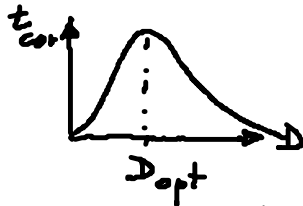


## English Summary:

### 3.3 Coherence resonance



e.g. FHN model

### 3.4 Correlation time and power spectral density

Self-consistent mean-field approximation for VolP oscillator:

$$t_{coh}(D) = \frac{4}{\pi |\tilde{E}(D)|}, \quad \tilde{E}(D) = \frac{\varepsilon}{2} \left( 1 + \sqrt{1 + \frac{2D^2}{\varepsilon^2 \omega_0^2}} \right)$$

Half-width of power spectral peak:  $\Delta\omega \approx \frac{|\tilde{E}|}{2} = \frac{2}{\pi t_{coh}(D)}$

### 3.5 Kohärenzresonanz in anregbaren Systemen vom Typ I

#### 3. Beispiel: SNIPER-Modell (Anregbarkeit Typ I)

(saddle-node infinite period bifurcation)

- knapp unterhalb einer globalen Bif. (Amplitude endl.,  $\omega \rightarrow 0$ )  
Hopf: Amplitude  $\rightarrow 0$ ,  $\omega$  endl.

$\equiv$  SNIC  $\triangleq$  saddle-node bif. on invariant cycle

$\equiv$  Sattel-Knoten-Bif. auf Grenzzyklus

gang et al.: PRL 71, 807 (1993): Kohärenzresonanz erstmals  
Aust, Hövel, Hizanides, Schöll: Eur. Phys. J - ST 127, 77 (2010) mit delay

$$\begin{cases} \dot{x} = x(1-x^2-y^2) + y(x-b) + D\zeta(t) \\ \dot{y} = y(1-x^2-y^2) - x(x-b) + D\zeta(t) \end{cases}$$

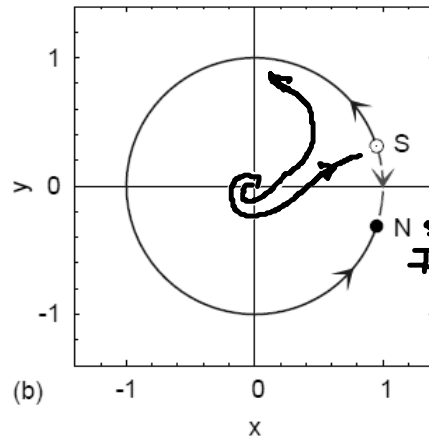
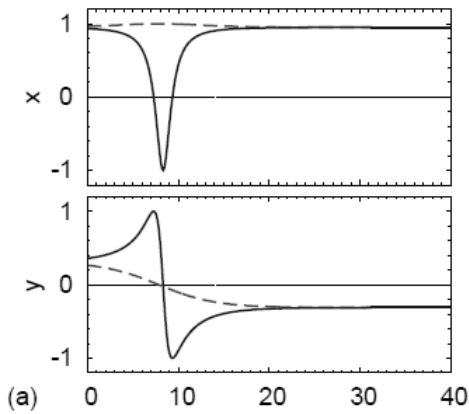
$D=0$ :  $\begin{cases} \dot{r} = r(1-r^2) \\ \dot{\varphi} = b - r\cos\varphi \end{cases} \quad \begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$

- Fixp. (i)  $r=0$  (instab. Fokus)  
 (ii)  $r=1, \varphi = \arccos b$  (Sattelp. S)  
 (iii)  $r=1, \varphi = -\arccos b$  (stab. Knoten N) } ex. falls  $b < 1$

