Superradianz II:

$$\dot{a}(t) = \frac{i}{\hbar}[\cdot\cdot,\cdot\cdot] +$$

$$+ \frac{\mu^2}{2\hbar\varepsilon_0}\frac{c}{(2\pi)^3}\sum_{\alpha,\beta=1}^{N}\int d\Omega_k\left[1-(\hat{k}\cdot\hat{\mu})\right] \circledast$$

$$\circledast = \int_0^t dt'' \int_0^\infty dk\, k^3\, e^{i\omega\left[t-t'-\frac{\hat{k}\cdot x_{\alpha\beta}}{c}\right]}\left[S_\alpha(t), a(t)\right] S_\beta(t')$$

$+ h.c.$

bislang lediglich Umformungen

Ziel: t' Integral loswerden

$$\int_0^t dt'\, e^{i\tilde{\omega}t'} S_\beta(t') = \int_0^t dt'\, e^{i\tilde{\omega}t'}\left[\sigma_\beta^-(t')e^{-i\omega_{eg}t'} + h.c.\right]$$

nehme nur Energie- oder Frequenzdifferenzen (Rotating Wave Approx. – Drehwellennäherung)

→ kann hier problematisch, da verstärkte Dynamik von $\sigma_\beta(t')$ (Rydberg-Superatome)

$$\overset{RWA}{\approx} \int_0^t dt'\, e^{i(\tilde{\omega}-\omega_{eg})t'}\sigma_\beta^-(t')$$

Markov: $t' = t - \tau$

$dt' = -d\tau$

$$= -\int_{\tau}^{0} d\tau\, e^{i(\hat{\omega}-\omega_{eg})(t-\tau)} \underbrace{\sigma_\beta^-(t-\tau)}_{\approx \sigma_\beta^-(t)}$$

Operator hat Gedächtnis

$$\overset{\text{Markov}}{\approx} e^{i(\hat{\omega}-\omega_{eg})t} \sigma_\beta^-(t) \underbrace{\int_0^t d\tau\, e^{i(\hat{\omega}-\omega_{eg})\tau}}_{\to\, t \to \infty}$$

$$= \int_0^\infty d\tau\, e^{i(\hat{\omega}-\omega_{eg})\tau}$$

$$= \pi\delta(\hat{\omega}-\omega_{eg}) - i P \frac{1}{\hat{\omega}-\omega_{eg}} = \zeta(\hat{\omega}-\omega_{eg})$$

(Heitler-Zeta-~~Fkt~~)

(Sokhotski-Plemelj-Theorem)

hier ist viel passiert

$$\dot{a}(t) = i[\cdot, \cdot] +$$

$$+ \frac{\mu^2 c}{2\varepsilon_0 \hbar}\left(\frac{1}{2\pi}\right) \sum_{\alpha,\beta}' \int_0^\infty dk\, k^3 \int d\Omega_k \left[1-(\hat{k}\cdot\hat{\mu})^2\right] \zeta(k-k_{eg})\, \circledast$$

$$+ h.c.$$

$$\circledast = [J_\alpha(t), a(t)]\, s_\beta(t)$$

$$F_{\alpha\beta}(k x_{\alpha\beta}) = \int d\Omega_k \left[1-(\hat{k}\cdot\hat{\mu})^2\right] e^{ik x_{\alpha\beta} \hat{\underline{k}}\cdot \hat{\underline{x}}_{\alpha\beta}}$$

$x_{\alpha\beta} = |x_\alpha - x_\beta|$ $\qquad k \hat{\underline{k}} = \underline{k}$

$$\hat{f}\, e^{ik x_{\alpha\beta} \hat{\underline{k}}\cdot\hat{\underline{x}}_{\alpha\beta}} = \left[1-(\hat{\underline{k}}\cdot\hat{\underline{\mu}})^2\right]$$

sollte Ortsableitung, ansonsten können wir es nicht aus dem Integral ziehen

Ausprobieren $\left[1+\frac{(\hat{\mu}\cdot\nabla_{x_{\alpha\beta}})^2}{k^2}\right] e^{ik x_{\alpha\beta}\,\hat{\underline{x}}_{\alpha\beta}\cdot\hat{\underline{k}}}$

$$= \left[1+\frac{1}{k^2}\left(\hat{\mu}\cdot\sum_{i=x,y,z}\partial_i \underline{e}_i\right)^2\right] e^{i\sum_{i=x,y,z} k_i x_i}$$

$$\nabla e^{i\underline{k}\cdot\underline{r}} = i\underline{k}\, e^{i\underline{k}\cdot\underline{r}}$$

$$(\mu\cdot\nabla)^2 e^{i\underline{k}\cdot\underline{r}} = (\mu\cdot\nabla)\, e^{i\underline{k}\cdot\underline{r}}\,(\hat{\underline{\mu}}\cdot i\underline{k})$$

$$= -(\mu\cdot\underline{k})^2$$

$$= \left[1-(\hat{\underline{k}}\cdot\hat{\underline{\mu}})^2\right] e^{i\underline{k}\cdot\underline{x}_{\alpha\beta}}$$

