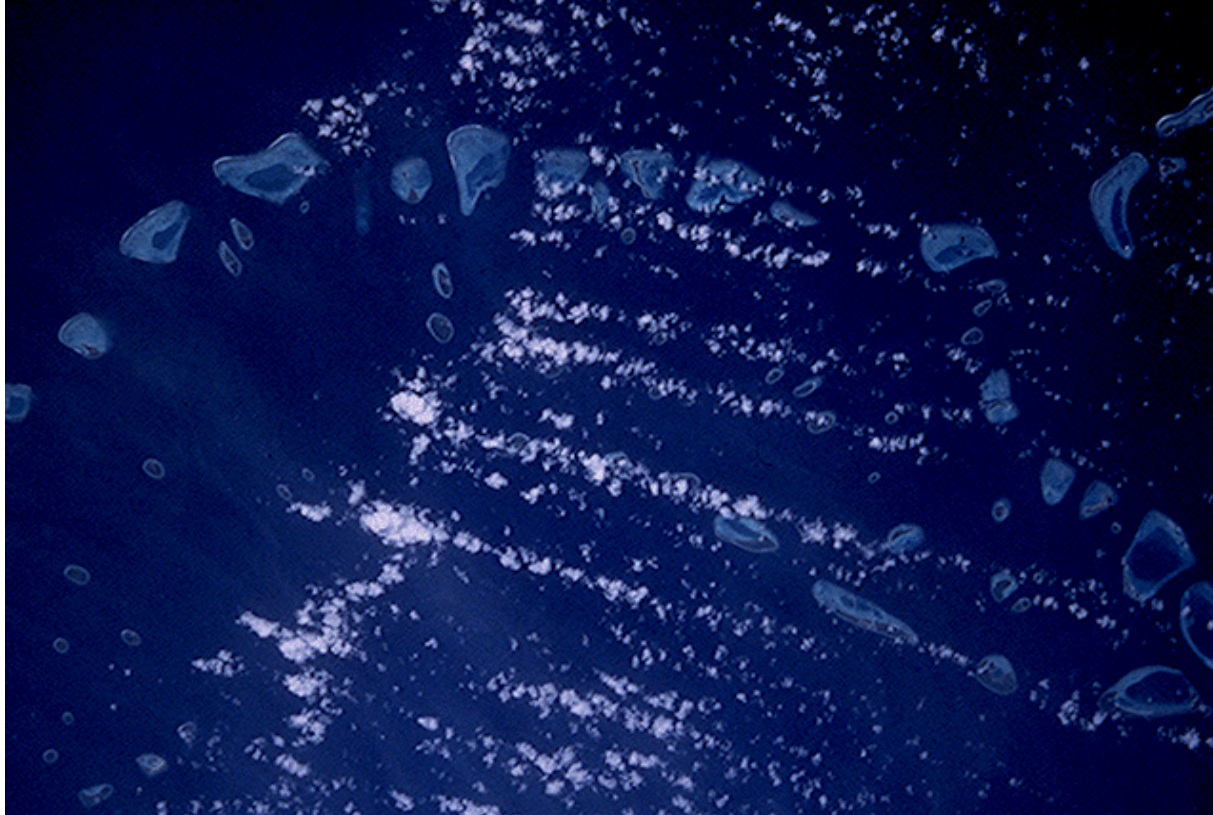
