ldots\ldots\ldots \left[-c c^+ - c^+ c \right] \qquad , \qquad \boxed{c c^+ = c^+ c + 1}$$

$$= \frac{1}{2} \frac{\hbar}{\cancel{\varepsilon}\omega} \frac{1}{V} (-\omega^2) \left(-(2c^+ c + 1) \right]$$

$$= \frac{1}{2} \frac{\hbar\omega}{\cancel{\varepsilon}} \frac{1}{V} (2 c^+ c + 1) = \frac{1}{\cancel{2}V} \hbar\omega \left(c^+ c + \frac{1}{2} \right)$$

$$[c^+ c, c] = -c \quad ; \quad [c^+ c, c^+] = c^+$$

$$\frac{\partial L}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + e\vec{A} = \vec{p} \quad , \quad \frac{\partial L}{\partial \vec{r}} = e \frac{\partial}{\partial \vec{r}} \dot{\vec{r}} \cdot \vec{A}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\vec{r}}} = m\ddot{\vec{r}} + e\dot{\vec{r}} \cdot \nabla \vec{A} - e \frac{\partial}{\partial \vec{r}} \dot{\vec{r}} \cdot \vec{A} \qquad \Big| \, m\ddot{\vec{r}} = e[\vec{E} + \dot{\vec{r}} \times \vec{B}]$$

$$e\dot{\vec{r}} \times (\nabla \times \vec{A}) = e\nabla \dot{\vec{r}} \cdot \vec{A} - e\dot{\vec{r}} \cdot \nabla \vec{A}$$

$$\int_V |\psi_n(\vec{k},\vec{r})|^2 \, d^3r = \frac{1}{N^3} \int_V |u_n(\vec{k},\vec{r})|^2 \, d^3r$$

$$= \frac{N^3}{N^3} \int_\Omega |u_n(\vec{k},\vec{r})|^2 \, d^3r = 1$$

Grundgebiet

Ω, $V = N^3 \Omega$