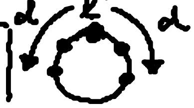
English Summary:

Decomposition of bosonic op. $\boxed{a(t) = \underbrace{\tilde{a}}_{\text{quantum fluct.}} + \underbrace{\alpha(t)}_{\text{mean-field } \alpha(t)\in\mathbb{C} \text{ semiclass. trajectory}}}$

Network of quantum Van der Pol osc.

semiclass. traj. $\boxed{\dot{\alpha}_\ell = \alpha_\ell(t)(\kappa_1 - 2\kappa_2 |\alpha_\ell(t)|^2) - i \sum_{\substack{s=1 \\ s\neq \ell}}^{N} K_{\ell s}\alpha_s(t)}$



nonlocal coupling $K_{\ell m} = \frac{V}{2d}\Theta(d - |\ell - m|)$

gaussian quantum fluctuations about $\alpha(t)$:
(density matrix in co-moving frame, $|\alpha_\ell(t)| \gg 1$)

$$\dot{\varrho}_\alpha \approx \underbrace{-\frac{i}{\hbar}[H_Q^{(\alpha)}, \varrho_\alpha]}_{\substack{\text{coherent dyn.} \\ \text{(incl. nonlocal coupl.)}}} + \underbrace{2\sum_{\ell=1}^{N}\left[\kappa_1\left(\tilde{a}_\ell^+ \varrho_\alpha \tilde{a} - \frac{1}{2}\{\varrho_\alpha, \tilde{a}_\ell \tilde{a}_\ell^+\}\right) + 4\kappa_2|\alpha_\ell|^2\left(\tilde{a}_\ell \varrho_\alpha \tilde{a}_\ell^+ - \frac{1}{2}\{\varrho_\alpha, \tilde{a}_\ell^+ \tilde{a}_\ell\}\right)\right]}_{\text{dissipative dynamics}}$$

4.7.4 Quantum signatures of chimera states

Chimera states of semiclassical trajectory

Dynamics of mean field $\alpha_\ell(t) = \frac{1}{\sqrt{2\hbar}}(Q_\ell(t) + iP_\ell(t)) = r_\ell(t)e^{i\phi_\ell(t)}$

$N = 50$ oscill., initial cond.: $r_\ell(0) = r_0 = \sqrt{\frac{\kappa_1}{2\kappa_2}}$ (limit cycle of single VdP)

phases randomly from Gaussian around $\ell = 25$ (in space)