Erzeugende $\hat{f}$ gefunden

$$F_{\alpha\beta}(kx_{\alpha\beta}) = \int_0^{2\pi} d\varphi_k \int_0^{\pi} d\Theta_k \,\sin\Theta_k \left[1+\frac{(\hat{\mu}\cdot\nabla_{\underline{x}})^2}{k^2}\right] e^{i\ldots}$$

keine Winkelabhängigkeit mehr

$$= [1+\ldots]\, 2\pi \int_0^{\pi} d\Theta_k \sin\Theta_k \; e^{ik x_{\alpha\beta}\,\underbrace{\hat{\underline{k}}\cdot\hat{\underline{x}}_{\alpha\beta}}_{=|\hat{k}||\hat{x}_{\alpha\beta}|\cos\Theta_k}}$$

ersetzt: $\xi = \cos\Theta_k$

$$d\xi = -\sin\Theta_k \, d\Theta_k$$

$$\xi = 1\ (\Theta_k = 0)\ ,\quad \xi = -1\ (\Theta_k = \pi)$$

$$= [1+\ldots]\, 2\pi \int d\xi\, e^{ik x_{\alpha\beta}\,\xi}$$

$$= \left[1 + \frac{(\hat{\mu}\cdot\nabla_{x_{\alpha\beta}})^2}{k^2}\right]^{-1} \frac{1}{ikx_{\alpha\beta}} \left[e^{ikx_{\alpha\beta}} - e^{-ikx_{\alpha\beta}}\right]$$

· wähle das Dipolmoment in z-Richtung

$$= \left[1 + \frac{(\hat{\mu}\cdot\nabla_{x_{\alpha\beta}})^2}{k^2}\right] 2\, \frac{\sin(kx_{\alpha\beta})}{k\, x_{\alpha\beta}}$$

Kugelkoordinaten (Gradient muss angepasst)

$$= \frac{\sin(kx_{\alpha\beta})}{kx_{\alpha\beta}} \left[1 - (\hat{\mu}\cdot\hat{x}_{\alpha\beta})^2\right]$$

$$+ \left[1 - 3(\hat{\mu}\cdot\underline{x}_{\alpha\beta})^2\right] \left[\frac{\cos(kx_{\alpha\beta})}{(k\alpha\beta)^2} - \frac{\sin kx_{\alpha\beta}}{(kx_{\alpha\beta})^3}\right]$$

$$\tilde{F}_{\alpha\beta}(kx_{\alpha\beta}) \longrightarrow F_{\alpha\beta}(0) = 1$$

$$\tilde{F}_{\alpha\beta}(0) = \frac{z}{\pi} \int d\Omega_k \left[1 - (\hat{k}\cdot\hat{\mu})^2\right] \underbrace{e^{ikx_{\alpha\beta}\hat{k}\cdot\hat{x}_{\alpha\beta}}}_{=1}$$

$$= z \int_0^{2\pi} d\varphi_k \int_0^{\pi} d\Theta_k \sin\Theta_k \underbrace{\left[1 - |\hat{k}||\hat{\mu}| \cos^2\Theta_k\right]}_{= \sin^2\Theta_k}$$

$$= z\, 2\pi \underbrace{\int_0^{\pi} d\Theta_k \sin^3\Theta_k}_{= 4/3}$$

$$z = \frac{3}{8\pi}$$

$$\dot a(t) = i\,[\ldots,\ldots] + (1+2)$$

$$(1+2) = \frac{\mu^2}{2\varepsilon_0\hbar}\frac{1}{(2\pi)^3}\sum_{\alpha\beta}\int_0^\infty dk\, k^3\,\frac{8\pi}{3}F_{\alpha\beta}(kx_{\alpha\beta})\,\delta(k-k_{eg})\,\otimes$$

$$\otimes = [s_\alpha(t), a(t)]\,s_\beta(t)$$

$$+\,h.c.$$

$$= \sum_{\alpha\beta}\left\{\frac{\mu^2}{2\varepsilon_0\hbar}\left(\frac{k_{eg}}{2\pi}\right)^3\frac{8\pi^2}{3}F_{\alpha\beta}(k_{eg}x_{\alpha\beta}) - i\,\frac{\mu^2 8\pi}{2\varepsilon_0\hbar\,3(2\pi)^3}\,P[\ldots\ldots]\right\}\otimes$$

$$\frac{\Gamma_{\alpha\beta}}{2} = \frac{1}{2}\,\frac{\mu^2 k_{eg}^3}{\hbar\,3\pi\,\varepsilon_0}\,F_{\alpha\beta}(k_{eg}x_{\alpha\beta})$$