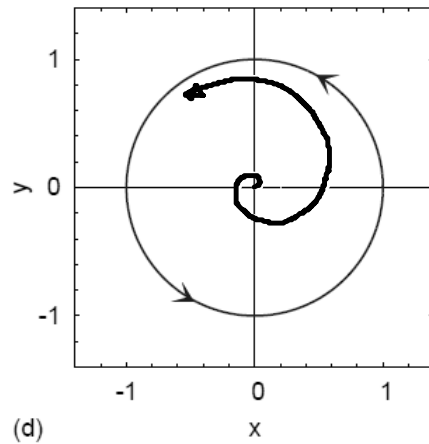
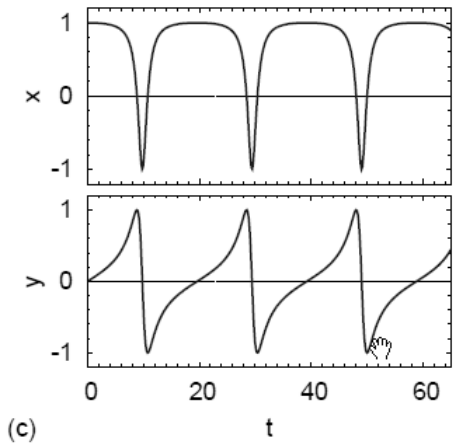
$b=1$ : Sattel-Knoten-Bif. auf dem invarianten Kreis  $r=1$   
 $(r=1, \dot{r}=0, \dot{\varphi} = b - \cos\varphi) \quad \dot{\varphi}=0$  für  $\varphi=0$

$b > 1$ : Grenzzyklus  $r=1$ ,  $\int \frac{d\varphi}{b - \cos\varphi} = t \Rightarrow \varphi(t)$  analyt.  
 $T = \frac{2\pi}{\sqrt{b^2-1}} \xrightarrow{b \rightarrow 1} \infty$

# Anregbares Regime : $b < 1$



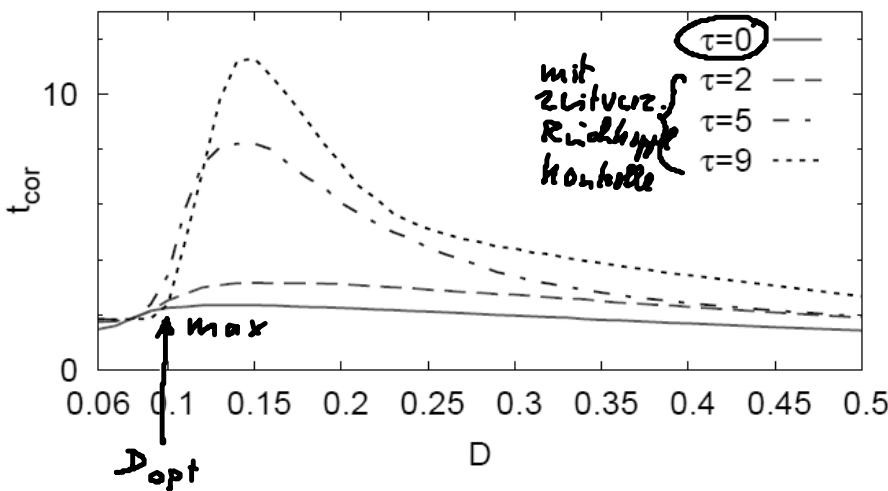
Anst et al. (2010)  
 Sattel = S  
 = Schwelle S  
 (für Anregung)  
 N stab. Fixp.  
 $b < 1$



LC  
 limit cycle  
 $b > 1$

Fig. 1. (Color online) Time series (a),(c) and phase portraits (b),(d) of the analytic solutions of Eqs. (2). Panels (a),(b) refer to  $b = 0.95 < 1$  with analytic solutions given by Eq. (7); full (blue) and broken (red) lines refer to two different initial conditions. Panels (c),(d) correspond to  $b = 1.05 > 1$ , see Eq. (8). ( $K = D = 0$ )

# $D \neq 0$ : Kohärenzresonanz



Anst et al (2010)

Fig. 4. (Color online) Correlation time in dependence on the noise intensity  $D$  for different time delays  $\tau$ . The solid (green) curve corresponds to the uncontrolled system ( $\tau = 0$ ). The dashed (red), dash-dotted (blue), and dotted (black) curves refer to values of  $\tau = 2, 5$ , and  $9$ , respectively. Other parameters:  $b = 0.95$  and  $K = 0.25$ .