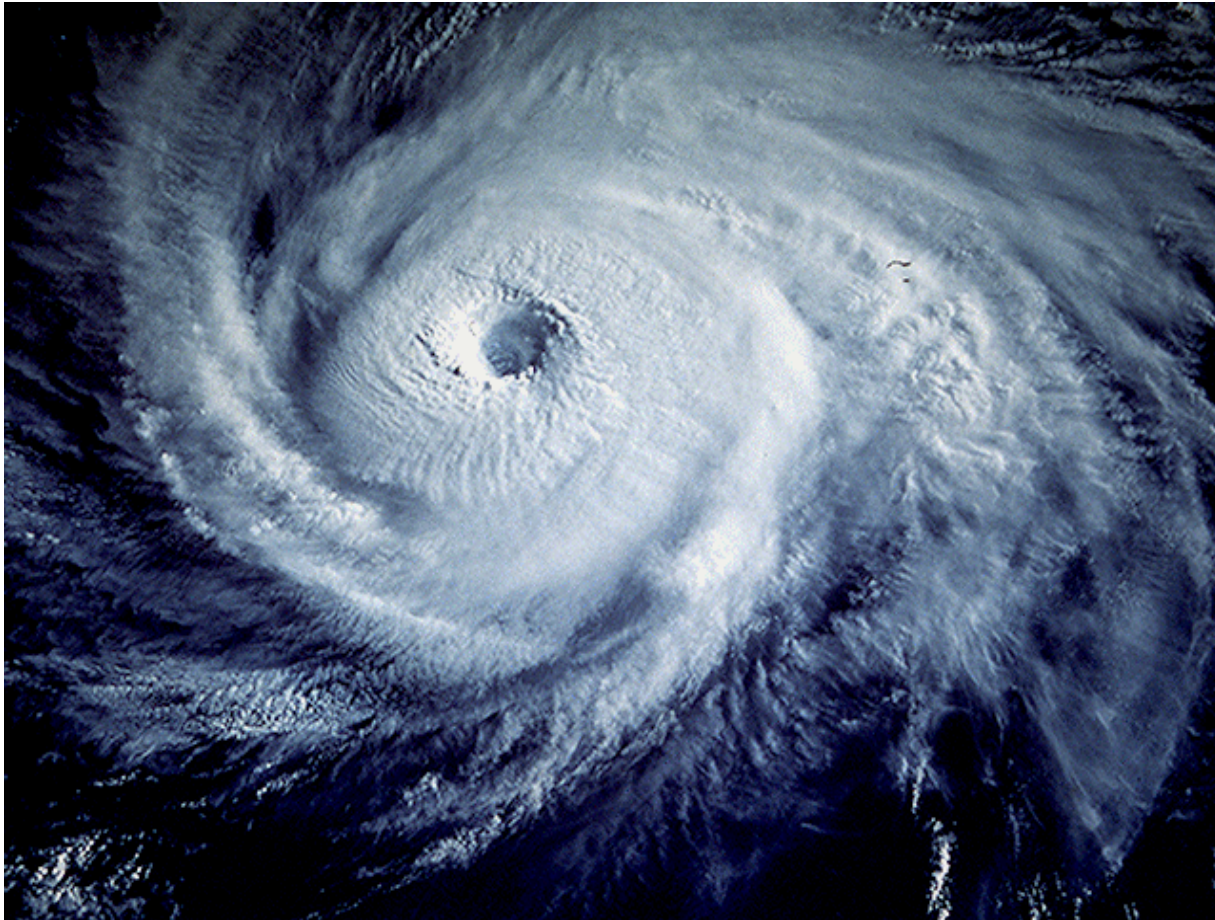