$\Gamma = \frac{\Gamma_{\alpha\beta}}{F_{\alpha\beta}}$ Einstein-Faktor der spontanen Emission + Formfaktor, der die Orientierung des Ensembles beschreibt

$$\Omega_{\alpha\beta} = -\frac{\Gamma}{k_{eg}^3}\,P\!\int_0^\infty dk\, k^3\,\frac{F_{\alpha\beta}(kx_{\alpha\beta})}{2\pi}\,\frac{1}{k-k_{eg}}$$

$$(1+2) = \sum_{\alpha\beta}\left[\frac{\Gamma_{\alpha\beta}}{2} - i\,\Omega_{\alpha\beta}\right][\sigma_\alpha^+ + \sigma_\alpha^-, a(t)]\,\sigma_\beta^-(t)$$

$$+\,h.c.$$

$$(1+2) = \sum_{\alpha\beta}[\ldots]\left\{\sigma_\alpha^+ a(t)\sigma_\beta^- + \sigma_\alpha^- a(t)\sigma_\beta^- - a(t)\sigma_\alpha^+\sigma_\beta^- - a(t)\sigma_\alpha^-\sigma_\beta^-\right\}$$

$$-\sum_{\alpha\beta}[\ldots]^*\left\{\sigma_\beta^+\sigma_\alpha^+ a + \sigma_\beta^+\sigma_\alpha^- a - \sigma_\beta^+ a\sigma_\alpha^+ - \sigma_\beta^+ a\sigma_\alpha^-\right\}$$

da $F_{\alpha\beta} = F_{\beta\alpha}$

$$= \sum_{\alpha\beta} \frac{\Gamma_{\alpha\beta}}{2} \{ 2\sigma_\alpha^+ a(t) \sigma_\beta^- - a(t)\sigma_\alpha^+ \sigma_\beta^- - \sigma_\alpha^+ \sigma_\beta^- a(t) \}$$

$$+ i \sum_{\alpha\beta} \Omega_{\alpha\beta} \underbrace{\{ -a(t)\sigma_\alpha^+ \sigma_\beta^- + \sigma_\alpha^+ \sigma_\beta^- a(t) \}}_{= [\sigma_\beta^+ \sigma_\alpha^-,\, a(t)]}$$

$$\dot{a}(t) = \frac{i}{\hbar} \sum_{\alpha=1}^{N} [\hbar(\omega_{eg} + \Omega_{\alpha\alpha}) \sigma_\alpha^+ \sigma_\alpha^-, a(t)]$$
$$+ \frac{i}{\hbar} \sum_{\alpha\neq\beta} \hbar \Omega_{\alpha\beta} [\sigma_\alpha^+ \sigma_\beta^-, a(t)]$$

$$+ \frac{1}{2} \sum_{\alpha\beta} \Gamma_{\alpha\beta} \{ 2\sigma_\alpha^+ a(t) \sigma_\beta^- - a(t) \sigma_\alpha^+ \sigma_\beta^- - \sigma_\alpha^+ \sigma_\beta^- a(t) \}$$

Grenzfälle: $N = 1$, d.h. $x_{\alpha\beta} = 0 = x_{\alpha\alpha}$
$F_{\alpha\beta}(0) = 1$ (so konstruiert)

$$\dot{a}(t) = \frac{i}{\hbar} [(\hbar\omega_{eg} + \Omega_{11}) \sigma_1^+ \sigma_1^-, a(t)]$$
$$+ \frac{\Gamma}{2} \{ 2\sigma_1^+ a(t) \sigma_1^- - a(t) \sigma_1^+ \sigma_1^- - \sigma_1^+ \sigma_1^- a(t) \}$$

Rein Lindblad, aber die Lindblad generierende Langevin Gl.

sei $a(t) = \sigma_1^+ \sigma_1^-$

$$\frac{d}{dt}\langle \sigma_{11} \rangle = \frac{i}{\hbar} \underbrace{[(\dots)\sigma_{11}, \sigma_{11}]}_{=0}$$

