Wdh

- Säkularnäherung: vernachlässige osz. Terme im WW-Bild

$$\tilde{A}_\alpha(t) = e^{+iH_S t} A_\alpha e^{-iH_S t}$$

=> Schwierig bei (Fast-) Entartungen

=> Born, Markov + Säkular => immer Lindblad

- Rezept (1) $H_I = \sum_\alpha A_\alpha \otimes B_\alpha$ $\quad (A_\alpha = A_\alpha^\dagger \quad B_\alpha = B_\alpha^\dagger)$ $\qquad Tr\{H_I \bar{\rho}_B\} = 0$

(2) $$\gamma_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} \underbrace{Tr\{e^{+iH_B\tau} B_\alpha e^{-iH_B\tau} B_\beta \bar{\rho}_B\}}_{C_{\alpha\beta}(\tau)} e^{+i\omega\tau} d\tau$$

[ $\sigma_{\alpha\beta}(\omega) = \frac{i}{\pi} \mathcal{P} \int \frac{\gamma_{\alpha\beta}(\omega')}{\omega - \omega'} d\omega'$ für Lamb-shift ]

(3) diagonalisiere System $H_S |a\rangle = E_a |a\rangle$

→ einsetzen

- Spezialfall falls $H_S$ nicht entartet: Ratengleichung für Populationen in E-EB

$$\dot{\rho}_{aa} = \sum_b \underline{\gamma_{ab,ab}} \rho_{bb} - \sum_b \gamma_{ba,ba} \rho_{aa}$$

$$\sum_{\alpha\beta} \underbrace{\gamma_{\alpha\beta}}_{\text{pos. definit}} (E_b - E_a) \langle a|A_\beta|b\rangle \langle a|A_\alpha|b\rangle^* \geq 0$$

- Beispiel → ME für HO

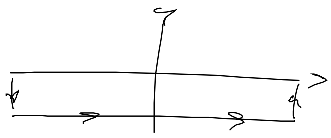
1.3.4. Gleichgewichts-TD

$\bar{\rho}_B = \frac{e^{-\beta H_B}}{Z_B}$ $\longrightarrow$ $\boxed{C_{\alpha\beta}(\tau) = C_{\beta\alpha}(-\tau - i\beta) \quad \text{KMS-Beziehung}}$

$$\gamma_{\alpha\beta}(-\omega) = \int_{-\infty}^{+\infty} C_{\alpha\beta}(\tau) e^{-i\omega\tau} d\tau = \int C_{\beta\alpha}(\underbrace{-\tau - i\beta}_{\tau'}) e^{-i\omega\tau} d\tau \qquad \tau = -\tau' - i\beta$$

$$= \int_{+\infty - i\beta}^{-\infty - i\beta} C_{\beta\alpha}(\tau') e^{+i\omega(\tau' + i\beta)} (-d\tau') = \int_{-\infty - i\beta}^{+\infty - i\beta} C_{\beta\alpha}(\tau') e^{+i\omega\tau'} d\tau' \, e^{-\beta\omega}$$

$$= \underbrace{\int_{-\infty}^{+\infty} C_{\beta\alpha}(\tau') e^{+i\omega\tau'} d\tau'}_{\gamma_{\beta\alpha}(\omega)} e^{-\beta\omega}$$



$$\boxed{\gamma_{\alpha\beta}(-\omega) = \gamma_{\beta\alpha}(+\omega) \cdot e^{-\beta\omega} \quad \text{KMS Beziehung}}$$

$$=> \frac{\gamma_{ab,ab}}{\gamma_{ba,ba}} = e^{\beta(E_b - E_a)}$$

aus $\bar{\rho}_B = \frac{e^{-\beta H_B}}{Z_B}$ $\quad \underset{t\to\infty}{\mathcal{L}_{BMS}} \Rightarrow \mathcal{L}_{BMS}\bar{\rho}_S = 0 \qquad \bar{\rho}_S = \frac{e^{-\beta H_S}}{Z_S}$

System thermalisiert (ergodisches Verhalten)

$\frac{n_B(\omega)}{1+n_B(\omega)} = e^{-\beta\omega}$

Bsp: $\gamma_{11}(\omega) = \tilde{\Gamma}(\omega)\,[1+n_B(\omega)]$

$$\tilde{\Gamma}(-\omega)\,[1+n_B(-\omega)] = [-\tilde{\Gamma}(+\omega)]\,[-n_B(+\omega)] = \tilde{\Gamma}(+\omega)\,[1+n_B(+\omega)]\cdot e^{-\beta\omega}$$

analog $\bar{\rho}_B = \frac{e^{-\beta(H_B-\mu N_B)}}{Z_B}$ $\qquad [H_S, N_S] = 0 = [H_B, N_B]$ $\qquad \bar{\rho}_S = \frac{e^{-\beta(H_S-\mu N_S)}}{Z_S}$