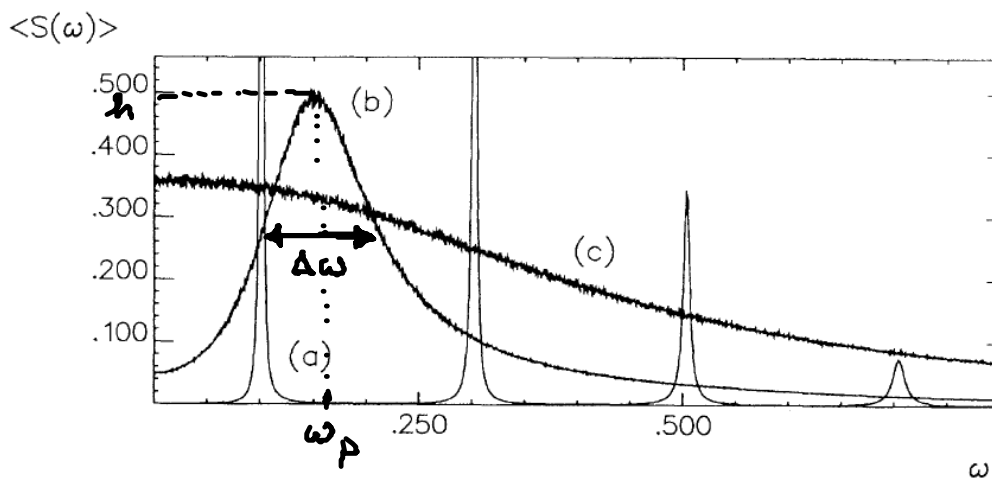


FIG. 1. The averaged power spectrum of  $y(t)$  for  $b=1.05$  and various  $D$ . A limit cycle exists for the deterministic system. (a)  $D=0.00003$ . (b)  $D=0.05$ . (c)  $D=0.9$ .

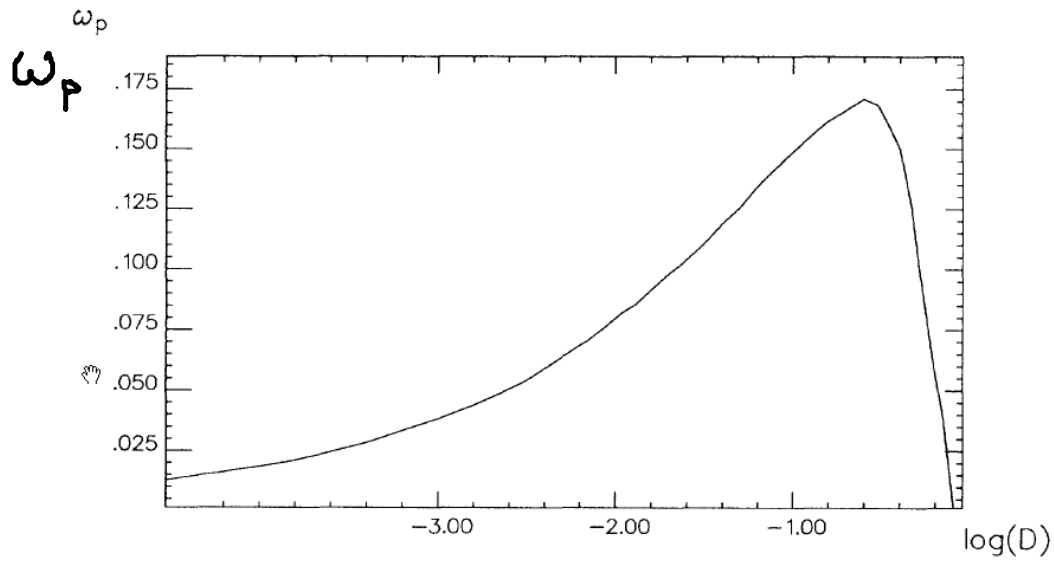


FIG. 3. The center frequency of the spectrum peak  $\omega_p$  vs  $\log(D)$  with  $b=1$  (the same in Figs. 4 and 5).