Space-time plot of chimeras

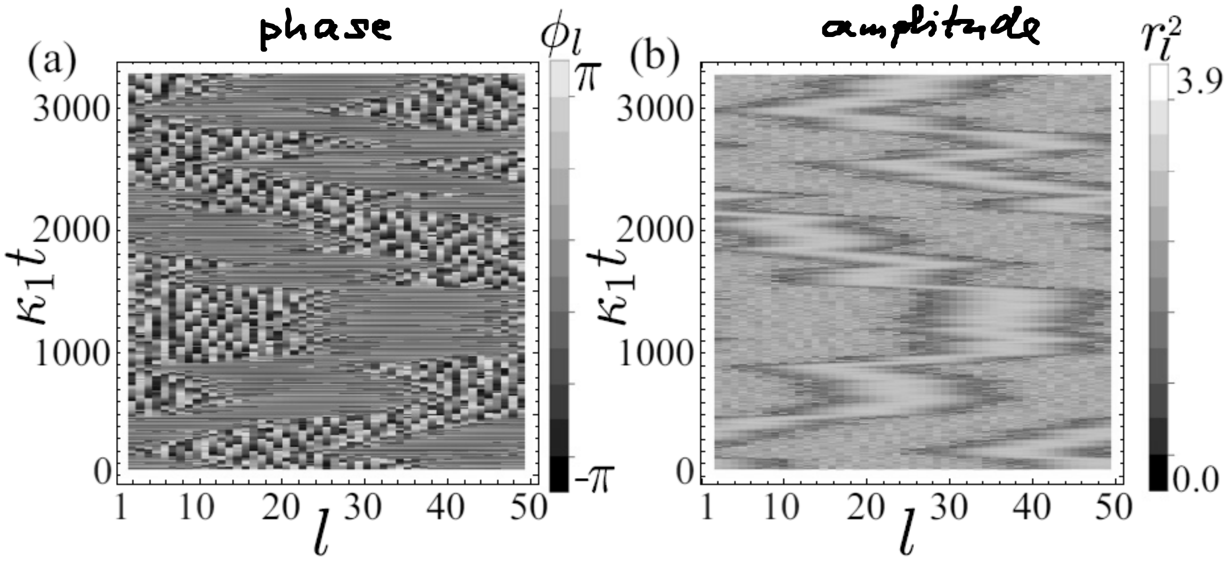


FIG. 1. (Color online) Space-time representation of the classical chimera state for oscillators $\alpha_l(t) = r_l(t)e^{i\phi_l(t)}$: (a) $\phi_l(t)$ and (b) $r_l^2(t)$. Parameters: $d = 10$, $\kappa_2 = 0.2\kappa_1$, $V = 1.2\kappa_1$, and $N = 50$.

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breathing and drifting chimeras (small system!)

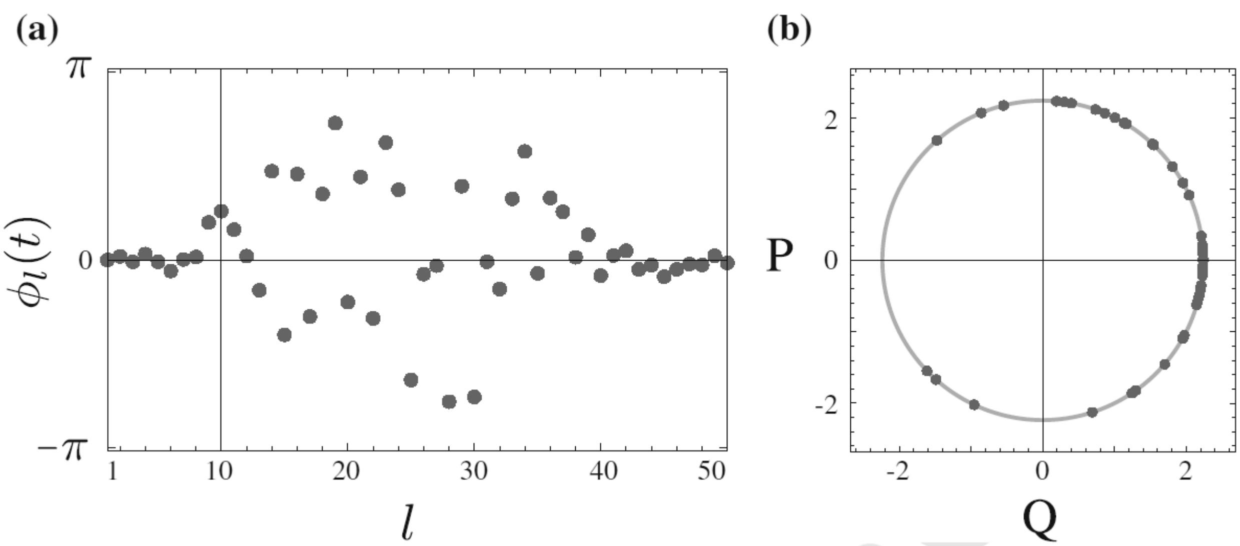


Fig. 16.1 Initial conditions used in the simulations. **a** Initial distribution of the phases $\phi_l(0)$ drawn randomly from a Gaussian distribution in space. **b** Phase-space representation of the initial conditions for the oscillators. The *green circle* represents the limit cycle with radius $|\alpha_l(0)| \approx 1.58$, where $\alpha_l(t) = \frac{Q_l(t) + iP_l(t)}{\sqrt{2\hbar}}$. Parameters: $\hbar = 1$, $d = 10$, $\kappa_2 = 0.2\kappa_1$, and $N = 50$

