Mollow Triplett: (Resonanzfluoreszenz II)

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Messung: $S(\omega) = \mathrm{Re}\left[\int_0^\infty d\tau \, \langle \sigma_+(0)\,\sigma_-(\tau)\rangle_{ss} \, e^{i\omega\tau}\right]$

im Steady State $\xrightarrow{\Omega_R}$ $|b\rangle$, $|a\rangle$, Γ (radiativer Zerfall)

$$\lim_{t\to\infty} \langle \sigma_-(t)\rangle = -i \frac{\Gamma \Omega_R}{\Gamma^2 + 2\Omega_R^2}$$

$$\lim_{t\to\infty} \langle \sigma_z(t)\rangle = \frac{-\Omega_R^2}{\Gamma^2 + 2\Omega_R^2}$$

Ziel: Zwei-Zeiten-Korrelationen

$$\lim_{t\to\infty} \langle \sigma_+(t)\,\sigma_-(t+\tau)\rangle = \langle \sigma_+(0)\,\sigma_-(\tau)\rangle_{ss}$$

Lösung

$$\begin{aligned} \langle \sigma_-(t)\rangle = {} & a_1(t) + a_2(t)\langle \sigma_-(0)\rangle \\ & + a_3(t)\langle \sigma_+(0)\rangle \\ & + a_4(t)\tfrac{1}{2}\left\{\langle \sigma_z(0)\rangle + 1\right\} \end{aligned}$$

$$a_1(t) = -i\frac{\Omega_R \Gamma}{\Gamma^2 + 2\Omega_R^2} \left\{1 - e^{-\frac{3}{4}\Gamma t}\left[\cos\mu t - \left(\frac{4\Omega_R^2 - \Gamma^2}{4\mu\Gamma}\right)\sin\mu t\right]\right\}$$

Hilft uns das für $\langle \sigma_+(0)\sigma_-(\tau)\rangle$?

Quantenregression (einfach!)

$$\frac{d}{d\tau}\langle A(\tau)\rangle = \langle \frac{d}{d\tau} A(\tau)\rangle$$

$$= -\frac{i}{\hbar}\langle [H, A]\rangle = a_1 \langle B(\tau)\rangle + a_2 \mathbb{1}$$

$$\frac{d}{d\tau}\langle B(\tau)\rangle = b_1 \langle A(\tau)\rangle + b_2$$

$\longrightarrow$ Lösung wäre dann

$$\langle A(\tau)\rangle = c_1(t)\langle A(0)\rangle + c_2(t)\langle B(0)\rangle + c_3(t)$$

$$\frac{d}{d\tau}\langle C(t) A(\tau)\rangle = \langle C(t)\frac{d}{d\tau} A(\tau)\rangle$$

$$= a_1 \langle C(t) B(\tau)\rangle + a_2 \langle C(t)\rangle$$

$$\frac{d}{d\tau}\langle C(t) B(\tau)\rangle = b_1 \langle C(t) A(\tau)\rangle + b_2 \langle C(t)\rangle$$

benenne um $C(t)A(\tau) = A'(\tau)$

$$\langle A'(\tau)\rangle = c_1(\tau)\langle A'(0)\rangle + c_2 \langle \mathbb{1}\rangle$$

$$A'(0) = C(t)A(0)$$

$$\langle A(t+\tau)\rangle = a_1(\tau)\langle A(t)\rangle + a_2(\tau)\langle B(t)\rangle + a_3(\tau)$$

$$\langle A'(t)\rangle = \langle C(t)\, A(t+\tau)\rangle$$

$$= \tilde{a}_1(\tau)\langle C(t)\, A(t)\rangle + \tilde{a}_2\langle C(t)\, B(t)\rangle + \tilde{a}_3(\tau)\langle C(t)\rangle$$

allg.: Onsager–Lax–Regressions Theorem

existiert $\langle \hat{O}(t+\tau)\rangle = \sum_j a_j(\tau)\langle O_j(t)\rangle$

dann $\langle \hat{O}_i(t)\,\hat{O}(t+\tau)\,\hat{O}_k(t)\rangle$

$$= \sum_j a_j(\tau)\langle \hat{O}_i(t)\,\hat{O}_j(t)\,\hat{O}_k(t)\rangle$$

$$\langle \sigma_+(t)\,\sigma_-(t+\tau)\rangle =$$

$$= a_1(\tau)\langle \sigma_+(t)\rangle + a_2(\tau)\langle \sigma_+(t)\sigma_-(t)\rangle$$

$$+ a_3(\tau)\langle \cancel{\sigma_+(t)\sigma_+(t)}\rangle$$

$$+ a_4(\tau)\frac{1}{2}\left\{\langle \cancel{\sigma_+(t)(\sigma_z+1)}\rangle\right\}$$

