English Summary:

Higher multipoles

$$\underline{A}^{(2)}(\underline{r},t) = \frac{\mu_0}{4\pi}\left\{ \underline{\nabla}_x \frac{1}{r}\underline{m}\left(t-\frac{r}{c}\right) + \frac{1}{6r^3}\dot{\underline{\underline{Q}}}\,\underline{r} + \frac{1}{6cr^2}\ddot{\underline{\underline{Q}}}\,\underline{r} \right\}$$

magnetic dipole electric quadrupole radiation

Wave optics and diffraction

$\varrho(\underline{r},t) = \varrho(\underline{r})e^{-i\omega t}$

Lorenz gauge

$$\Rightarrow \boxed{(\Delta + k^2)\,\phi(\underline{r}) = -\frac{1}{\epsilon_0}\varrho(\underline{r})} \qquad k = \frac{\omega}{c}$$

(homog. wave eq.: Helmholtz eq.)

boundary value problem:

Scalar Kirchhoff identity

$$\boxed{\phi(\underline{r}') = \int_{\partial V} d\underline{f}\left[\tilde{G}(\underline{r}-\underline{r}')\underline{\nabla}_r\phi(\underline{r}) - \phi(\underline{r})\underline{\nabla}_r\tilde{G}(\underline{r}-\underline{r}')\right]} \quad \underline{r}'\in V,\ \underline{r}\in\partial V$$

(a) Green'sche Fkt. des unendl. Raumes:

Randbed. $\tilde{G}(\underline{r}-\underline{r}') \to 0$ für $r\to\infty$

$\Rightarrow$ retardiertes Pot. (§4.2)

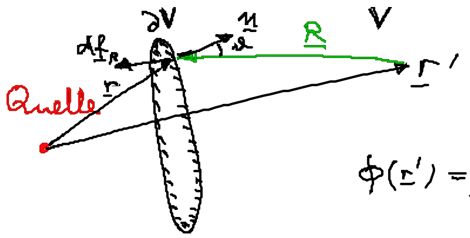
$$G(\underline{r}-\underline{r}', \underbrace{t-t'}_{\tau}) = \begin{cases} \frac{1}{4\pi|\underline{r}-\underline{r}'|}\,\delta\left(t-t'-\frac{|\underline{r}-\underline{r}'|}{c}\right) & t-t'>0 \\ 0 & t-t'<0 \end{cases}$$

$$\boxed{\tilde{G}(\underline{r}-\underline{r}') := \int_0^\infty d\tau\, G(\underline{r}-\underline{r}',\tau)e^{i\omega\tau} = \frac{e^{ik|\underline{r}-\underline{r}'|}}{4\pi|\underline{r}-\underline{r}'|}} \quad \text{mit } k := \frac{\omega}{c}$$

$$\phi(\underline{r},t) = \int_{\mathbb{R}^3} d^3r'\,\tilde{G}(\underline{r}-\underline{r}')e^{-i\omega t}\varrho(\underline{r}')/\epsilon_0$$

$$= \int d^3r'\,\frac{e^{i(k|\underline{r}-\underline{r}'|-\omega t)}}{4\pi|\underline{r}-\underline{r}'|}\varrho(\underline{r}')/\epsilon_0$$

beschreibt Überlagerung auslaufender Kugelwellen (Ausstrahlbed.)



$\underline{R} := \underline{r} - \underline{r}'$

Kirchhoff-Identität:

$$\phi(\underline{r}') = \frac{1}{4\pi}\int_{\partial V} d\underline{f}_R \left\{ \frac{e^{ikR}}{R} \nabla_r \phi(\underline{r}) - \phi(\underline{r}) \underbrace{\nabla_r \frac{e^{ikR}}{R}}_{\frac{e^{ikR}}{R}\left(ik - \frac{1}{R}\right)\frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|}} \right\}$$

mit $d\underline{f}_R \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} = df_R \cos\alpha$

und Beschränkung auf Fernzone von ∂V (d.h. $R \gg \frac{1}{k}$):

$$\boxed{\phi(\underline{r}') \approx \frac{1}{4\pi}\int_{\partial V} df_R \underbrace{\underbrace{\left\{\frac{\partial}{\partial n}\phi(\underline{r}) - ik\phi(\underline{r})\cos\alpha\right\}}_{\text{richtungsabh. Amplitude}} \underbrace{\frac{e^{ikR}}{R}}_{\text{Kugelwelle}}}_{\text{„Sekundärwelle"}}}$$

Exakte Formulierung des Huygens'schen Prinzips
(Jeder Pkt. der Oberfläche eines Hindernisses ist Ausgangspkt. einer Kugelwelle; deren phasengerechte Überlagerung ergibt das Wellenfeld in $\underline{r}'$.)