$h$

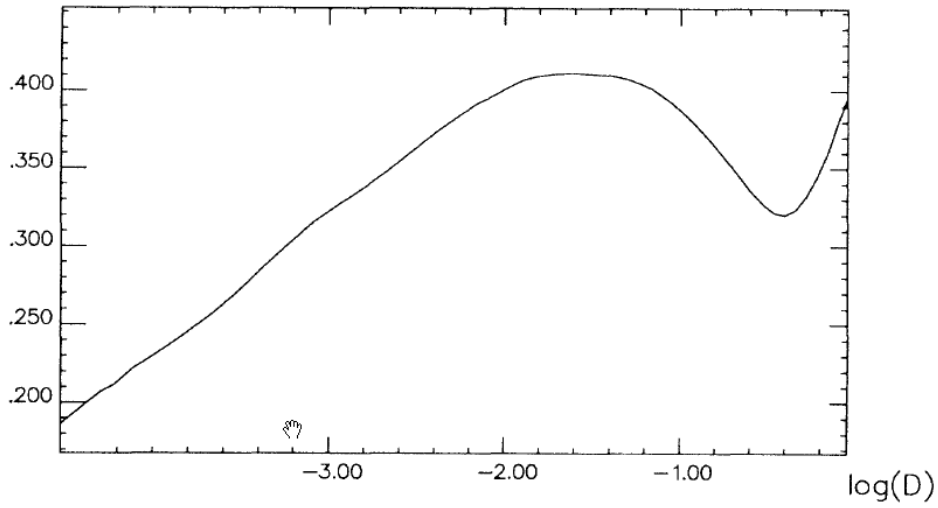
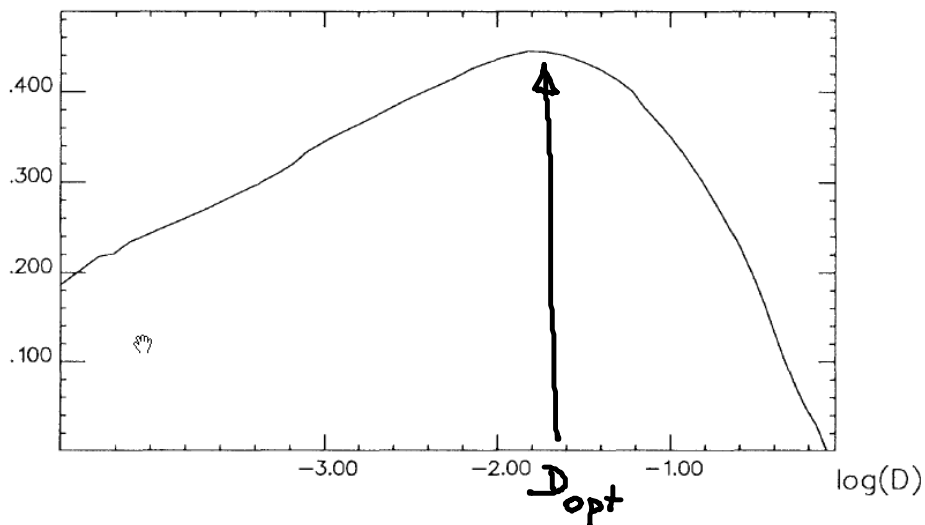


FIG. 4. The spectrum peak height  $h$  plotted against  $\log(D)$ . A resonancelike curve is obvious. However,  $h$  goes up again for very large  $D$ .

