

## Superradianz:

aber zuerst, Skizze der Quantisierung des Lichtfeldes, ausgehend von den Maxwell-Gl.:

$$\underline{\nabla} \cdot \underline{E} = \frac{1}{\epsilon_0} \underline{\rho}, \quad \underline{\nabla} \cdot \underline{B} = 0$$

$$\underline{\nabla} \times \underline{B} = \frac{1}{c^2} \dot{\underline{E}} + \frac{1}{\epsilon_0 c^2} \underline{j}, \quad \underline{\nabla} \times \underline{E} = -\dot{\underline{B}}$$

(Cohen-Tannoudji, Dupont-Roc, Jynberg: Atom-Photon-Interactions, Appendix)

$$\text{Fourier-Transformation: } \underline{E}(\underline{r}, t) = \int d^3r' e^{i\underline{k} \cdot \underline{r} - i\underline{k} \cdot \underline{r}'} \underline{E}(\underline{k}, t)$$

$$\underline{\underline{also}}: \underline{\underline{j}} \cdot \underline{E} = \frac{1}{\epsilon_0} \underline{\rho}(\underline{k}, t)$$

$$\underline{\underline{j}} \cdot \underline{B} = 0$$

$$\underline{\underline{j}} \times \underline{E} = -\frac{\partial}{\partial t} \underline{B}$$

$$\underline{\underline{j}} \times \underline{B} = \frac{1}{c^2} \dot{\underline{E}} + \frac{1}{\epsilon_0} \frac{1}{c^2} \underline{j}$$

$$\underline{k} \cdot \underline{E} = \epsilon_{||} = \underline{\rho}$$

$$\underline{k} \cdot \underline{B} = 0 \quad (\text{rein transversal})$$

$$\underline{B} = \underline{B}_\perp$$

$$\text{Vektorpotential: } \underline{E}(\underline{r}, t) = -\underline{\nabla}\phi - \dot{\underline{A}}(\underline{r}, t)$$

$$\underline{B}(\underline{r}, t) = \underline{\nabla} \times \underline{A}$$

$$\underline{E}(\underline{r}, t) = \dot{\underline{r}} \hat{\phi}(\underline{r}, t) - \frac{\partial \hat{A}(\underline{r}, t)}{\partial t}$$

$$\underline{B}(\underline{r}, t) = \dot{\underline{r}} \times \hat{A}(\underline{r}, t)$$

Verwende Eichfreiheit  $\nabla \cdot \underline{A} = 0$   
 $A_{\perp}$  invariant (Coulombbeziehung)

Dynamik des Feldes:

$$\underline{\alpha}(\underline{r}, t) = \chi(\underline{r}) [\underline{E}_{\perp}(\underline{r}, t) + \hat{f} \underline{B}(\underline{r}, t)]$$

$\hat{f}$  wirkt auf  $\underline{B}$

$$\underline{\alpha}(\underline{r}, t) = \chi(\underline{r}) [\underline{E}_{\perp}(\underline{r}, t) - c \frac{\underline{r}}{|\underline{r}|} \underline{B}(\underline{r}, t)]$$

$$\dot{\underline{\alpha}}(\underline{r}, t) = \chi(\underline{r}) \left[ \dot{\underline{E}}_{\perp} - \frac{\underline{r}}{|\underline{r}|} \dot{\underline{B}} \right]$$

mit  $\dot{\underline{B}} = -\dot{\underline{r}} \times \underline{E}_{\perp}(\underline{r}, t)$

$$\dot{\underline{E}}_{\perp} = c^2 \dot{\underline{r}} \times \underline{B} - \frac{\Delta}{\epsilon_0} \underline{j}$$

$$\dot{\underline{\alpha}}(\underline{r}, t) = \chi(\underline{r}) \left[ -\frac{\Delta}{\epsilon_0} \underline{j} + c^2 \dot{\underline{r}} \times \underline{B} - \frac{c}{r} (\underline{r} \cdot \underline{E}_{\perp}) \right]$$

$$\left[ \underline{r} \cdot \underline{E}_{\perp} + \underline{r} \cdot \underline{E}_{\perp} \right]$$

$$= \chi(\underline{r}) \left[ \frac{\Delta}{\epsilon_0} \underline{j} + \underbrace{\dot{\underline{r}} c}_{= \omega} \underline{r} \times \underline{B} - \underline{E}_{\perp} \right] = 0$$

es folgt  $\dot{\underline{\alpha}}(\underline{r}, t) = -\dot{\omega} \underline{\alpha}(\underline{r}, t) + \frac{\dot{\omega}}{\sqrt{2\epsilon_0 \omega}} \underline{j}$

$$\chi(\underline{r}) = -\dot{\omega} \sqrt{\frac{\epsilon_0}{2\omega}}$$

(ohne Inhomogenität  $\dot{}$ ,  
einfacher harmonischer  
Oszillation)

$$\text{Lagrangendichte } \mathcal{L} = \frac{\epsilon_0}{2} E^2 - \frac{1}{2\mu_0} B^2$$

$$\mathcal{L} = \frac{1}{2} \sum_{i=x,y,z} \left\{ \epsilon_0 \dot{A}_i^2 - \mu_0^{-1} (\nabla \times \underline{A})_i^2 \right\}$$

einsetzen  $\frac{\partial \mathcal{L}}{\partial f} = \frac{\partial \mathcal{L}}{\partial t} \frac{\partial t}{\partial f} + \sum_j \frac{\partial \mathcal{L}}{\partial x_j} \frac{\partial x_j}{\partial f}$