Wolkenstraßen über den Malediven



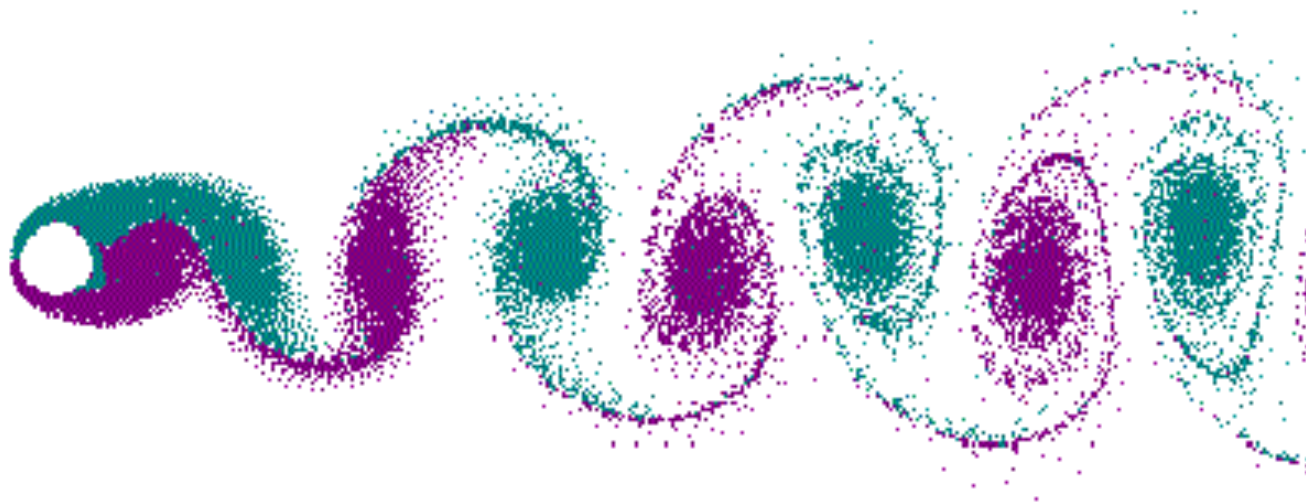
Taifun „Odessa“



Gasatmosphäre des Jupiter

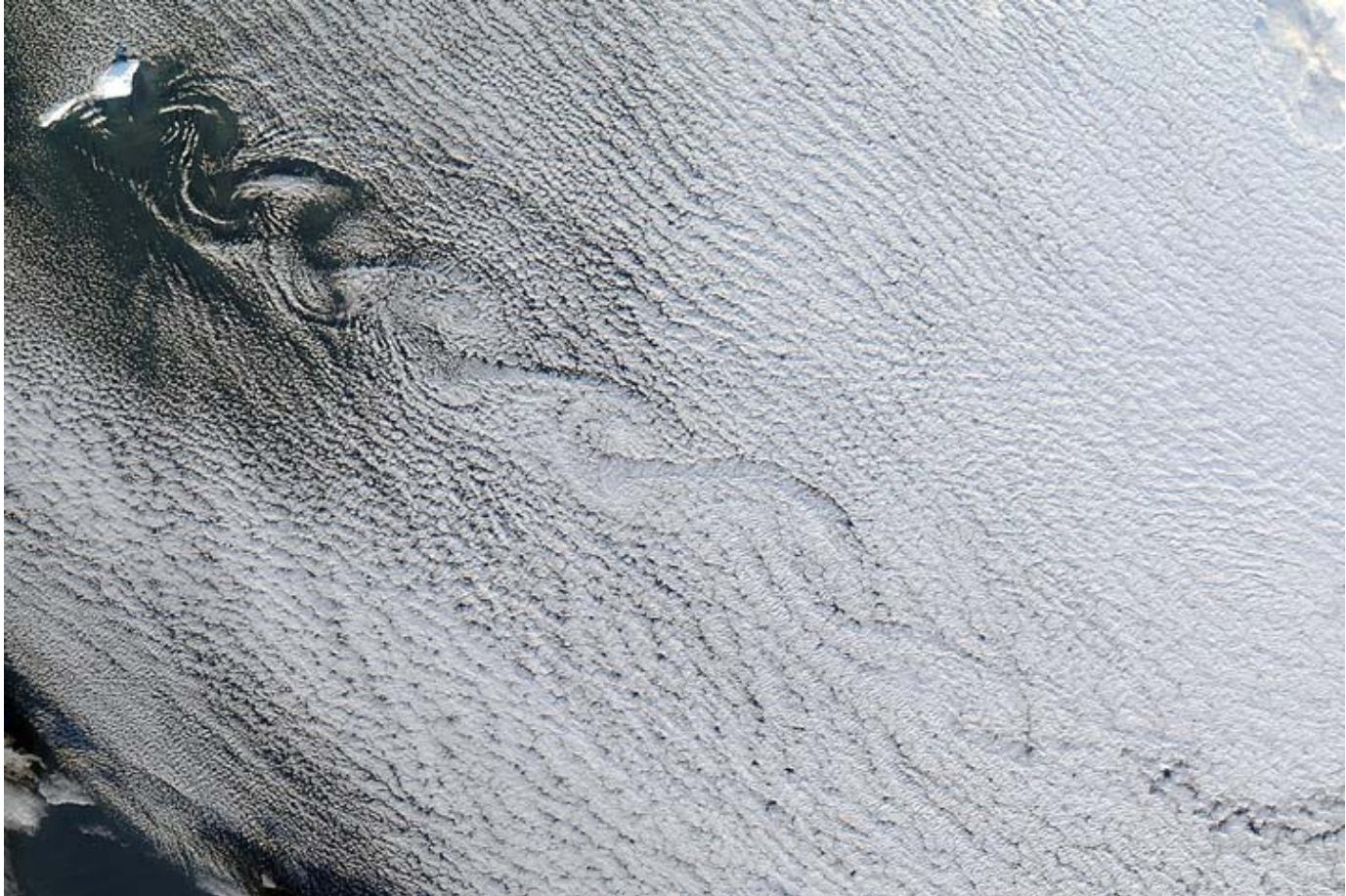


