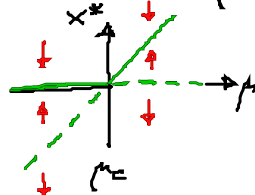
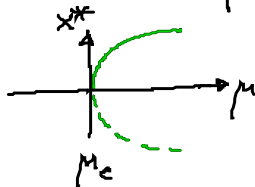


English Summary:

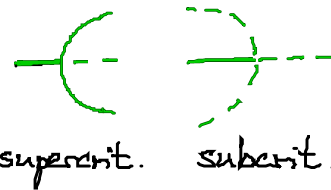
1.3 Bifurcations

(A) Zero-eigenvalue bifurcation: $\lambda < 0 \rightarrow \lambda > 0$
 $\det A > 0 \quad \det A < 0$

saddle-node bifurcation transcrit. bif.



pitchfork bif.



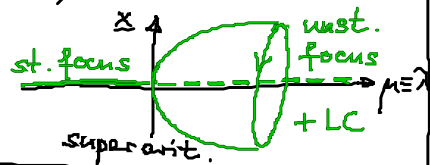
(B) Hopf bifurcation: $\lambda_{1,2} = \lambda_0 \pm i\omega$, $\lambda_0 < 0 \rightarrow \lambda_0 > 0$

Hopf normal form

(Stuart-Landau osc.) $z = r e^{i\varphi} \in \mathbb{C}$

$$\dot{z} = (\lambda + i\omega \mp (1 + i\gamma)|z|^2) z$$

$$\begin{cases} \dot{r} = (\lambda \mp r^2) r \\ \dot{\varphi} = (\omega \mp \gamma r^2) \end{cases} \quad \text{Super-/subcrit.}$$



$$\begin{cases} \text{amplitude } r \rightarrow 0 \\ \text{frequency } \omega \neq 0 \end{cases}$$

linear stab.:
Floquet modes
 $\delta z = e^{\lambda t} u(t)$

Floquet multiplier $\mu = e^{\lambda T}$

$u(t+T)$

(C) Lokale Bifurkation von Grenzzyklen

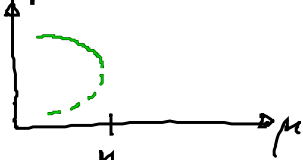
Startpt. jetzt: Grenzzyklus, nicht Fixpt.

keine einfache lin. Stabilitätsanalyse

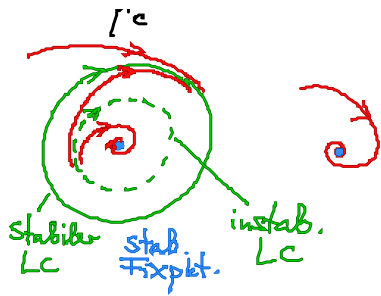
(C1) Sattel-Knoten-Bifurkation eines Grenzzyklus

(Kondensation von Pfaden, fold bifurcation of limit cycles)
LC

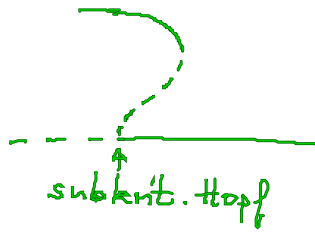
Amplitude



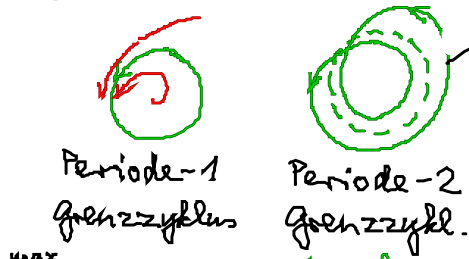
$$\begin{cases} \text{Amplitude } \neq 0 \\ \text{Frequenz } \neq 0 \end{cases}$$