Erkennung:  $f_i = \{A_x, A_y, A_z\}$

Impulsvariable  $\pi_j = \pi_{A_i} = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = \epsilon_0 \dot{A}_i$

Kanonische Quantisierung

$$\{ A_i(\underline{R}), \pi_j^*(\underline{R}') \}$$

$$\rightarrow [\hat{A}_i(\underline{R}), \hat{\pi}_j^*(\underline{R}')] = \delta_{ij} \delta(\underline{R} - \underline{R}')$$

Diskutiere für Bosonen  $[A, B]_- = AB - BA$   
für Fermionen  $[A, B]_+ = AB + BA$

Experimente: Bosonen zeigen  
Bose-Einstein-Kondensat  
(-)  
Pauli-Prinzip für  
Fermionen (+)

$$\underline{\alpha} \rightarrow \hat{a}, \quad [\hat{a}_i(\underline{k}), \hat{a}_j(\underline{k}')] = \delta_{ij} \delta_{\underline{k}-\underline{k}'}$$

womit gilt:

$$H = \int d^3r \mathcal{H} = \int d^3r \left\{ \sum_i \left[ \frac{\partial \hat{f}_i}{\partial x} \right] \hat{\pi}_i - \mathcal{L} \right\}$$

$$= \dots = \sum_{\underline{k}} \hbar \omega_{\underline{k}} \left( \hat{\alpha}_{\underline{k}}^\dagger \hat{\alpha}_{\underline{k}} + \frac{1}{2} \right)$$

Anwendung auf die Spontane Emission von  $N$ -Atomen und dies im Operatorbild

→ Superradianz  
und Subradianz

$$H = \sum_{\alpha=1}^N \hbar \omega_{\alpha} \hat{\sigma}_{\alpha}^{+} \hat{\sigma}_{\alpha}^{-} \quad \leftarrow \quad \hat{\sigma}_{\alpha}^{-} \equiv |g\rangle_{\alpha} \langle e|$$

Flip-Operator  
 $\hbar \omega_{\alpha} \equiv$  Übergangsfrequenz

$$+ \sum_{\lambda=1,2} \int d^3r \hbar \omega_{\underline{k}} b_{\lambda \underline{k}}^{+} b_{\lambda \underline{k}}$$

$\lambda =$  Polarisation  
 $\underline{k} =$  Wellenzahl  
 $\underline{k} = \frac{2\pi}{\lambda}$

$$- \sum_{\alpha=1}^N \mu_{\alpha} \cdot \underline{\hat{E}}(\underline{x}_{\alpha}) (\hat{\sigma}_{\alpha}^{+} + \hat{\sigma}_{\alpha}^{-}) \quad \mu_{\alpha} \equiv \text{vekt}$$

zerlegt das  $\underline{E}$ -Feld in (+), (-)

$$\underline{E}(\underline{x}_\alpha) = \underline{E}^{(+)}(\underline{x}_\alpha) + \underline{E}^{(-)}(\underline{x}_\alpha)$$

$$\underline{E}^{(+)} = \sum_{\underline{k}} \int d^3R \sqrt{\frac{\hbar \omega}{2 \epsilon_0 (2\pi)^3}} \underline{\epsilon}_{\underline{k}R} b_{\underline{k}R} e^{i \underline{k} \cdot \underline{x}_\alpha}$$

$$\underline{E}^{(-)} = (\underline{E}^{(+)})^\dagger$$

$\nearrow$  Vakuum-Feldamplitude  
 $\nwarrow$  Polarisations-tensor

Annahme: identische Atome  $\omega_\alpha = \omega_{eg}$   
 ohne Überlapp  $[\sigma_\alpha^{(\pm)}, \sigma_\beta^{(\pm)}] = 0$   
 $(\alpha \neq \beta)$

Selbe Orientierung  $\underline{r}_\alpha = \underline{r}$

$\rightarrow$  alle Atome koppeln an dasselbe Photoreservoir

Vorgehen: Herleitung der BWG. einer beliebigen Operators  $a$ , über die Heisenberg BWG. (Photoreservoir ausgepurt)

$$\begin{aligned} \dot{a} &= \frac{i}{\hbar} [H, a] = \\ &= i \sum_{\alpha=1}^N [\omega_{eg} \sigma_\alpha^+ \sigma_\alpha, a] + \\ &+ \sum_{\underline{k}} \int d^3R \left\{ \frac{i}{\hbar} \frac{\hbar \omega}{2\pi} \left[ \sum_{\alpha=1}^N e^{i \underline{k} \cdot \underline{x}_\alpha} S_{\alpha, a} \right] \underline{\epsilon}_{\underline{k}R} b_{\underline{k}R} + \right. \\ &\quad \left. + \underline{\epsilon}_{\underline{k}R}^* b_{\underline{k}R}^\dagger \left[ \sum_{\alpha=1}^N e^{i \underline{k} \cdot \underline{x}_\alpha} S_{\alpha, a} \right] \right\} \end{aligned}$$

