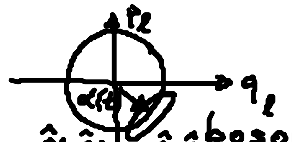
English Summary:

Quantum Signatures of chimera states

semiclass. trajectory (mean-field) $\alpha_\ell(t) = \frac{1}{\sqrt{2\hbar}}(Q_\ell + iP_\ell) = r_\ell(t)\, e^{i\phi_\ell(t)}$

quantum fluctuations $\rho_\alpha(t_0) = \bigotimes_{\ell=1}^{N} |0\rangle_\ell \langle 0|_\ell \rightarrow W_\alpha(\tilde{z},t)$ Wigner fct.

$\tilde{R} = (\tilde{q}_1, \tilde{p}_1, \dots, \tilde{q}_N, \tilde{p}_N)$ $\qquad \tilde{z}_\ell = \frac{1}{\sqrt{2\hbar}}(\tilde{q}_\ell + i\tilde{p}_\ell)$



bosonic squeezing

covariance matrix $C_{ij} := \left\langle \frac{\hat{\tilde{R}}_i \hat{\tilde{R}}_j + \hat{\tilde{R}}_j \hat{\tilde{R}}_i}{2} \right\rangle_\alpha - \langle \hat{\tilde{R}}_i \rangle_\alpha \langle \hat{\tilde{R}}_j \rangle_\alpha$

quantum mutual Rényi information, Rényi entropy $S_\mu = \frac{1}{1-\mu} \ln \mathrm{tr}\, \rho^\mu$ $\quad \mu \in \mathbb{N}$

5. Boltzmann-Gleichung

Ziel: Beschreibung von Transport, z.B.
- Ladungstransport in Halbleitern,
- Massentransport in Gasen, Flüssigkeiten
- Wärmetransport (Wärmeleitung)

Response von Teilchen (Elektronen, Flüssigkeitsmoleküle) auf äußere Felder (el., magn. Felder, Dichtegradienten, Temperaturgradient):

- Beschleunigung
- Dissipation: Energie-, Impulsrelax. durch Stöße (Phononen, Störstellen, andere Teilchen/Elektronen)

$\Big\}$ Nichtgleichgewichtsdynamik, räumliche Propagation

fern vom thermodynamischen Gleichgewicht

$\swarrow$ thermisches Gleichgewicht (bzgl. Temperatur T) Austausch mit Wärmebad

$\searrow$ chemisches Gleichgewicht (bzgl. chem. Potenzial μ) Austausch mit Teilchenreservoir

Beschreibung durch Verteilungsfkt. $f(\underline{r}, \underline{p}, t)$

oder $f(\underline{r}, \underline{k}, t)$ mit (Kristall-)Quasiimpuls $\underline{p} = \hbar \underline{k}$

5.1 Hierarchie von Transportgleichungen

E. Schöll (ed.): Theory of Transport Properties of Semiconductor Nanostructures (Springer 1998)

Drift-diffusion:

classical reaction-diffusion-advection eqs.

$$\boxed{\dot{n} = \underbrace{f(n)}_{\text{react.}} + \underbrace{D\Delta n}_{\text{diff.}} + \underbrace{\underline{\nabla}(n\mu\underline{\mathcal{E}})}_{\text{drift}}}$$

el. Field $\mathcal{E}$
part. density $n(\underline{r},t)$
mobility $\mu = \frac{e}{m^*}\tau_m$ $(e>0)$
Diff. const. $D = \frac{\mu k T_e}{e}$
(Einstein relation)

⇑ adiabatic elimin. of fast momentum and energy balance

hydrodynamic:

balance equations

$$\boxed{\begin{aligned} &\dot{n} + \underline{\nabla}\cdot(n\underline{v}) = \varphi(n, \bar{E}) \\ &\dot{\underline{p}} + (\underline{v}\cdot\underline{\nabla})\underline{p} + \frac{1}{n}\underline{\nabla}(nkT_e) - \underbrace{q(\underline{\mathcal{E}} + \underline{v}\times\underline{B})}_{\text{Lorentz force}} = -\frac{\underline{p}}{\tau_m} \\ &\dot{\bar{E}} + (\underline{v}\cdot\underline{\nabla})\bar{E} + \frac{1}{n}\underline{\nabla}\cdot(nkT_e\underline{v}) - \underbrace{\frac{\kappa}{n}\Delta T_e}_{\text{heat cond.}} - \underbrace{q\underline{v}\cdot\underline{\mathcal{E}}}_{\text{Joule heating}} = \frac{\bar{E} - \bar{E}_0}{\tau_e} \end{aligned}}$$

$n(\underline{r},t) = \int f(\underline{r},\underline{k},t)\, 2d^3k$
particle density
$\underline{p}(\underline{r},t) = \langle\hbar\underline{k}\rangle = m^*\underline{v}$
momentum density
$\bar{E}(\underline{r},t) = \langle E(\underline{k})\rangle = \frac{m^*}{2}v^2 + \frac{3}{2}kT_e$ (T_e el. temp.)
energy density
$\underline{v}$ velocity
φ gen.-recomb. rate

macroscopic

⇑ moment expansion

τ_m momentum relaxation time
τ_e energy relaxation time

kinetic:

semiclass. Boltzmann eq.