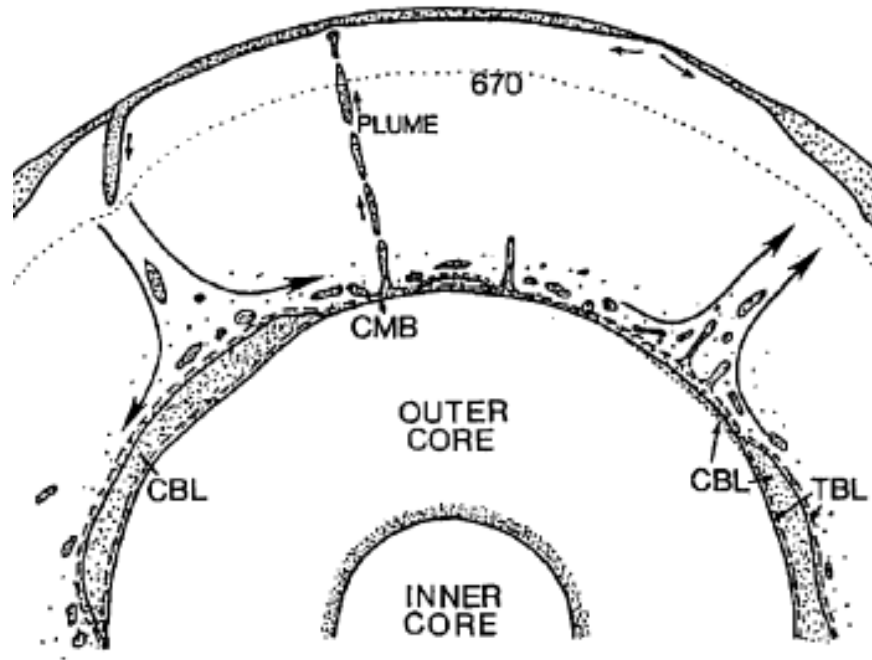
Von Karman'sche Wirbelstraße



ISLA DE GUADALUPE

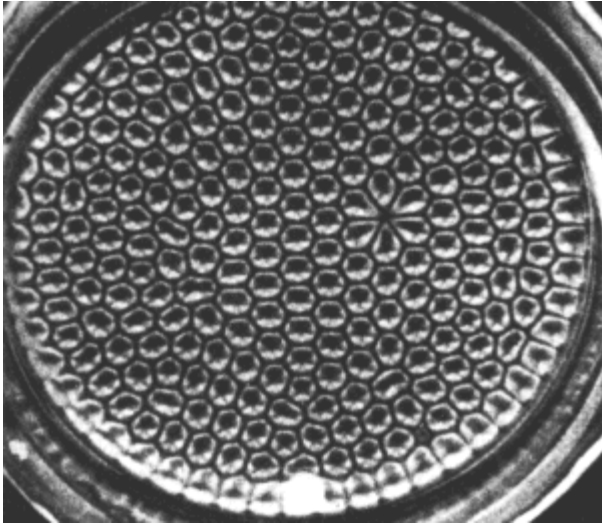
BAJA CALIFORNIA





Geophysikalische Beispiele:

- Zirkulation in Atmosphäre und Ozeanen
- Kontinentalverschiebung durch Strömungen im Erdmantel
- Schwerelosigkeit / Züchtung reiner Kristalle



Henri Bénard (1874-1934)

1900 Dissertation: „Les tourbillons cellulaires dans une nappe liquide“

Wärmetransport durch 0,81 mm dicke Schicht von 20 cm Durchmesser aus Walrat-Fett

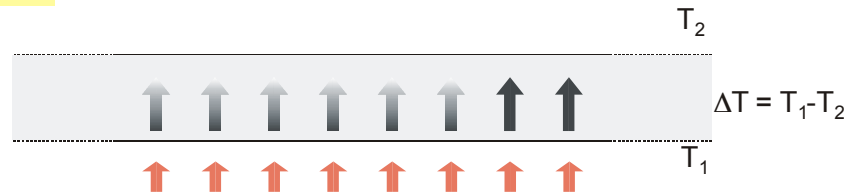
bienenwabenartige Struktur der Konvektionszellen

1916 Lord Rayleigh Erklärung der Musterbildung beim konvektiven Wärmetransport

viskose Flüssigkeit (Schichtdicke H) im Schwerfeld (g):

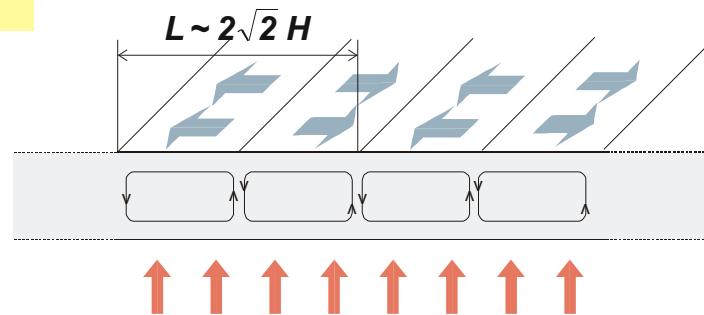