$\beta$



$$\beta = \frac{h}{\Delta\omega/\omega_p}$$

FIG. 5. The SNR  $\beta = h(\Delta\omega/\omega_p)^{-1}$  vs  $\log(D)$ . A stochastic resonance maximum can be seen.

### 3.6 Kohärenzresonanz in nicht anregbaren Systemen

Kohärenzresonanz in der Nähe einer subkrit. Hopf-Bif.:

- Geffert, Zakharova, Vüllings, Just, Schöll: Eur. Phys. J. B 87, 291 (14)
- Ushakov, Wünsche, Henneberger, Khovanov, Schimansky-Gies, Zakharova: PRL 95, 123903 (2005) exp. + theor. } gener. Stuart-Landau
- Zakharova, Vadivasova, Anishchenko, Koseska, Kurths: PRE 81, 011106 (2010) } gener. VdP Osc.
- Zakharova, Fedtistov, Vadivasova, Schöll: Eur. Phys. J. ST 222, 2481 (13)
- Semenov, Fedtistov, Vadivasova, Schöll, Zakharova: Chaos 25, 033111 (15)

Normalform einer superkrit. Hopf-Bif.:

$$\dot{z} = (\lambda + i\omega - |z|^2)z + \sqrt{2D} \zeta(t) \quad z = r e^{i\varphi} \in \mathbb{C}$$

Stuart-Landau-Oszillator

$$\begin{cases} \dot{r} = (\lambda - r^2)r \\ \dot{\varphi} = \omega \end{cases} \Rightarrow \varphi = \omega t$$

D=0 : Fixp.  $z=0$

Eigenwert  $\lambda + i\omega$

$\lambda < 0$  : stabiler Fokus

Hopf-Bif. bei  $\lambda=0$

$\lambda > 0$  : instab. Fokus + stabiler Grenzzyklus  $r = \sqrt{\lambda}$

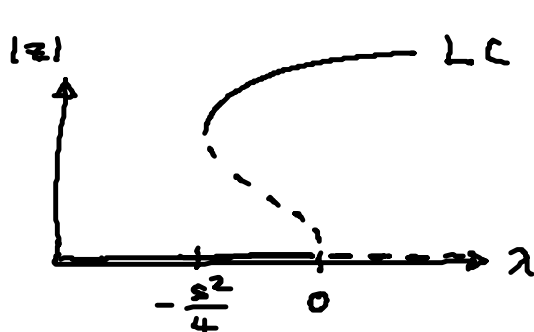
Normalform einer subkrit. Hopf-Bif.:

$$\dot{z} = (\lambda + i\omega + s|z|^2 - |z|^4)z + \sqrt{2D} \zeta(t) \quad s=1$$

verallgem. Stuart-Landau-OSE.

D=0 :  $\begin{cases} \dot{r} = (\lambda + sr^2 - r^4)r \\ \dot{\varphi} = \omega \end{cases}$  Fixp.  $r=0$  stabil für  $\lambda < 0$

Grenzzyklen ( $r \neq 0$ ):  $r^4 - sr^2 - \lambda = 0 \Rightarrow r^2 = \frac{s}{2} \left( 1 \pm \sqrt{1 + \frac{4\lambda}{s^2}} \right)$



Sattel-Knoten-Bif. eines stabilen u. instab. LC bei  $\lambda = -\frac{s^2}{4}$

Bistabilität zwischen Fixp.  $z=0$  und LC mit  $r^2 = \frac{s}{2} \left( 1 + \sqrt{1 + \frac{4\lambda}{s^2}} \right)$

für  $-\frac{s^2}{4} \leq \lambda \leq 0$

$D \neq 0$  : stationäre Lösung der FP-Gl.

$$P(r) = \sqrt{r} \exp\left\{ \frac{r^2}{D} \left( \frac{\lambda}{2} + \frac{sr^2}{3} - \frac{r^4}{6} \right) \right\}$$

wegen Normierung  
in Polarkoord.