$f(\underline{r},\underline{k},t)$ distribution fct.

$$\boxed{\dot{f} + \underline{v}_g\cdot\underline{\nabla}_r f - \frac{e}{\hbar}\underline{\mathcal{E}}\cdot\underline{\nabla}_k f = \left(\frac{\partial f}{\partial t}\right)_{\text{collision}}}$$

$\underline{\mathcal{E}}$ el. field
$\underline{v}_g = \frac{1}{\hbar}\underline{\nabla}_k E(\underline{k})$
group velocity

mesoscopic

⇑ Markov approx., elim. of coherences

Quantum kinetics:

Density matrix eqs.

$$\boxed{\frac{d}{dt}\langle c^+_{i,k}c_{j,k}\rangle = \frac{i}{\hbar}\langle[H, c^+_{i,k}c_{j,k}]\rangle + \text{dissipation}}$$

$f_{ij}(\underline{k}) = \langle c^+_{i,k}c_{j,k}\rangle$
distrib. fct. $f_{ii}(\underline{k}) = \langle c^+_{i,k}c_{i,k}\rangle$
polarization $f_{ij}(\underline{k})$ $i\neq j$
(coherences)
spatially inhomog.: Wigner fct.

microscopic

Ableitung der makroskop. Rategl. aus Quantenkinetik

Halbleiter-Bloch-Gln. §4.2 + phänomenolog. Stoßterme:

El. vert. fkt.
Polar,
Lochvert. fkt

$$\boxed{\begin{aligned} \dot f_e(\underline{k},t) &= \frac{1}{i}\Omega_p\left(p^*(\underline{k},t)-p(\underline{k},t)\right) - \frac{f_e}{T_1} \\ \dot p(\underline{k},t) &= \frac{1}{i}\omega_p(\underline{k})\,p(\underline{k},t) + \frac{1}{i}\Omega_p(f_e+f_h-1) - \frac{p}{T_2} \\ \dot f_h &= \dot f_e \end{aligned}}$$

$\Omega_p = \frac{\mu\cdot\mathcal{E}}{\hbar}$ Rabi-frequ.

$\omega_p = \frac{E_c(\underline{k})-E_v(\underline{k})}{\hbar}$ opt. Übergangs-frequ.

T_1 Lebensdauer bzgl. Rekomb.

T_2 Lebensdauer der Polarisation (Phasenrelax. zeit)

Adiabat. Elim. der Polarisation:

$$0 \overset{!}{=} \dot p = \frac{1}{i}\omega_p p + \frac{1}{i}\Omega_p \underbrace{(f_e+f_h-1)}_{\text{Inversion}} - \frac{p}{T_2}$$

$$\Rightarrow p = \frac{T_2}{1+i\omega_p T_2}\left[-i\Omega_p(f_e+f_h-1)\right]$$

$$= \frac{-i-\omega_p T_2}{1+\omega_p^2 T_2^2}\,\Omega_p T_2 (f_e+f_h-1)$$

$$p^* - p = -\frac{2i}{1+\omega_p^2 T_2^2}\,\Omega_p T_2(f_e+f_h-1)$$

$$\Rightarrow \boxed{\dot f_e = -\underbrace{\frac{2\Omega_p^2 T_2}{1+\omega_p^2 T_2}}_{g_0 N_{photon}}(f_e+f_h-1) - \frac{f_e}{T_1}}$$

, da $\Omega_p \sim \underset{\text{Intensität}}{|\mathcal{E}|^2} \sim \underset{\text{Photonenzahl}}{N_{photon}}$

mittlere El. dichte $n := \int f_e\, 2d^3k$ im Leitungsband

" " $n_0 := \int (1-f_h)\, 2d^3k$ im Valenzband

$$\Rightarrow \boxed{\dot n_e = -\underbrace{g_0(n_e-n_0)N_{photon}}_{\text{gain}} - \underbrace{\frac{n_e}{T_1}}_{\text{Rekomb.}} \quad \underbrace{\left(+\frac{j}{ed}\right)}_{\text{Pumpstrom}}}$$

Laser-Rategl.