$$0 < \Delta T < (\Delta T)_{cr}$$



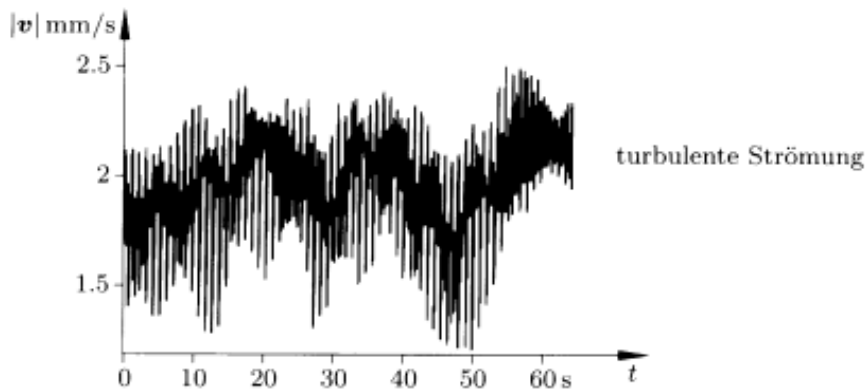
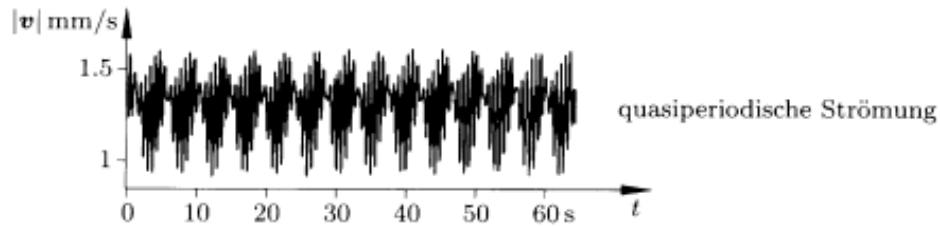
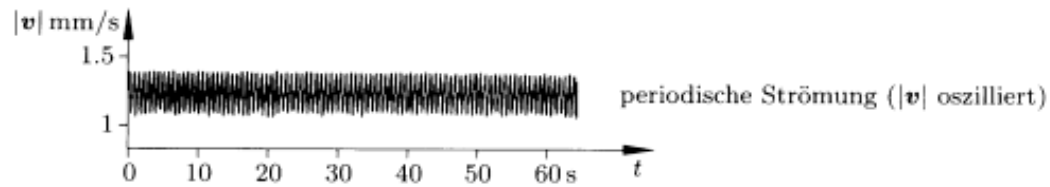
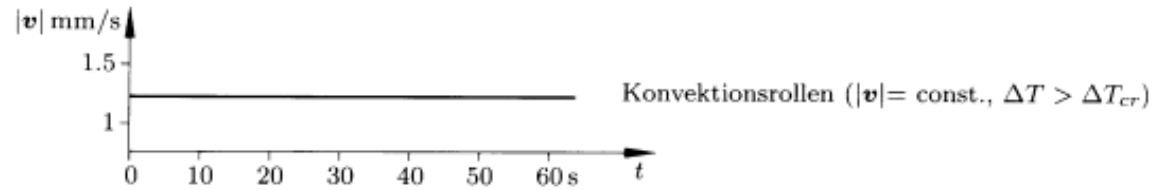
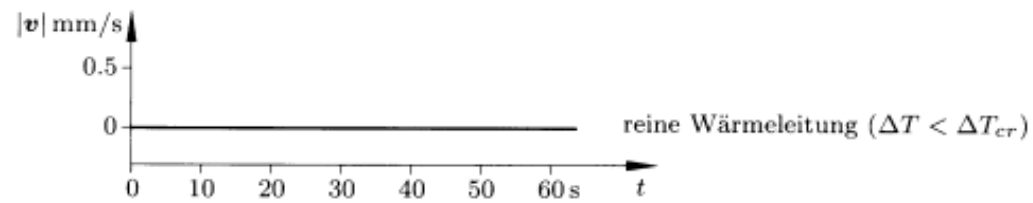
potentiell instabile Dichteschichtung (Auftrieb)
Wärmeleitung, Flüssigkeit ruht

$$(\Delta T)_{cr} < \Delta T$$

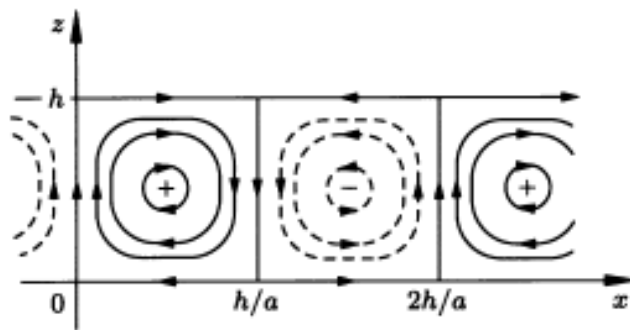


stationäre Konvektionsrollen (2d)

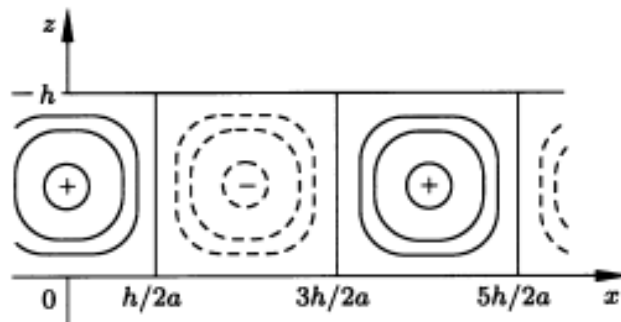
$$Ra = \frac{\alpha H^3 g \Delta T}{\nu \kappa} \sim \frac{\text{Antrieb}}{\text{Dissipation}}$$