$[H_I, N_S+N_B] = 0$

$\beta$ & $\mu$ equilibrieren

BMS: ME respektieren die TD:

betrachte Entropie des Systems

$\boxed{\begin{array}{l}\text{Sei } \rho \text{ eine DM} \\ S(\rho) = -\mathrm{Tr}\{\rho \ln \rho\} \\ \text{heißt von-Neumann-Entropie}\end{array}}$

EW von $\rho$

$$\rho = \sum_n \rho_n |n\rangle\langle n| \qquad \langle n|m\rangle = \delta_{nm}$$

$$S(\rho) = -\sum_n \rho_n \cdot \ln \rho_n = S_{Sh}(\{\rho_n\})$$

$\rho_n \in [0,1] \qquad \sum_n \rho_n = 1$

- $S(\rho) \geq 0$
- Bsp: bipartites System

$S(\rho_{12}) = 0$

a) $\rho_{12} = |\Psi^a\rangle\langle\Psi^a| \qquad |\Psi^a\rangle = \frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$

$$\rho_1 = \mathrm{Tr}_2\{\rho_{12}\} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \text{ (gemischt)}$$
$$= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \qquad S(\rho_1) = \ln 2$$

$$\rho_{12} = |\Psi^b\rangle\langle\Psi^b| \qquad |\Psi^b\rangle = \underbrace{\frac{1}{\sqrt{2}}[|0\rangle + |1\rangle]}_{|\Psi_1^b\rangle} \otimes [|0\rangle + |1\rangle]\frac{1}{\sqrt{2}}$$

$$\longrightarrow \rho_1 = |\Psi_1^b\rangle\langle\Psi_1^b|$$

$$\longrightarrow S(\rho_1) = 0$$

$\boxed{\begin{array}{l}\text{Seien } \rho \text{ und } \sigma \text{ zwei DM} \\ \text{Dann ist die relative Quantenentropie zwischen } \rho \,\&\, \sigma \text{ gegeben} \\ \text{durch} \\ \qquad D(\rho\|\sigma) = \mathrm{Tr}\{\rho\ln\rho - \rho\ln\sigma\} \geq 0\end{array}}$ Skript FK-Physik

- $D(\rho\|\rho) = 0$
- $D(\rho\|\sigma) \neq D(\sigma\|\rho)$

Lindblad: Kraus-Abb. sind kontraktiv

$\rho' = \mathcal{K}\rho$

$$D(\mathcal{K}\rho\|\mathcal{K}\sigma) \leq D(\rho\|\sigma)$$

$$\mathcal{K} = e^{\mathcal{L}\cdot\Delta t} \qquad \mathcal{L}\bar{\rho} = 0$$

$\sigma = \bar{\rho}$

$$D(\rho\|\bar{\rho}) - D(e^{\mathcal{L}\Delta t}\rho\|\bar{\rho}) \geq 0$$

entwickle für kleine $\Delta t \to 0$

=> $\boxed{\begin{array}{l} \text{Spohn'sche Ungleichung} \quad \mathcal{L}\bar\rho = 0 \\ \quad \mathcal{L} \text{ sei Lindblad-Generator} \\ -Tr\{(\mathcal{L}\rho)[\ln\rho - \ln\bar\rho]\} \geq 0 \end{array}}$

Interpretation ?

Änderung von $S(\rho)$

① Term

$$\frac{d}{dt}S(\rho) = \dot S(\rho) = -Tr\{\dot\rho \ln\rho\} - \underbrace{Tr\{\rho \frac{d}{dt}\ln\rho\}}_{=0} = -Tr\{(\mathcal{L}\rho)\ln\rho\}$$

$$\Rightarrow Tr\{\rho\frac{d}{dt}\ln\rho\} = Tr\{U\rho_0 U^\dagger[\dot U \ln\rho_0 U^\dagger + U\ln\rho_0\dot U^\dagger + U\rho_0^{-1}\cdot\dot\rho_0 U^\dagger]\}$$

$$= Tr\{\rho_0 U^\dagger \dot U \ln\rho_0 + U\rho_0\cdot\ln\rho_0\dot U^\dagger + \dot\rho_0\}$$

$\rho = U\rho_0 U^\dagger$

Diagonalmatrix

$$= Tr\{\rho_0\underbrace{(U^\dagger\dot U + \dot U^\dagger U)}_{=0}\} + \underbrace{Tr\{\dot\rho_0\}}_{=0} = 0$$

