SUMMARY

Networks of excitable systems

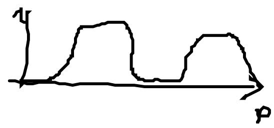
Excitability type I SNIPER $\boxed{\begin{aligned}\dot r &= r(1-r^2)\\ \dot\varphi &= b - r\cos\varphi\end{aligned}}$

" II FHN $\boxed{\begin{aligned}\varepsilon\dot u &= u - \frac{u^3}{3}\\ \dot v &= u + a\end{aligned}}$

Desync in small-world networks:

- with additional links p

- SNIPER with small σ



5. WECHSELSPIEL VON DELAY UND RAUSCHEN

$\left.\begin{array}{ll}\text{BISHER :} & \text{DETERMINIST.}\\ \text{NUN :} & \text{STOCHAST.}\end{array}\right\}$ dyn. Systeme

STOCHAST. PROZESS

ZEITENTWICKL.

Zufallsvariable $X(t)$

($\longleftrightarrow$ thermodyn. Gleichgewicht)

Zeit$\underline{\text{unabhängig}}$

$$P(x_1, t_1; x_2, t_2; \dots) = P(x_1, t_1)\, P(x_2, t_2 | \dots)$$

Realisationen von $X(t)$

MARKOFF PROZESS

verbundswahrscheinl.

$$P(x_1, t_1 | x_2, t_2; \dots) := \frac{P(x_1, t_1; x_2, t_2; \dots)}{P(x_2, t_2; \dots)}$$

bedingte Wahrscheinl.

$t_1 \geq t_2$

AMNESIE

$$:= P(x_1, t_1 | x_2, t_2)$$

LANGEVIN - GLEICHUNG

$$m\ddot{x} = -\eta\dot{x} + \xi(t)$$

Reibung Rauschen

$\boxed{\text{Gauß'sches white noise}}$

$\langle \dots \rangle$ stat. Mittel

- $\langle \xi(t) \rangle = 0$
- $\langle \xi(t)\, \xi(t') \rangle = \delta(t - t')$ unkorrel.
- zentraler Grenzwertsatz: $\xi(t) \rightarrow$ Gauß. verteilung

AUTOKORREL. FKT

$$\Psi(s) := \langle (x(t) - \langle x \rangle)\, (x(t+s) - \langle x \rangle) \rangle$$

$\langle x \rangle = 0$

ergod. System

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} dt\, x(t)\, x(t+s)$$

WIENER – KHINCHIN – THEOREM

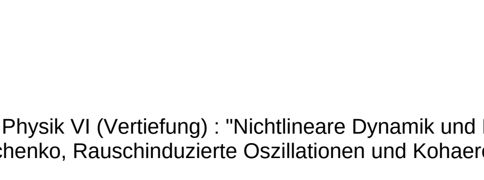
Fourier Trafo von $\Psi(s)$ $\hat{=}$ Spektrale Leistungsdichte

$$S(\omega) := \lim_{T\to\infty} \frac{\pi}{T} \left|\hat{x}(\omega, T)\right|^2$$

$$\hat{x}(\omega, T) = \frac{1}{2\pi}\int_{-T}^{T} dt\, e^{i\omega t} x(t)$$

$$\boxed{\Rightarrow S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\langle x(t)\, x(t+s)\rangle}_{\delta(s)} e^{i\omega s} ds}$$

$$\xrightarrow{\text{white noise}} S(\omega) = \frac{1}{2\pi}$$



Rauschinduzierte Oszillationen

1. Bsp.

$$\boxed{\begin{aligned}\dot{x} &= y\\ \dot{y} &= (\varepsilon - x^2)\,y - \omega_0^2 x + D\,\xi(t)\end{aligned}}$$

VdP:

$\varepsilon > 0$ inst. Fokus + LC

$\varepsilon = 0$ Hopf Bif.

$\varepsilon < 0$ stab. Fokus

$$D = 0 \left\{ \begin{aligned} &\varepsilon = -0{,}01,\ \omega_0 = 1\\ &\lambda = \frac{\varepsilon}{2} \pm i\sqrt{\omega_0^2 - \left(\frac{\varepsilon}{2}\right)^2} \approx \frac{\varepsilon}{2} \pm i\omega_0 \end{aligned}\right.$$