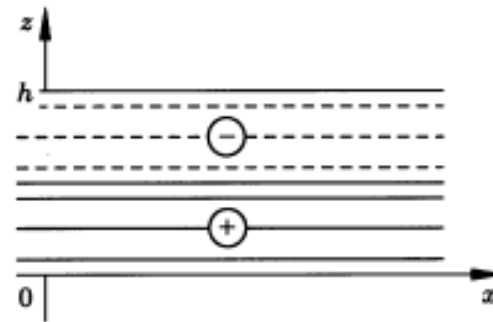
Fünf unterschiedliche Strömungszustände des Bénard-Experiments bei steigender Temperaturdifferenz ΔT (nach Graham, 1982)



a)



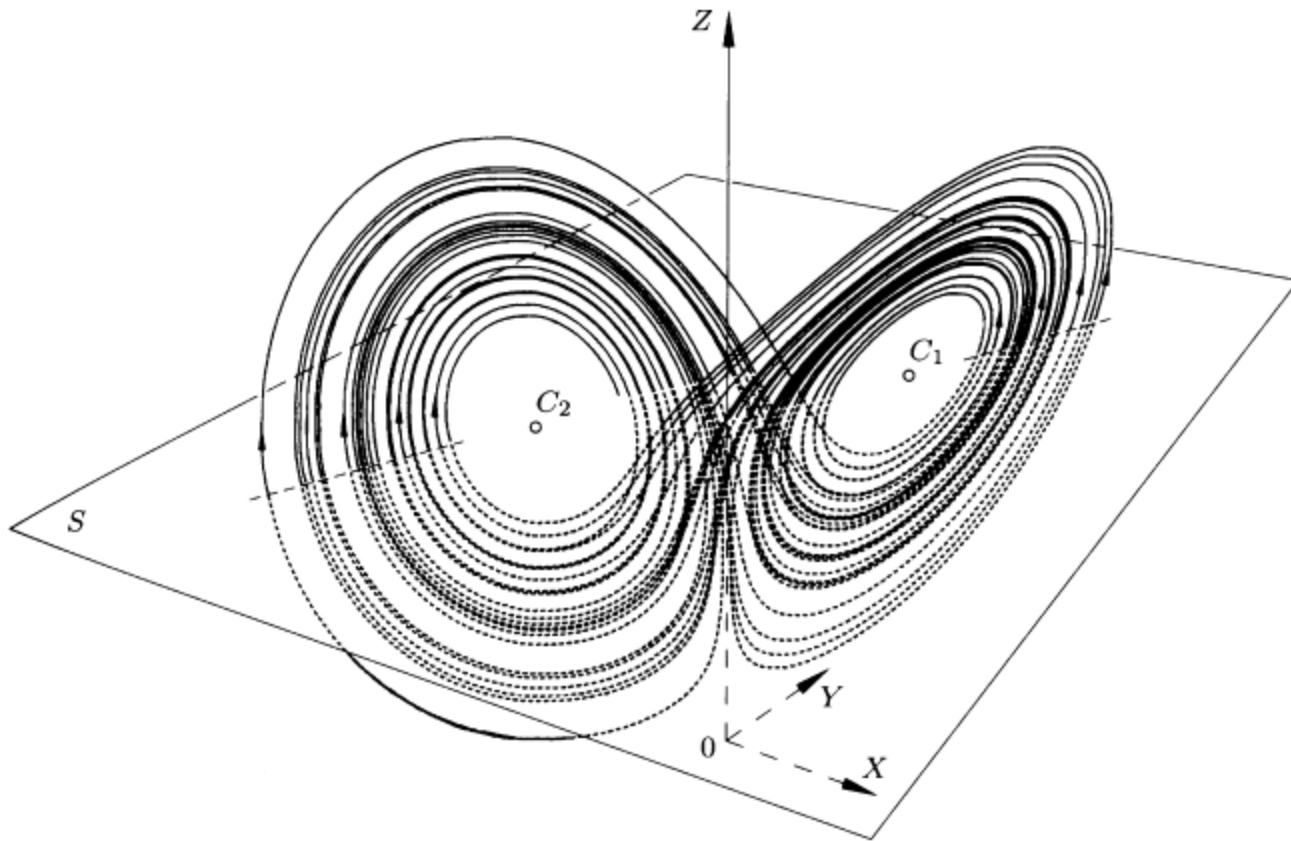
b)