$|b\rangle\langle a|\left(|b\rangle\langle b| - |a\rangle\langle a|\right) = -|b\rangle\langle a|$

$$= a_1(\tau)\langle \sigma_+(t)\rangle + a_2(\tau)\underbrace{\langle \sigma_+(t)\sigma_-(t)\rangle}_{\text{auch bekannt}}$$

$$\langle \sigma_+(t)\sigma_-(t)\rangle = \frac{1}{2}\left\{\langle \sigma_z(t)\rangle + 1\right\}$$

$$= s_1(t) = \frac{\Omega_R^2}{\Gamma^2 + 2\Omega_R^2}\left\{1 - \left(\cos\mu t + \frac{3\Gamma}{4\mu}\sin\mu t\right)e^{-\frac{3}{4}\Gamma t}\right\}$$

für $\langle \sigma_z(0)\rangle = -1$, $\langle \sigma_-(0)\rangle = 0 = \langle \sigma_+(0)\rangle$

$$\lim_{t\to\infty}\langle \sigma_+(t)\sigma_-(t+\tau)\rangle = \left[a_1(\tau)\, a_1^*(\infty) + a_2(\tau)\, s_1(\infty)\right]e^{i\omega_0\tau}$$

$$:= e^{i\omega_{ba}\tau}\left(\frac{\Omega_R^2}{\Gamma^2+2\Omega_R^2}\right)\Bigg\{\frac{\Gamma^2}{\Gamma^2+2\Omega_R^2}+\frac{e^{-\Gamma\tau/2}}{2}+$$

$$+\frac{e^{-3/4\Gamma\tau}}{4}\Bigg[e^{-i\overbrace{\sqrt{\Omega_R^2-\left(\frac{\Gamma}{4}\right)^2}}^{}\tau}\left\{\frac{2\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2}+i\frac{\Gamma}{4\mu}\frac{10\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2}\right\}$$

$$\underbrace{}_{=\mu}$$

$$+e^{i\mu\tau}\left\{\frac{2\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2}-i\frac{\Gamma}{4\mu}\frac{10\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2}\right\}\Bigg]\Bigg\}$$

Diskussion:

(1) schwache Anregung $\Omega_R \ll \frac{\Gamma}{4}$:

also für $\mu = \sqrt{\Omega_R^2-\left(\frac{\Gamma}{4}\right)^2} \approx i\,\Gamma/4$

$$\Leftrightarrow\infty$$

$$\langle\sigma_+(t)\,\sigma_-(t+\tau)\rangle =$$

$$= e^{-i\omega_{ba}\tau}\left(\frac{\Omega_R^2}{\Gamma^2+\cancel{2\Omega_R^2}}\right)$$

$$\Bigg\{\underbrace{\frac{\Gamma^2}{\Gamma^2+\cancel{2\Omega_R^2}}}_{=1}+\frac{e^{-\Gamma\tau/2}}{2}+$$

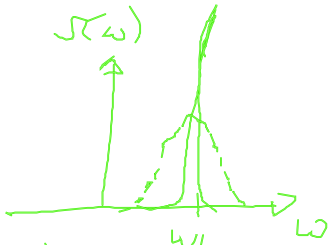
$$+e^{-3\Gamma/4}\Bigg[e^{\Gamma\tau}\left(\frac{\cancel{2\Omega_R^2}-\Gamma^2}{\underbrace{\cancel{2\Omega_R^2}+\Gamma^2}_{=-1}}+i\frac{\Gamma}{i4\Gamma}\frac{10\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2}\right)$$

$$+e^{-\Gamma\tau}\left(\frac{\cancel{2\Omega_R^2}-\Gamma^2}{\underbrace{\cancel{2\Omega_R^2}+\Gamma^2}_{=-1}}-i\frac{\Gamma}{4i\Gamma}\frac{10\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2}\right)\Bigg]$$

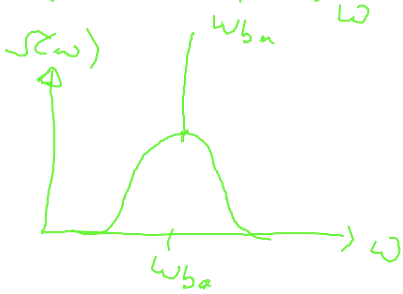
$$= e^{-i\omega_{ba}}\left(\frac{\Omega_R}{\Gamma}\right)^2\left\{\underline{\underline{1}}+\cancel{\frac{e^{-\Gamma\tau/2}}{2}-\frac{e^{-\Gamma/2\,\tau}}{2}}\right\}$$

$$S(\omega) = \mathrm{Re}\left[\int_0^\infty d\tau \, \langle \sigma_+(t)\sigma_-(t+\tau)\rangle_{ss} \, e^{i(\omega-\omega_{ba})\tau}\right]$$