(b) Greenfkt. zu Randbed. $\tilde{G}(\underline{r}-\underline{r}')\Big|_{\substack{\underline{r}\in\partial V \\ \underline{r}'\in V}} = 0$

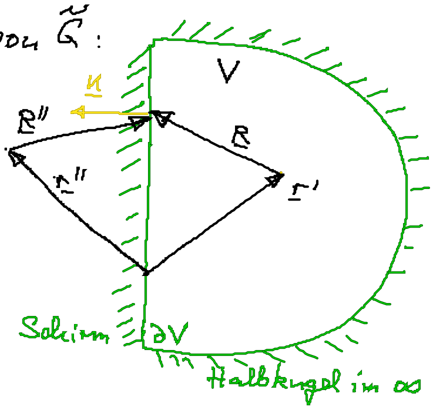
$$\Rightarrow \boxed{\phi(\underline{r}') = -\int_{\partial V} d\underline{f}\, \phi(\underline{r}) \nabla_r \tilde{G}(\underline{r}-\underline{r}')}$$

neue Greenfkt. $\tilde{G}(\underline{R}) = g(\underline{R}) + \frac{1}{4\pi}\frac{e^{ikR}}{R}$

$$\boxed{\begin{aligned}&(\Delta + k^2) g = 0\\ &\text{mit } g\Big|_{\partial V} = -\frac{1}{4\pi}\frac{e^{ikR}}{R}\Big|_{\partial V}\end{aligned}}$$

alte Greenfkt. $\to 0$ für $r \to \infty$

Beispiel für die Konstruktion von $\tilde{G}$:



Ebener Schirm

Spiegelladungsmethode:

$$\tilde{G}(\underline{r}-\underline{r}') = \frac{1}{4\pi}\left(\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} - \frac{e^{ik|\underline{r}-\underline{r}''|}}{|\underline{r}-\underline{r}''|}\right)$$

$$\nabla_r \tilde{G}(\underline{r}-\underline{r}') = \frac{1}{4\pi}\left[\nabla_{\underline{r}} \frac{e^{ikR}}{R} - \nabla_r \frac{e^{ikR''}}{R''}\right]$$

$$= \frac{1}{4\pi}\left[\frac{e^{ikR}}{R}\left(ik - \frac{1}{R}\right)\frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} - \frac{e^{ikR''}}{R''}\left(ik - \frac{1}{R''}\right)\frac{\underline{r}-\underline{r}''}{|\underline{r}-\underline{r}''|}\right]$$

mit $R = R''$, $\quad d\underline{f}\cdot\frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}''|} = -d\underline{f}\cdot\frac{\underline{r}-\underline{r}''}{|\underline{r}-\underline{r}''|} = df \cos\vartheta$

folgt

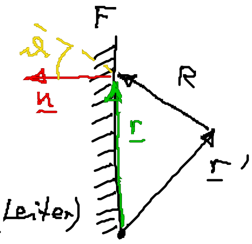
$$d\underline{f}\cdot\nabla_r\tilde{G} = df\,\frac{1}{2\pi}\,\frac{e^{ikR}}{R}\left(ik - \cancel{\frac{1}{R}}\right)\cos\vartheta$$

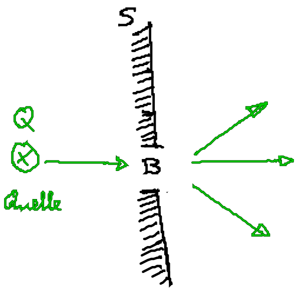
Fernzone $\lambda \ll R$

Fernzone:

$$\boxed{\phi(\underline{r}') = -\int_{\partial V}\left[d\underline{f}\cdot\nabla_r\tilde{G}(\underline{r}-\underline{r}')\right]\phi(\underline{r}) = -\frac{i}{\lambda}\int_F df\,\phi(\underline{r})\,\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}\cos\vartheta}$$

Randwerte $\phi(\underline{r})\Big|_{\underline{r}\in F}$ erraten!