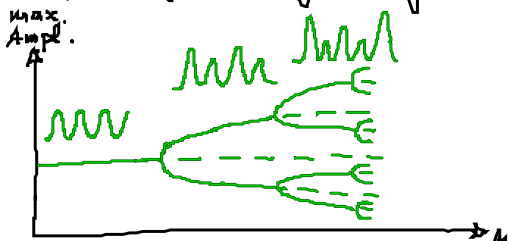
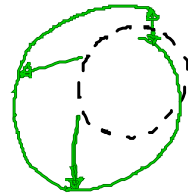
künftig Bistabilität zwischen Fixpt. u. LC



(C2) Perioden-Verdopplung (flip-Bifurkation, subharmon. Bif.)



mind. 3D Phasenraum



Torsion benachbarter Traj.!
Floquet-Exp. $\Lambda = \lambda + i\omega$

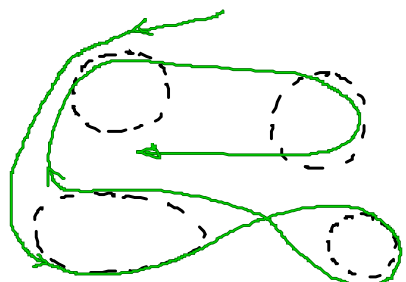
$n=0$ $n=1$ $n=2 \dots n$
 T $2T$ $4T$ $2^n T$

Bif. : $\lambda = 0$, $\omega T = \pi$
 \Rightarrow Floquet-Multiplikator

$$\mu = e^{\Lambda T} = e^{i\pi} = -1$$

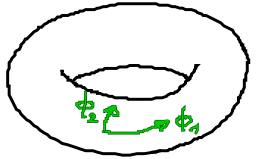
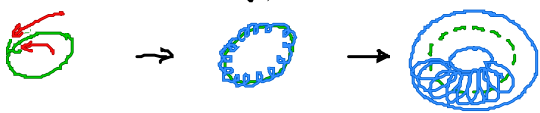
häufig Periodenverdopplungskaskade
 \rightarrow Chaos (Feigenbaum-Szenario)

\rightarrow unendlich viele instabile period. Orbits der Periode $2^n T$ ($n=0, 1, 2, \dots$)



(C3) Sacker-Neimark-Bifurkation

(Sekundäre Hopf-Bif. eines LC)



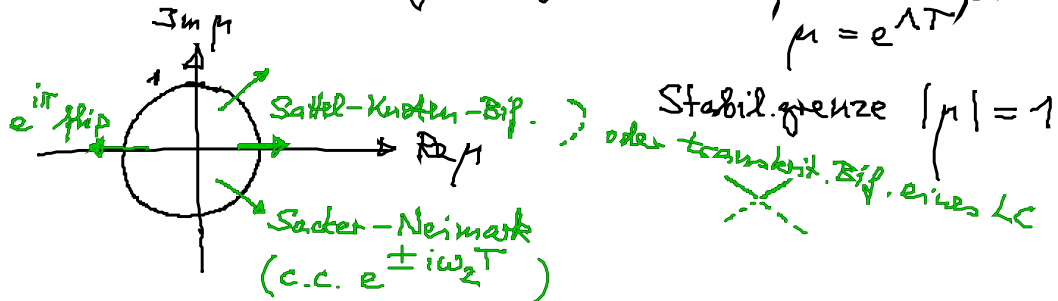
1. Hopf-Bif. des Fixp. : $\phi_1 = \omega_1 t$
2. Hopf-Bif. des LC : $\phi_2 = \omega_2 t$

LC \longrightarrow 2-Torus mind. 3D Phasenraum
 inkommensurable Frequenzen $\frac{\omega_1}{\omega_2} \notin \mathbb{Q}$
 (quasiperiodisch) irrational