$-\frac{U(r)}{D}$  ,  $-U'(r) = A(r)$  Drift

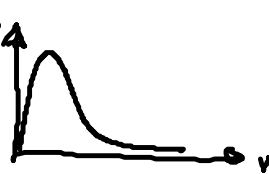
Amplituden-  
Wahrscheinl. verteilung  
(radialsymm.)

$$\iint p(r, \varphi) r dr d\varphi = \int p 2\pi r dr$$

$P(r)$  Amplituden-Wahrscheinl. dichte für Ring mit Radius  $r$

Stochast. Bifurkation :  $P(r)$  unimodal  
(Phänomenol. Bif. = P-Bif.)

→ binodal



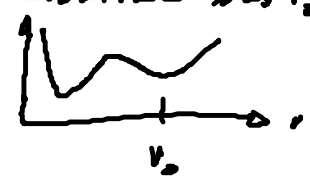
Übergang:

Wendepkt. v.  $U$  mit hor. Tang.

transkrit. DSE.  
im Ursprung



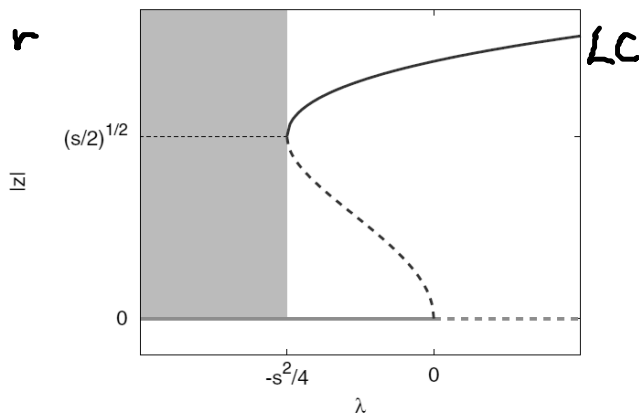
transkrit. stoch. LC bei  $r_0$



$$(\ln P)' = 0 : -\frac{D}{r} - 2r - sr^3 + r^5 = 0$$

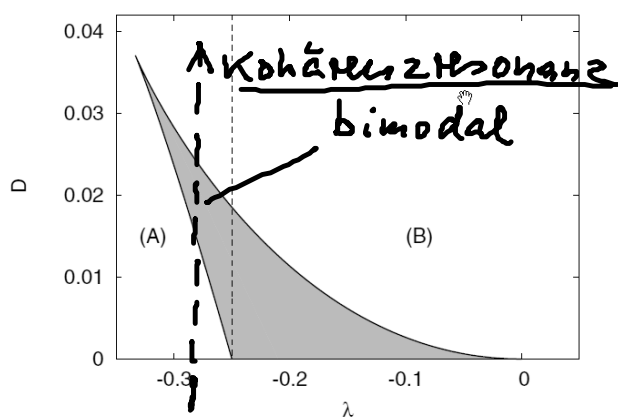
$$(\ln P)'' = 0 : \frac{D}{r^2} - 2 - 3sr^2 + 5r^4 = 0$$

$$\rightarrow r^2 = -\frac{9D + 2s}{6r + 2s^2} > 0$$

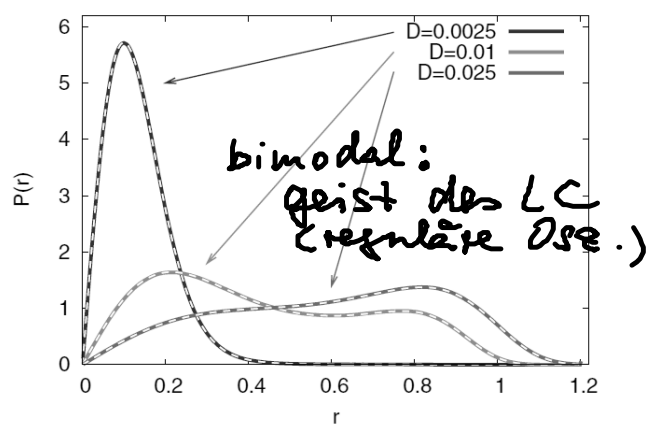


Geffest et al (2014)