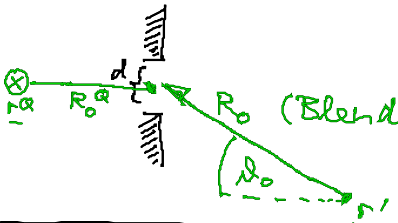


Annahme: $\phi(\underline{r})\Big|_S = 0$ (Leiter)

$$\phi(\underline{r})\Big|_B = \frac{e^{ikR^Q}}{R^Q}$$ freie einfallende Welle

$$\Rightarrow \phi(\underline{r}') = -\frac{i}{\lambda} \int_B df \, \frac{e^{ik(R+R^Q)}}{R R^Q} \cos\vartheta \qquad \begin{aligned} \underline{R} &= \underline{r} - \underline{r}' \\ \underline{R}^Q &= \underline{r} - \underline{r}^Q \\ df &= d^2 r \end{aligned}$$

Kleine Blende
(aber $\lambda \ll d$)

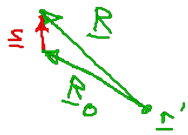


$\cos\vartheta \approx \cos\vartheta_0 \approx \text{const.}$

$$\boxed{\phi(\underline{r}') \approx -\frac{i}{\lambda} \frac{\cos\vartheta_0}{R_0 R_0^Q} \int_B df \, e^{ik(R+R^Q)}}$$

Grenzfälle:

(i) Fraunhofer'sche Beugung (Fernzone: $\lambda \ll d \ll R$)



Setze $\underline{R} = \underline{R}_0 + \underline{s}$

$$R^2 \approx R_0^2 + 2\underline{R}_0 \cdot \underline{s}$$

$$R \approx R_0 + \underline{\alpha} \cdot \underline{s} \qquad \underline{\alpha} := \frac{\underline{R}_0}{R_0}$$

analog $$R^Q \approx R_0^Q + \underline{\alpha}_0 \cdot \underline{s} \qquad \underline{\alpha}_0 := \frac{\underline{R}_0^Q}{R_0^Q}$$

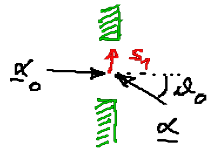
$$\Rightarrow \boxed{\phi(\underline{r}') \approx -\frac{i}{\lambda} \frac{e^{ik(R_0 + R_0^Q)}}{R_0 R_0^Q} \cos\vartheta_0 \int_B d^2 s \, e^{ik(\underline{\alpha} + \underline{\alpha}_0) \cdot \underline{s}}}$$

(ii) Fresnel'sche Beugung (Mittelzone $\lambda \ll R \approx d$)

$$R^2 = R_0^2 + 2\underline{R}_0 \cdot \underline{s} + s^2$$ nicht genähert!

Beispiel: Fraunhofer'sche Beugung am Spalt
(1-dim.)

Senkrechter Einfall: $\underline{\alpha}_0 \cdot \underline{s} = 0$



$$\phi(\underline{r}') = C \int_{-\frac{d}{2}}^{\frac{d}{2}} ds_1\, e^{ik\alpha s_1}$$

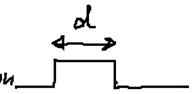
$$= \frac{C}{ik\alpha}\left(e^{ik\alpha\frac{d}{2}} - e^{-ik\alpha\frac{d}{2}}\right)$$

$\underline{\alpha}\cdot\underline{s} = s_1 \sin\vartheta_0$

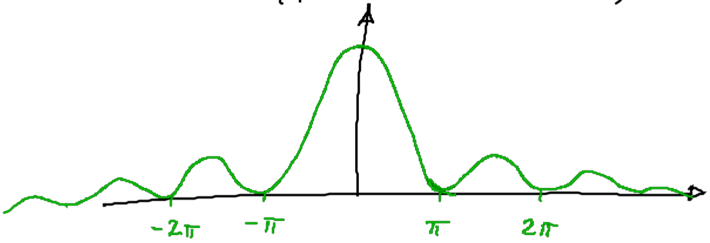
$\alpha := \sin\vartheta_0$

$$\boxed{\phi(\underline{r}') = C d \, \frac{\sin\left(k\alpha\frac{d}{2}\right)}{k\alpha\frac{d}{2}}}$$

„Spaltfkt."

Fourier-trafo von 

$|\phi|^2 \sim I$ (Intensität)



$k\alpha\frac{d}{2} = k\frac{d}{2}\sin\vartheta_0$

(Beob.richtung)

Beugungsminima bei $\boxed{\sin\vartheta_0 = n\frac{\lambda}{d}}$

$$\frac{2\pi}{\lambda}\frac{d}{2}\cdot\sin\vartheta_0 = n\pi$$