$$S_\alpha := \sigma_\alpha^+ + \sigma_\alpha^-$$

$$b_{\underline{x}R} = \frac{y}{\pi} [H_1 b_{\underline{x}R}] =$$

$$= -i\omega_R b_{\underline{x}R} + \frac{i}{\hbar} \epsilon_R^* M \cdot \underline{e}_{\underline{x}R} \left[ \sum_{\alpha=1}^N e^{-i\vec{k} \cdot \underline{x}} s_{\alpha} \right]$$

Lösung über formale Integration

$$b_{\underline{x}R}(t) = b_{\underline{x}R}(0) e^{-i\omega_R t} + \frac{i}{\hbar} \epsilon_R^* M \cdot \underline{e}_{\underline{x}R} \int_0^t dt' e^{-i\omega_R(t-t')} \sum_{\alpha=1}^N e^{-i\vec{k} \cdot \underline{x}} s_{\alpha}(t')$$

Einsetzen in die Bwgl. für  $a(t)$ .  
Wir sind sowieso nur an den  
Vakuumfall interessiert

$$\dot{a}(t) =$$

$$= i \sum_{\alpha=1}^N \omega_{\alpha} [\sigma_{\alpha}^+ \sigma_{\alpha}^- a] +$$

$$+ \frac{1}{\hbar} \sum_{\underline{x}} \int d^3\underline{x} \epsilon_R |M \cdot \underline{e}_{\underline{x}R}|^2 \int_0^t dt' e^{-i\omega_R(t-t')} \left[ \sum_{\alpha, \beta} e^{-i\vec{k} \cdot \underline{x}_{\alpha\beta}} [s_{\alpha}(t')] s_{\beta}(t') \right]$$

$$+ h.o.a., \quad \underline{x}_{\alpha\beta} = \underline{x}_{\alpha} - \underline{x}_{\beta}$$

Auswerten  $|M \cdot \underline{e}_{\underline{x}R}|^2$ , hierfür die  
Orthogonalität der Polarisationsrichtung  
verwenden [Schedt, Bd. 4, Quantisierung  
der Felder]

$$|M \cdot \underline{e}_{\underline{x}R}|^2 = \sum_{\underline{x}\underline{x}'} \sum_{\underline{e}_i \underline{e}_j} M_i M_j e_{\underline{x}R}^i e_{\underline{x}'R}^j$$

$$= \sum_{ij} \mu_i \mu_j \underbrace{\sum_{\underline{x}, \underline{x}'} e^{i \underline{x} \cdot \underline{r}} e^{i \underline{x}' \cdot \underline{r}}}_{= \delta_{\underline{x}, \underline{x}'}}$$

$$= \sum_{ij} \mu_i \mu_j \underbrace{\sum_{\underline{x}=\underline{x}'} e^{i \underline{x} \cdot \underline{r}} e^{i \underline{x} \cdot \underline{r}}}_{= \delta_{ij} - \frac{\underline{x}_i \underline{x}_j}{r^2}}$$

$$= \sum_i \mu_i^2 - \sum_{ij} \frac{\underline{x}_i \underline{x}_j \mu_i \mu_j}{r^2}$$

$$= \mu^2 - (\hat{\underline{x}} \cdot \underline{\mu})^2 \quad \hat{\underline{x}} = \frac{\underline{x}}{|\underline{x}|}$$

$$= \mu^2 [1 - (\hat{\underline{x}} \cdot \underline{\mu})^2]$$

$$\int d^3 \underline{r} \rightarrow \int_0^\infty d\underline{r} \underline{r}^2 \underbrace{\int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin \theta}_{\int d\Omega_{\underline{r}}}$$

$$|\underline{E}| = \frac{\mu \omega^2}{2 \epsilon_0 (2\pi)^3}$$

[Cohenberg PRA 2, 883 (1970)]

$$\dot{\underline{a}}(t) = \dot{\underline{y}}[\dots, \dots]$$

$$+ \frac{\mu^2}{2 \epsilon_0} \frac{c}{(2\pi)^3} \sum_{\alpha, \beta} \int d\Omega_{\underline{r}} [1 - (\hat{\underline{r}} \cdot \underline{\mu})^2] \quad \text{⊗}$$

$$\text{⊗} = \int_0^+ dt' \int_0^\infty d\underline{r} \underline{r}^3 \frac{j\omega [t-t' - \frac{\hat{\underline{r}} \cdot \underline{x}_{\alpha\beta}}{c}]}{[\underline{r} \cdot \underline{x}(t), \underline{r} \cdot \underline{x}(t')] f_{\beta}(t')}$$

+ h. a.