\rightarrow Trajektorie schließt sich nicht, liegt dicht auf dem Torus

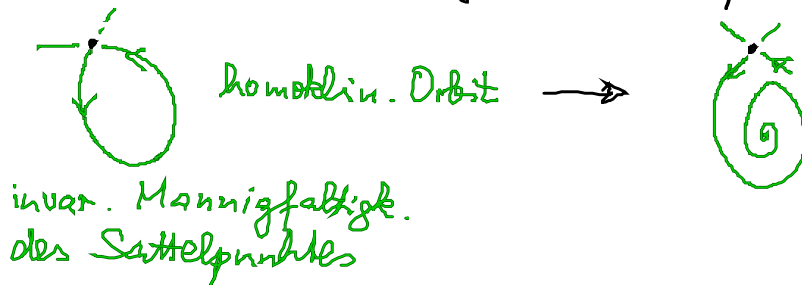
falls $\frac{\omega_1}{\omega_2} \in \mathbb{Q} \Rightarrow$ geschlossener Orbit = LC
 (frequency locking)
 Modenkopplung

• Instabilität von Grenzzyklus = Floquet-Multiplikator $\mu = e^{\lambda T}$



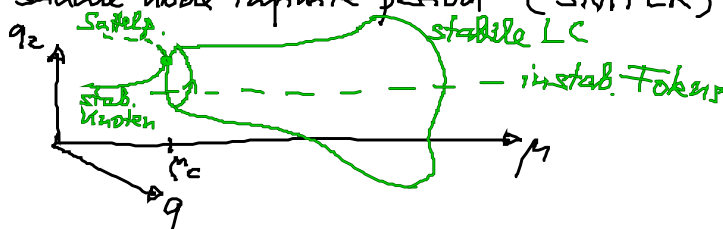
(D) globale Bifurkation von Grenzzyklen

globale qualitative Änderung des Phasenporträts,
 z.B.

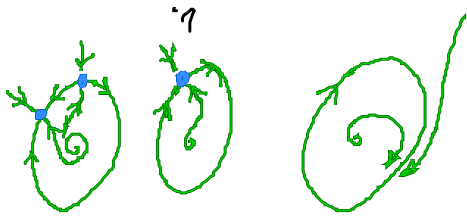


(D1) Sattel-Knoten-Bif. auf einem Grenzzyklus (Omega explosion)

saddle-node on invariant cycle (SNIC)
 saddle-node infinite period (SNIPER)

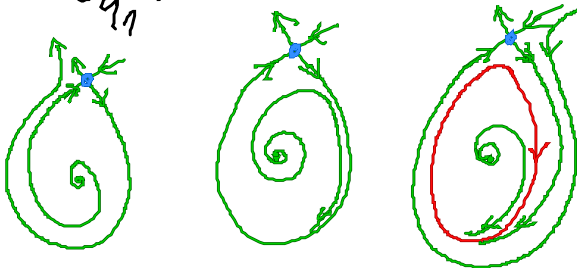
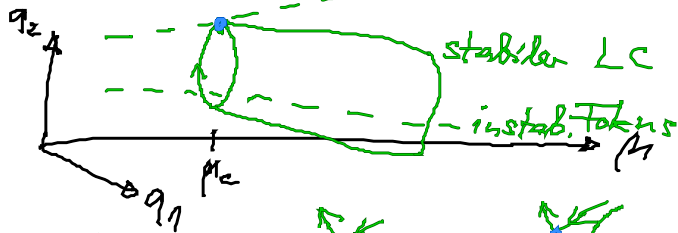


Amplitude $\neq 0$
 Freq. $\rightarrow 0$



(D2) Homokline Bif. (blue-sky catastrophe)

Sattelpkt. kollidiert mit LC: homokliner Orbit (saddle-to-saddle loop)

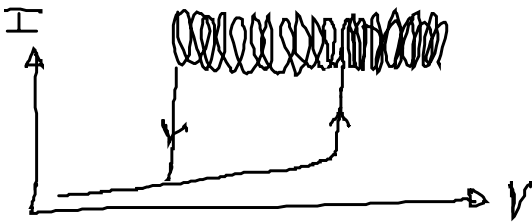


Ampl. $\neq 0$
Frequ. $\rightarrow 0$

häufig Bistab. zwischen Osz. u. Fixpkt., z.B.



stabile Fixpkt.



z.B. nichtlinearer Halbleiter
Hysteresis von
Osz. u. stationärem
Strom