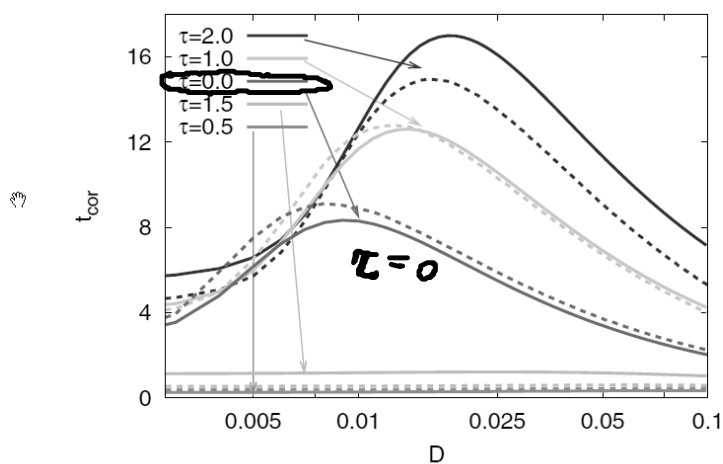
Fig. 1. The bifurcation diagram of the deterministic system without time delay, equation (1) for  $K = 0$  and  $D = 0$ , in the subcritical regime  $s > 0$ . Bronze (grey): trivial fixed point (solid: stable, dashed: unstable), blue (black): periodic orbit (solid: stable, dashed: unstable), dotted line: location of the saddle-node bifurcation of periodic orbits. The shaded region is the parameter region of interest for coherence resonance.



**Fig. 2.** Stochastic bifurcation diagram of the stochastic differential equation without delay equation (1), corresponding to equation (5) for  $s = 1$ . The blue shaded area denotes the parameter values, where the probability distribution has a bimodal shape. The dashed line shows the border between the deterministic monostable (A) and the deterministic bistable (B) regime.



**Fig. 3.** Stationary amplitude probability distribution of the stochastic delay differential equation for  $\omega = 2\pi$ ,  $\lambda = 0.18$ ,  $s = 1$ ,  $\tau = 0.25$ ,  $K = 0.5$  and three values of the noise strength  $D$  as obtained from numerical simulations of equation (1) (solid lines). Dashed lines (white) show the respective result of an analytical approximation scheme (Eq. (9)). The analytical approximation is in excellent agreement with the simulation results.



kernelzeit

**Fig. 8.** Influence of the time delay on the correlation time  $t_{cor}$ . The dashed lines correspond to numerical simulations of equation (1), the solid lines show the approximation, calculated from equation (27) with  $\ell = \ell' = 0$ . Parameters:  $\omega = 2\pi$ ,  $\lambda = -0.26$ ,  $K = 0.5$ ,  $s = 1$ .