$D \neq 0$ rauschind. Osz

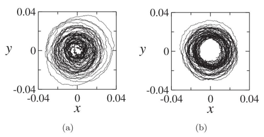
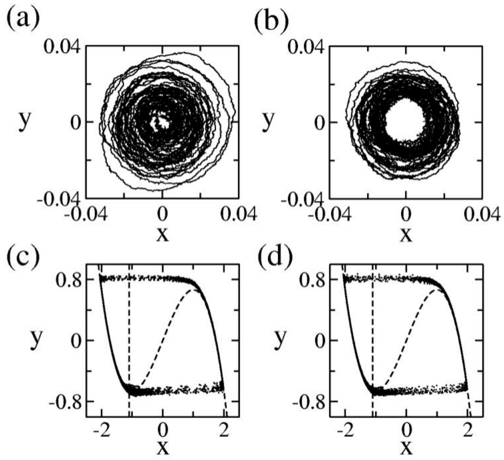


Fig. 1. Numerically simulated phase portraits of noise-induced oscillations of the Van der Pol system at $\omega_0 = 1$, $\varepsilon = -0.01$, $D = 0.003$: (a) without feedback $K = 0$; (b) with feedback $K = 0.2$, $\tau = T_0$. In both cases the system was integrated during 300 time units.

2. Bsp. FHN

$$\begin{aligned}\dot{x} &= x - \frac{x^3}{3} - y\\ \dot{y} &= x + a + D\,\xi(t)\end{aligned}$$

$|a| > 1$



Vdp

FHN

FIG. 1. Phase portraits of noise-induced motion: (a),(b) Van der Pol oscillator at $D = 0.003$; (c),(d) FitzHugh-Nagumo system at $D = 0.09$ (the dashed lines denote the null isoclines), (a),(c) $K = 0$; (b),(d) $K = 0.2$, $\tau = T_0$.

KOHÄRENZ RESONANZ

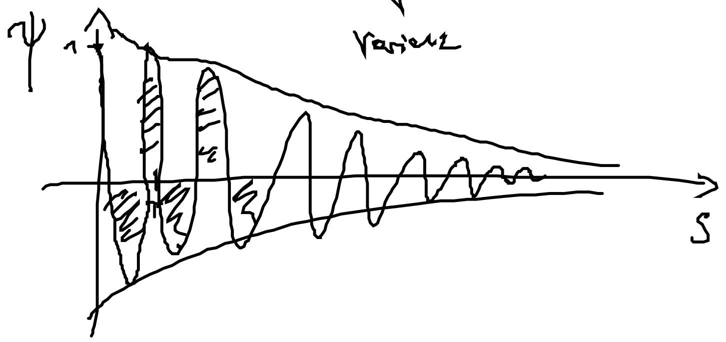
(PIKOVSKY, KURTHS, PRL 78 (1997))

konstruktive Einfluss von noise:

KOHÄRENZ (REGULARITÄT) optimal
für ein $D_{OPT} > 0$

$\boxed{\text{Maß für KOHÄRENZ: KORRELATIONSZEIT } t_{COR}}$

$$t_{cor} = \frac{1}{\underbrace{\psi(0)}_{\text{Varianz}}} \int_0 |\psi(s)| \, ds$$



Bsp. $\dot{x} = -(\lambda - i\omega_0)x + \xi(t)$

$$\psi(s) = \psi(0) \, e^{-\lambda s} \cos(\omega_0 s)$$

$$t_{cor} = \int_0^\infty e^{-\lambda s} \underbrace{|\cos(\omega_0 s)|}_{} \, ds$$

<u>$\lambda \ll \omega_0$</u>

$$\frac{1}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\phi \, d\phi = \frac{2}{\pi}$$

$$\approx \frac{2}{\pi} \int_0^\infty e^{-\lambda s} \, ds = \frac{2}{\pi \lambda}$$

$$\psi(s) = \psi(0) \, e^{-\frac{2}{\pi}\frac{s}{t_{cor}}} \cos(\omega_0 s)$$

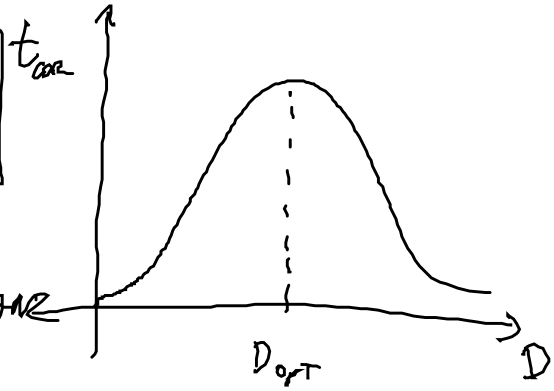
exponentielle abklingende Korrelation

linear

$\Updownarrow$

FHN (non linear)

$|a| > 1$



KOHÄRENZRESONANZ

für optimale Rauschintensität D_{opt}