Quantum fluctuations ($|\alpha_l| \gg 1$)

solve Master eq. (4) with pure coherent state as initial cond.

$$\rho(t_0) = \bigotimes_{l=1}^{N} |\alpha_l(t_0)\rangle\langle\alpha_l(t_0)| \qquad t_0 = 3000/\kappa_1$$

in co-moving frame

$$\rho_\alpha(t_0) = \bigotimes_{l=1}^{N} |0\rangle_l \, {}_l\langle 0|$$

phase-space representation (co-moving frame $\sim$)

$z = \alpha(t) + \tilde z$

$\hat a_l = \frac{1}{\sqrt{2\hbar}} (\hat q_l + i \hat p_l)$ of op.

$\tilde z_l = \frac{1}{\sqrt{2\hbar}} (\tilde q_l + i \tilde p_l) \in \mathbb{C}$

Wigner representation of $\rho_\alpha(t)$:

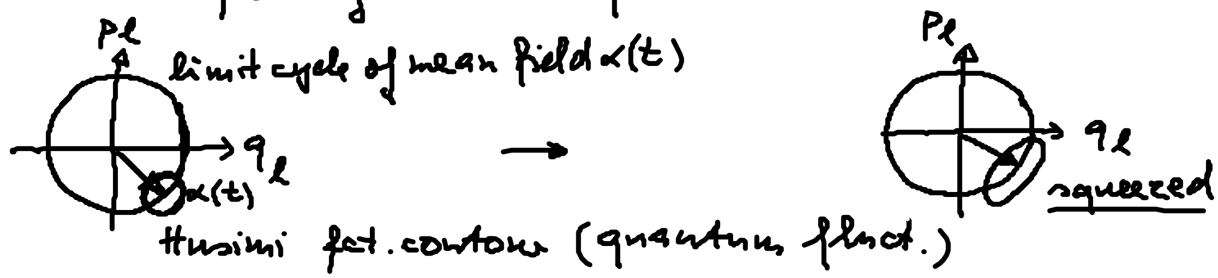
$$W_\alpha(\tilde z) = \int \frac{d^{2N}\lambda}{\pi^{2N}} e^{-\lambda \tilde z^* + \lambda^* \tilde z} \, \mathrm{tr}\left[\rho_\alpha(t) e^{-\lambda \tilde a^+ + \lambda^* \tilde a}\right]$$

$\hat{=}$ semiclassical prob. distribution fct $W_\alpha(\tilde q, \tilde p)$, but may be < 0

Husimi function $Q(z) := \frac{1}{\pi} \langle z | \rho(t) | z \rangle$:

$$Q_\alpha(\tilde z) = \frac{2}{\pi} \int W_\alpha(\tilde x) \, e^{-2|\tilde z - \tilde x|^2} d^{2N} \tilde x$$

bosonic squeezing due to quantum correlations:



- synchronized quantum osc. $\Rightarrow$ squeezing in the same direction
- desynchronized quantum osc. $\Rightarrow$ squeezing dir. random

<u>Chimera state</u>:

$t_0 = 3000/\kappa_1$ (initial cond.) $\quad t_0 + 0.5/\kappa_1$ (quantum corr. build up)