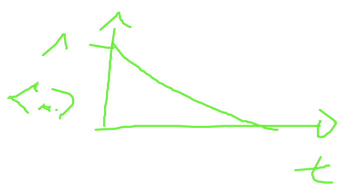
$$+ \frac{\Gamma}{2}\, 2 \, \langle \underbrace{\sigma_1^+ \sigma_1^+}_{=0} \sigma_1^- \sigma_1^+ \rangle$$

$$- \frac{\Gamma}{2} \underbrace{\langle \sigma_1^+ (\sigma_1^- \sigma_1^+) \sigma_1^- \rangle}_{} + h.c.$$

$$= \langle \sigma_1^+ (\mathbb{1} - \sigma_1^+ \sigma_1^-) \sigma_1^- \rangle$$

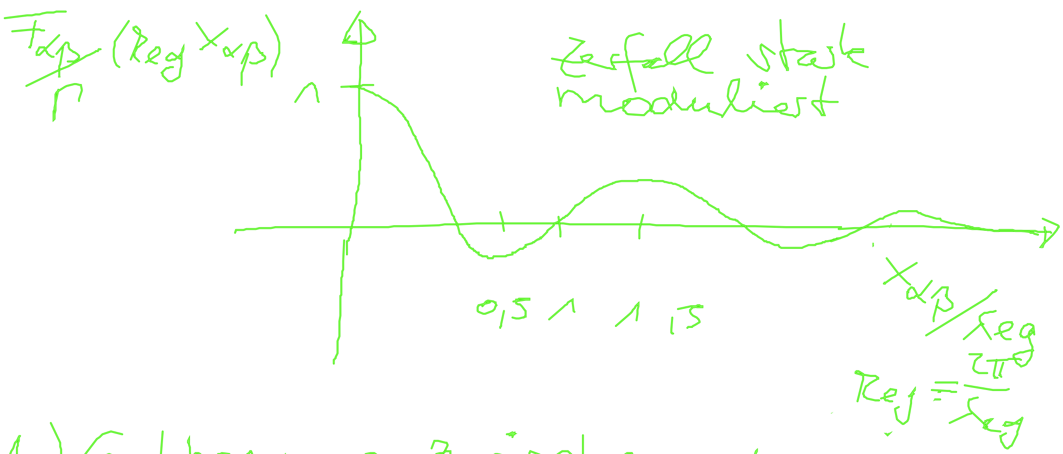
$$= \langle \sigma_1^+ \sigma_1^- \rangle - \langle \sigma_1^+ \sigma_1^+ \dots \rangle$$

$$\frac{d}{dt}\langle \sigma_1^+ \sigma_1^- \rangle = -\Gamma \langle \sigma_1^+ \sigma_1^- \rangle$$



ab $N \geq 2$, spielt $F_{\alpha\beta}$ eine Rolle

Winkelabhängigen Formfaktor



(1.) Entfernung zwischen den Emittern ist viel geringer als die Wellenlänge

$k_{eg} x_{\alpha\beta} = \frac{2\pi}{\lambda_{eg}} x_{\alpha\beta} \ll 1$

d.h. $F_{\alpha\beta} \longrightarrow 1$

$\hbar \Omega_{\alpha\beta} \longrightarrow \frac{1}{4\pi\varepsilon_0} \frac{\underline{\mu} \cdot \underline{\mu} - 3(\underline{\mu} \cdot \underline{x}_{\alpha\beta})}{x_{\alpha\beta}^3}$

und damit

$\dot{a}(t) = -\frac{i}{\hbar}[\ldots, \ldots]$

$+\frac{1}{2}\Gamma\Big\{\underbrace{\sum_\alpha \sigma_\alpha^+}_{=S^+} a \underbrace{\sum_\beta \sigma_\beta^-}_{=S^-} - S^+ S^- a(t) - a(t) S^+ S^-\Big\}$

$\hat{=}$ das genau nennt man Superradianz $\rightarrow$ kollektive Prozesse

$S^{\pm}$ sind kollektive Jumps

$\Rightarrow$ d.h. stark verschränkte und korrelierte Dynamik

(2.) $k_{eg} x_{\alpha\beta} = \frac{2\pi}{\lambda_{eg}} x_{\alpha\beta} \gg 1$

$F_{\alpha\beta}(k_{eg} x_{\alpha\beta}) \longrightarrow \delta_{\alpha\beta}$

(schon alleine physikalisch spielen die anderen Emitter keine Rolle)

also $\Gamma_{\alpha\beta} \longrightarrow \Gamma \delta_{\alpha\beta}$

$\Sigma' \Gamma_{\alpha\beta} \to \Omega$

$$\dot{a}(t) = [\dots] + \frac{\Gamma}{\Sigma} \sum_{\alpha \geq 1}^{N} \Big(2\sigma_\alpha^+ a \sigma_\alpha^- - \sigma_\alpha^+ \sigma_\alpha^- a - a \sigma_\alpha^+ \sigma_\alpha^- \Big)$$

$\longrightarrow$ keine kollektiven Prozesse

$\longrightarrow$ Hilbertraum direkte Summe