Spektrale Leistungsdiichte

$$\lambda = -0.26$$

$$\lambda = -0.35$$

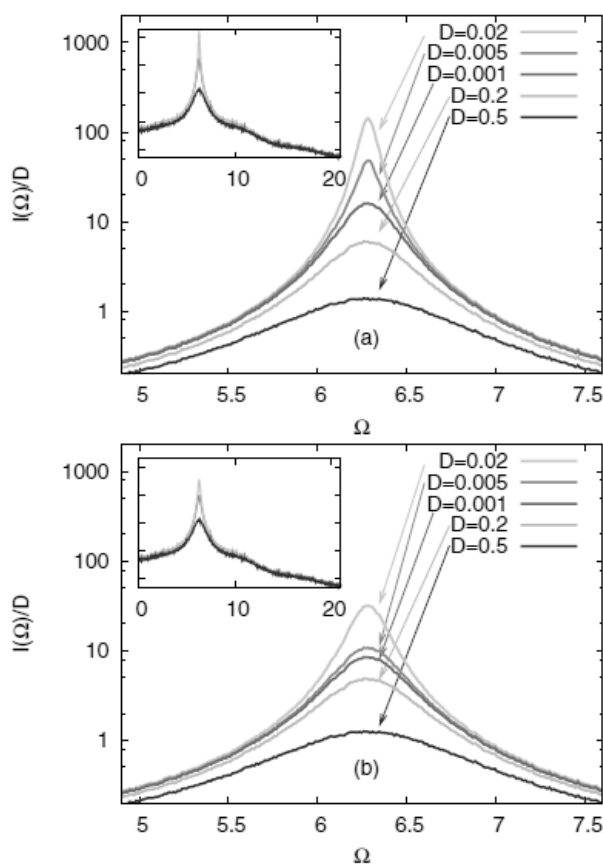


Fig. 6. Normalised power spectral density  $I(\Omega)/D$  in the vicinity of the central peak at  $\Omega = 2\pi$ , as obtained from a numerical simulation of equation (1) for increasing values of the noise intensity  $D$ ,  $\tau = 1.0$ ,  $\omega = 2\pi$ ,  $K = 0.5$ ,  $s = 1$  and two different values of  $\lambda$  (a)  $\lambda = -0.26$ , (b)  $\lambda = -0.35$  (see Fig. 5). The insets show the structure of the power spectral densities for  $D = 0.001$ ,  $D = 0.01$  and  $D = 0.1$  on a larger scale.

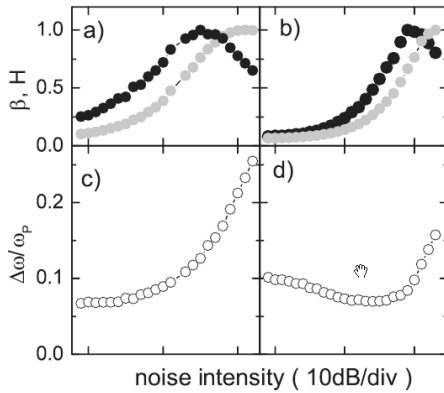


FIG. 2. Peak height (gray dots), normalized width (open dots), and signal-to-noise ratio (black dots) versus noise intensity as deduced from the power spectra by Lorentzian fits. Note the log scale. (a),(c) Supercritical bifurcation, (b), (d) Subcritical bifurcation. Maxima of curves are normalized to unity.

Exp.  
(Ushakov et al 2005)

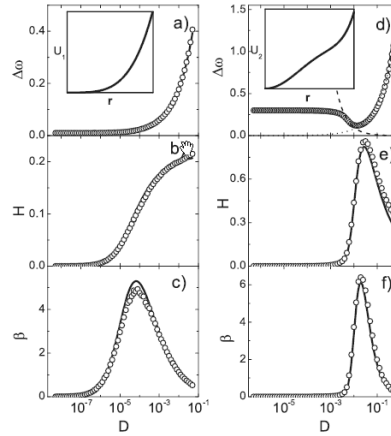


FIG. 3. Width ( $\Delta\omega$ ), peak height ( $H$ ), and signal-to-noise ratio ( $\beta$ ) versus noise intensity ( $D$ ) as obtained from the theoretical analysis. Left column (a)–(c) Supercritical bifurcation ( $F = F_1$ ). Right column (d)–(f) Subcritical bifurcation ( $F = F_2$ ). Symbols “○” represent the numerical solution of (2), lines are obtained from the analytical treatment. Insets: Shape of the potentials  $U_1(r)$  and  $U_2(r)$ . Dashed lines in (d) show the asymptotic behavior  $D^{-1}$  and  $D^{2/3}$ , respectively (see text for details).

then.