$$= \left(\frac{\Omega_R}{\Gamma}\right)^2 \delta(\omega - \omega_0)$$



elastische Streuung



(2) starke Anregung $\Omega_R \gg \frac{\Gamma}{4}$

$\rightarrow$ die komplette 2-Zeiten-Korrelation muss mitgenommen werden

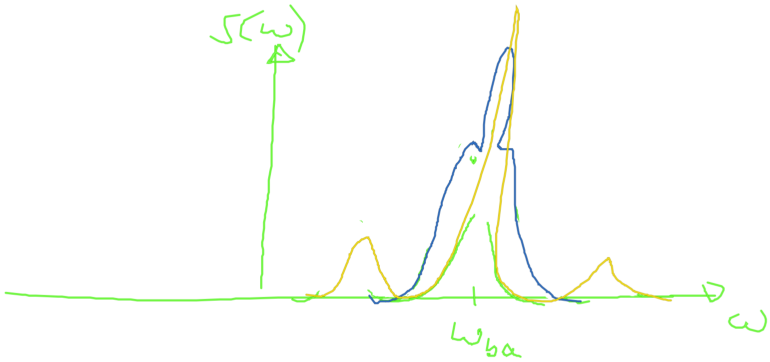
$S(\omega)$ konvergiert, da $\mu = \sqrt{\Omega_R^2 - \left(\frac{\Gamma}{4}\right)^2}$ reell, $\Gamma\tau$ ist vorhanden

$$S(\omega) = \left(\frac{\Omega_R^2}{2\Omega_R^2+\Gamma^2}\right)\Bigg[\frac{4\pi\Gamma^2}{\Gamma^2+2\Omega_R^2}\,\delta(\omega_{ba}-\omega) + \qquad A = \frac{10\Omega_R^2 - \Gamma^2}{2\Omega_R^2+\Gamma^2}$$

$$+ \frac{\Gamma^2}{(\omega_{ba}-\omega)^2+\left(\frac{\Gamma}{2}\right)^2} + \frac{\frac{3\Gamma}{4}\cdot\frac{2\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2} + (\omega_{ba}+\mu-\omega)\frac{\Gamma}{4\mu}A}{(\omega_{ba}-\omega+\mu)^2+\left(\frac{3\Gamma}{4}\right)^2}$$

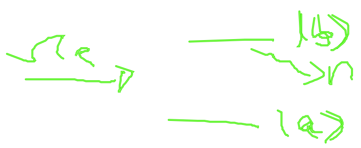
$$+ \frac{(3\Gamma/4)\,\frac{2\Omega_R^2-\Gamma^2}{2\Omega_R^2+\Gamma^2} - (\omega_{ba}-\mu-\omega)\frac{\Gamma}{4\mu}\,\frac{10\Omega_R^2-\Gamma^2}{4\mu(2\Omega_R^2+\Gamma^2)}}{(\omega_{ba}-\mu-\omega)^2+(3\Gamma/4)^2}\Bigg]$$

→ zusätzliche Features erscheinen



⟹ Mollow-Triplett

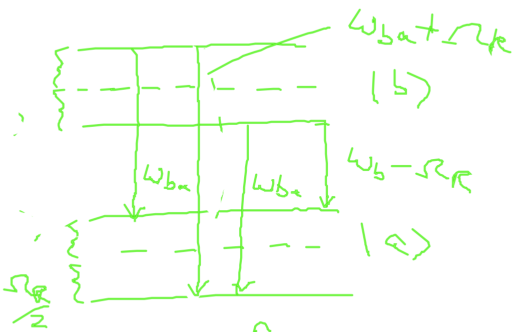
$\Omega_R \gg \frac{\Gamma}{4}$ Peaks (relativ) 1 : 3 : 1



$\xrightarrow{\text{lineares}}$ Spektrum

Start anlegen

"dressed state"



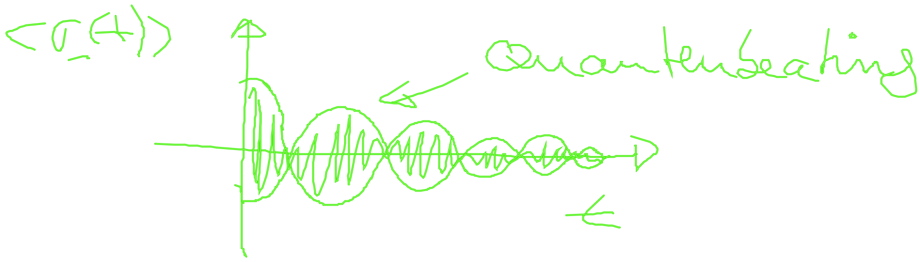
gemessen Grove et al. PRA 19, 227 (1977)

→ Beweis der Rabioszillation,
da Peaks mit Ω_R shiften

Erinnere: $|c_b|^2 = \sin[\Omega_R t]$, $c_b = 0$

das ist der neue Beitrag

≙ AC-Stark-Effekt
(dynamischer Stark-Effekt)



gemessen (u.a.) Na $3^2S_{1/2} \to 3^2P_{3/2}$
Übergang optisch aktiv

$$\frac{\Omega_R}{2\pi} = 78\,\mathrm{MHz}$$