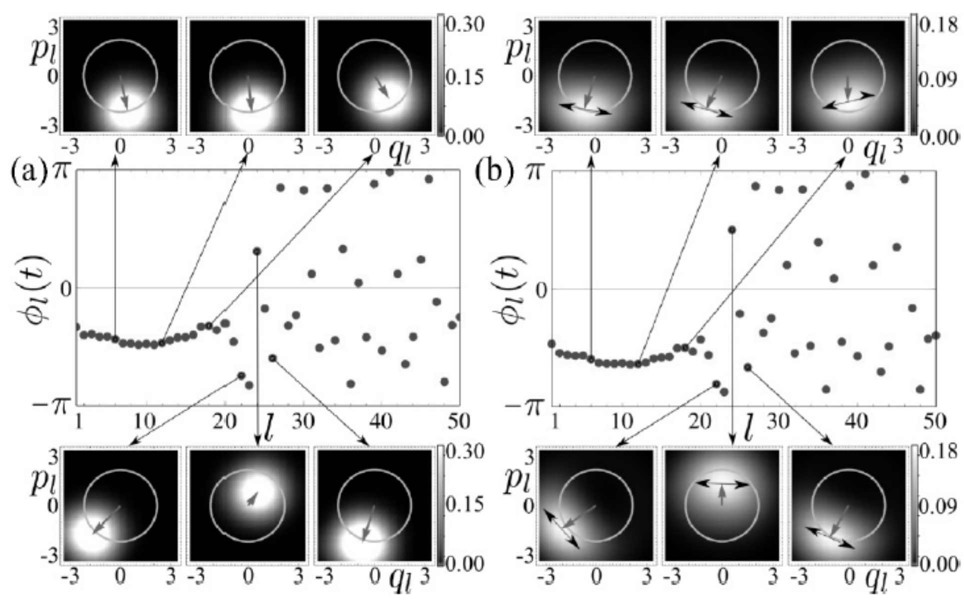


FIG. 2. (Color online) Quantum signatures of the classical chimera state. (a) Snapshot of the phase chimera depicted in Fig. 1 at $\kappa_1 t_0 = 3000$. We consider an initial density matrix $\rho(t_0)$, which is a tensor product of coherent states centered around the positions of the individual oscillators as depicted in the insets (Husimi function). (b) After a short-time interval $\kappa_1 \Delta t = 0.5$, quantum correlations appear in the form of squeezing (black double arrows in the insets). Parameters: $d = 10$, $\kappa_2 = 0.2\kappa_1$, $V = 1.2\kappa_1$, and $N = 50$.

Short-time evolution ($\sim 1/\kappa_1$)

Master eq. (4) $\Rightarrow$ Fokker–Planck eq. for Wigner fct. W_α, depends on mean-field sol. $\alpha(t)$

$$\frac{\partial W_\alpha}{\partial t} = -\sum_{i=1}^{2N} A_{ij}(t)\,\partial_{\tilde{R}_i}(\tilde{R}_j W_\alpha) + \frac{1}{2}\sum_{i=1}^{2N} B_{ij}(t)\,\partial^2_{\tilde{R}_i\tilde{R}_j} W_\alpha$$

$$\tilde{R} = (\tilde{q}_1, \tilde{p}_1, \dots, \tilde{q}_N, \tilde{p}_N)$$

Exact sol.

$$W_\alpha(\tilde{R}, t) = \frac{\exp\left(-\frac{1}{2}\tilde{R}^T . C^{-1}(t) . \tilde{R}\right)}{(2\pi)^N \sqrt{\det C(t)}}$$

with covariance matrix

$$C_{ij} := \left\langle \frac{\hat{\tilde{R}}_i \hat{\tilde{R}}_j + \hat{\tilde{R}}_j \hat{\tilde{R}}_i}{2} \right\rangle_\alpha - \langle \hat{\tilde{R}}_i \rangle_\alpha \langle \hat{\tilde{R}}_j \rangle_\alpha$$

(correlations of quantum fluctuations $\hat{\tilde{R}}_{2\ell-1} = \hat{\tilde{q}}_\ell$ and $\hat{\tilde{R}}_{2\ell} = \hat{\tilde{p}}_\ell$)

$$\langle \hat{O} \rangle_\alpha \equiv \mathrm{tr}(\rho_\alpha \hat{O})$$

<u>Chimera-like quantum correlations</u>:

Semiclass trajectory $\alpha(t) \Rightarrow C(t)$

initial state $\rho_\alpha(t_0) = \bigotimes_{\ell=1}^{N} |0\rangle_\ell \langle 0|_\ell \Rightarrow C(t)$ diagonal

coherent pure state

$$C_{2\ell-1,2\ell-1} \equiv \langle \hat{\tilde{q}}_\ell^2 \rangle_\alpha = \frac{\hbar}{2}$$

$$C_{2\ell,2\ell} \equiv \langle \hat{\tilde{p}}_\ell^2 \rangle_\alpha = \frac{\hbar}{2}$$

(Heisenberg uncertainty principle ✓)

after Δt: quantum correlations in C (nondiagonal)

chimera : synchr. domain $l = 1,..,20 \triangleq$ 40 x 40 block homog.
desynchr. " $l = 21, ..., 50 \triangleq$ 60 x 60 block irregular

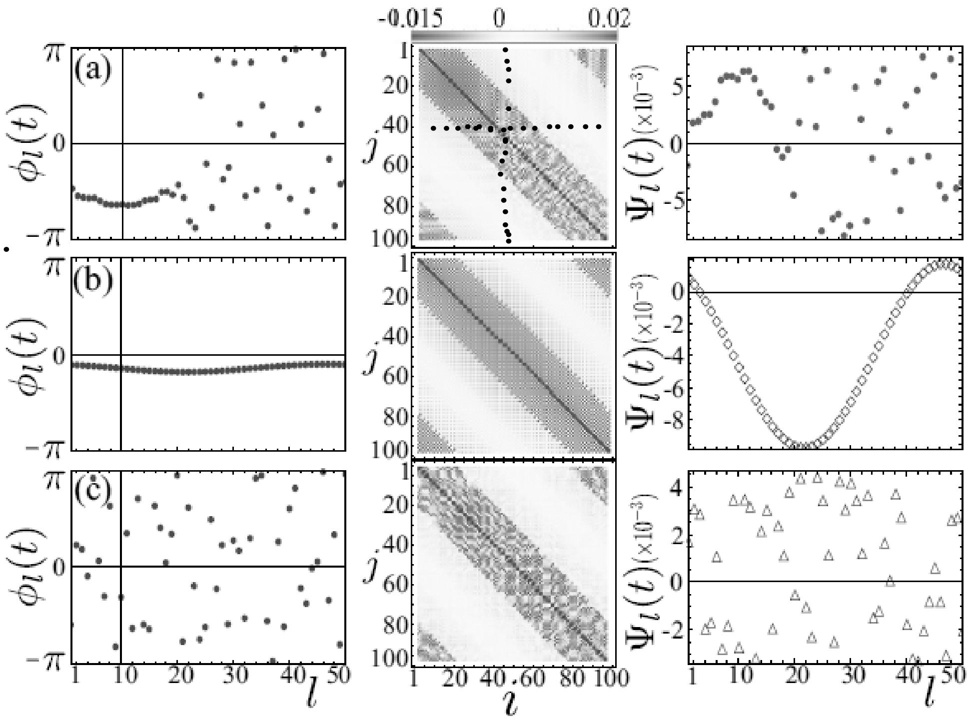


FIG. 3. (Color online) Quantum fluctuations after a short-time evolution. Similarly to Fig. 2, we consider an initial density matrix $\rho(t_i)$, which is a tensor product of coherent states centered around the classical positions of the oscillators. Snapshots of the phase (left column) and covariance matrices (central column) after short-time evolution $\kappa_1 \Delta t = 0.5$ of the states: (a) chimera for $V = 1.2\kappa_1$, (b) synchronized state for $V = 1.6\kappa_1$, and (c) desynchronized state for $V = 0.8\kappa_1$. Right column: Weighted spatial average $\Psi_l(t)$ of the covariance matrix for the states shown in (a), (b), and (c), respectively. Parameters $d = 10$, $\kappa_2 = 0.2\kappa_1$, and $N = 50$.