$\ln\rho = U\ln\rho_0 U^\dagger$

$$= \frac{d}{dt}U^\dagger U = \frac{d}{dt}\mathbb{1} = 0$$

1. Term entspricht $\frac{d}{dt}S(\rho)$

② Term $+Tr\{(\mathcal{L}\rho)\cdot\ln\bar\rho\}$

$$\bar\rho = \frac{e^{-\beta(H_S-\mu N_S)}}{Z_S}$$

$$Tr\{(\mathcal{L}\rho)\ln\bar\rho\} = -\beta\, Tr\{(\mathcal{L}\rho)(H_S-\mu N_S)\} - \ln(Z_S)\underbrace{Tr\{(\mathcal{L}\rho)\}}_{=0}$$

$$= \beta\cdot(I_E - \mu I_N) = \beta\cdot\dot Q$$

Wärmestrom in das System hinein

$$\frac{d}{dt}Tr\{H_S\rho\} = Tr\{H_S\dot\rho\} = Tr\{H_S(\mathcal{L}\rho)\} = I_E$$

$$\frac{d}{dt}Tr\{N_S\rho\} = Tr\{N_S(\mathcal{L}\rho)\} = I_N$$

=> Spohn'sche Ungl. bedeutet: $\dot S - \beta\dot Q \geq 0$

Vernachlässigung der WW-Energie

$$dU_{res} = T\,dS_{res} + \mu\,dN_{res} \qquad \frac{dS_{res}}{dt} = \beta\left[\frac{dU_{res}}{dt} - \mu\frac{dN_{res}}{dt}\right] \approx -\beta(I_E - \mu I_N)$$
$$= -\beta\dot Q$$

$\boxed{\begin{array}{l}\dot S + \dot S_{res} \geq 0 \\ \text{2. Hauptsatz}\end{array}}$

- $\dot S < 0$ ist möglich (Abkühlung des Systems)
- in SS: $\dot S \to 0$ & $\dot Q \to 0$
  $\dot S_i \to 0$

$$\dot S_i = \dot S + \dot S_{res} = \dot S - \beta\dot Q \geq 0$$

irreversible Entropie-Produktionsrate

=> BMS-Mastergleichung respektiert die TD

## 1.3.5 Coarse graining

+ einfachere Herleitung
+ lediglich 1 freier Parameter
+ immer Lindblad-Form

① Trafo in das WW-Bild

$$\dot{\tilde{\rho}} = -i\left[\tilde{H}_I(t), \tilde{\rho}(t)\right] \qquad \tilde{\rho}(t) = \tilde{U}(t)\,\rho_0\,\tilde{U}^+(t)$$

$$U(t) = \hat{\tau}\exp\left\{-i\int_0^t \tilde{H}_I(t')\,dt'\right\} \qquad \frac{d}{dt}U = -i\,H(t)\,U(t) \quad : \int_0^t dt'$$

Zeitordnung

$$\tilde{U}(t) = \mathbb{1} - i\int_0^t \tilde{H}_I(t')\,\tilde{U}(t')\,dt'$$

wieder einsetzen

$$\tilde{U}(t) = \mathbb{1} - i\int_0^t \tilde{H}_I(t')\,dt' - \int_0^t dt_1\int_0^{t_1} dt_2\,\tilde{H}_I(t_1)\,\tilde{H}_I(t_2) + \mathcal{O}(H_I^3)$$

$t_2 < t_1$ "Zeitordnung"

$$\tilde{U}^+(t) = \mathbb{1} + i\int_0^t \tilde{H}_I(t')\,dt' - \int_0^t dt_1\int_0^{t_1} dt_2\,\tilde{H}_I(t_2)\,\tilde{H}_I(t_1) + \dots$$

$$\mathrm{Tr}_B\left\{\tilde{U}(t)\,\rho_S^0\otimes\rho_B\,\tilde{U}^+(t)\right\} = e^{\mathcal{L}\cdot t}\rho_S^0 = \left[\mathbb{1} + \mathcal{L}\cdot t + \dots\right]\rho_S^0$$

bei $t = \tau$ $\qquad \mathcal{O}(\lambda^2)$

$$\mathrm{Tr}\{H_I(t)\,\rho_B\} = 0 \qquad H_I = \mathcal{O}(\lambda)$$

Problem: $\mathcal{L}$ hängt von $t$ ab

wähle: $\tilde{H}_I = \sum_\alpha \tilde{A}_\alpha(t)\otimes\tilde{B}_\alpha(t) = \tilde{H}_I^+$

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Coarse-graining Mastergleichung bei

$$\mathcal{L}_\tau\,\rho_S = -i\left[\frac{1}{2i\tau}\int_0^\tau dt_1\int_0^\tau dt_2\sum_{\alpha\beta} C_{\alpha\beta}(t_1,t_2)\cdot\mathrm{sgn}(t_1-t_2)\,\tilde{A}_\alpha(t_1)\,\tilde{A}_\beta(t_2),\ \rho_S\right]$$

$$+\frac{1}{\tau}\int_0^\tau dt_1\int_0^\tau dt_2\sum_{\alpha\beta} C_{\alpha\beta}(t_1,t_2)\left[A_\beta(t_2)\,\rho_S\,A_\alpha(t_1) - \frac{1}{2}\left\{A_\alpha(t_1)\,A_\beta(t_2),\rho_S\right\}\right]$$

$$C_{\alpha\beta}(t_1,t_2) = \mathrm{Tr}\left\{\tilde{B}_\alpha(t_1)\,\tilde{B}_\beta(t_2)\,\bar{\rho}_B\right\}$$

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- immer Lindblad