# 6.3 Zeitabhängige Feldoperatoren

Bisher Teilchenzahlzustände $|n_1 n_2 \dots\rangle$
und Erzeugungs- und Vernichtungsoperatoren $a_\lambda^+, a_\lambda$
und Feldoperatoren $\hat\psi(\underline{x})$ und $\hat\psi^+(\underline{x})$.

$[H, U] = 0 = [H, U^+]$

$$\dot\rho = \frac{\partial}{\partial t}\left(U \rho_0 U^+\right) = \dot U \rho_0 U^+ + U \rho_0 \dot U^+$$

$$= -\frac{i}{\hbar} H U \rho_0 U^+ + U \rho_0 \frac{i}{\hbar} H U^+ = -\frac{i}{\hbar} H\rho + \frac{i}{\hbar} H\rho = \frac{i}{\hbar}[H,\rho]$$

Falls $H(\underline{x},t)$ $\exists$ $U$ mit $U H U^+ \psi = E U U^+ \psi$

$H\psi = E\psi$

$\underline{U U^+} H\psi = E\psi$

$H U^+\psi = E U^+\psi$

$\mathbb{1}\; H\psi = E\psi$

$[H, U] = 0$

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$$\frac{\partial}{\partial t} A_H = \frac{\partial}{\partial t}\left(U^+ A U\right) = \dot U^+ A U + U^+ \dot A U + U^+ A \dot U$$

$$= \frac{i}{\hbar} H U^+ A U + U^+ \dot A U + U^+ A \left(\frac{i}{\hbar}\right) H U$$

$$= \frac{i}{\hbar} H A_H - \frac{i}{\hbar} A_H H + U^+ \dot A U = \frac{i}{\hbar}[H, A_H] + U^+ \dot A U$$

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$$\left(-\frac{\hbar}{i}\frac{\partial}{\partial t}\hat\psi^+\right)^+ = \frac{\hbar}{i}\frac{\partial}{\partial t}\hat\psi = -\left([\hat\psi^+, \hat A]\right)^+ = \left(\hat\psi^+\hat A - \hat A\hat\psi^+\right)^+ \qquad (A^+ = A)$$

$$= \hat A^+\hat\psi - \hat\psi\hat A^+ = \hat A\hat\psi - \hat\psi\hat A = -[\hat\psi, \hat A]$$

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$$[\hat{\psi}, \hat{A}] = \int [\hat{\psi}, \hat{\psi}^+(x') A(x',t) \psi(x')] d\tau'$$

$$= \int \hat{\psi}\,\hat{\psi}^+(x') A(x') \psi(x') d\tau' - \int \hat{\psi}^+(x') A(x') \psi(x') \hat{\psi}(x) d\tau'$$

$$= \int \hat{\psi}^+(x') \hat{\psi}(x) A(x') \psi(x) d\tau' + \int A(x') \psi(x') \delta(x-x') d\tau - \downarrow$$

$$= \int \hat{\psi}^+(x') A(x') \psi(x') \hat{\psi}(x) d\tau' + A(x) \psi(x) - \downarrow = A(x)\hat{\psi}(x)$$