Def. weighted correlation $\Psi_l(t) := \frac{V}{2d} \sum_{\substack{m=l-d \\ m \neq l}}^{l+d} C_{2l,2m}(t)$
(sliding spatial average)

chimera $\longrightarrow$ regular + irregular

Quantum mutual information:

partition into 2 spatial domains A and B

A: L elements, B: $N-L$ elements

Rényi entropies $S_\mu(\varrho) = \frac{1}{1-\mu} \ln \mathrm{tr}(\varrho^\mu)$, $\mu \in \mathbb{N}$

$\mu = 2$: $$S_2(\varrho_\alpha) = -\ln\left[\int W_\alpha^2(\tilde{R},t)\, d^{2N}\tilde{R}\right]$$

bipartite gaussian state $\varrho_{AB} = \varrho_\alpha$

reduced density matrix $\varrho_A = \mathrm{tr}_B\, \varrho_{AB}$

$\varrho_B = \mathrm{tr}_A\, \varrho_{AB}$

uncorrelated reference state $\varrho_{Ref} = \varrho_A \otimes \varrho_B$

$\mu = 2$: Rényi mutual information

$$I_2(\varrho_{A:B}) := S_2(\varrho_A) + S_2(\varrho_B) - S_2(\varrho_{AB})$$
$$= \frac{1}{2} \ln\left(\frac{\det C_A \cdot \det C_B}{\det C}\right)$$

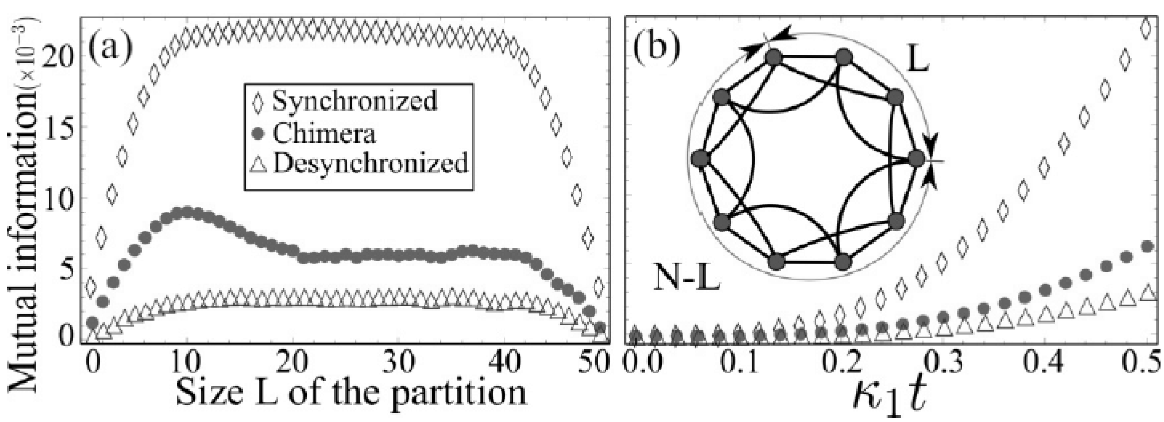


FIG. 4. (Color online) Rényi quantum mutual information for the states shown in Fig. 3. The green dots, blue diamonds, and purple triangles represent the chimera, synchronized, and desynchronized states, respectively. (a) Gaussian Rényi-2 mutual information $\mathcal{I}_2(\rho_{\mathrm{A:B}})$ as a function of the size L of *Alice* after an evolution time $\Delta t = 0.5/\kappa_1$. (b) The time evolution of the mutual information during the time interval Δt for a fixed size $L_c = 20$. Inset: scheme of the nonlocally coupled network. Parameters: $d = 10$, $\kappa_2 = 0.2\kappa_1$, and $N = 50$.

<u>Chimera</u> : $L_c = 2D$ critical size $\triangleq$ size of coherent domain

$L > L_c$: change of mutual inform., I_2 small
(change of correl.)

I_2 between sync and desync.