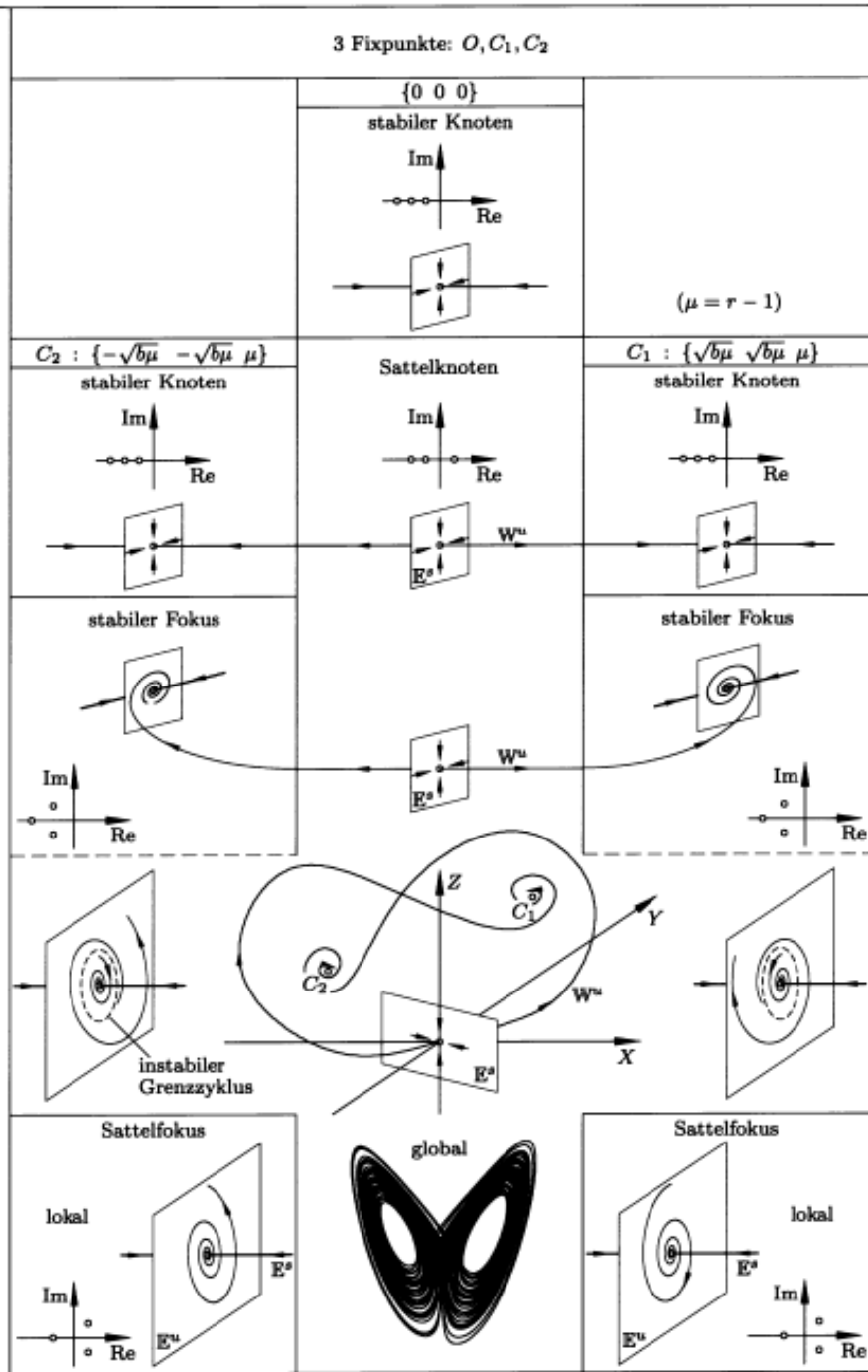
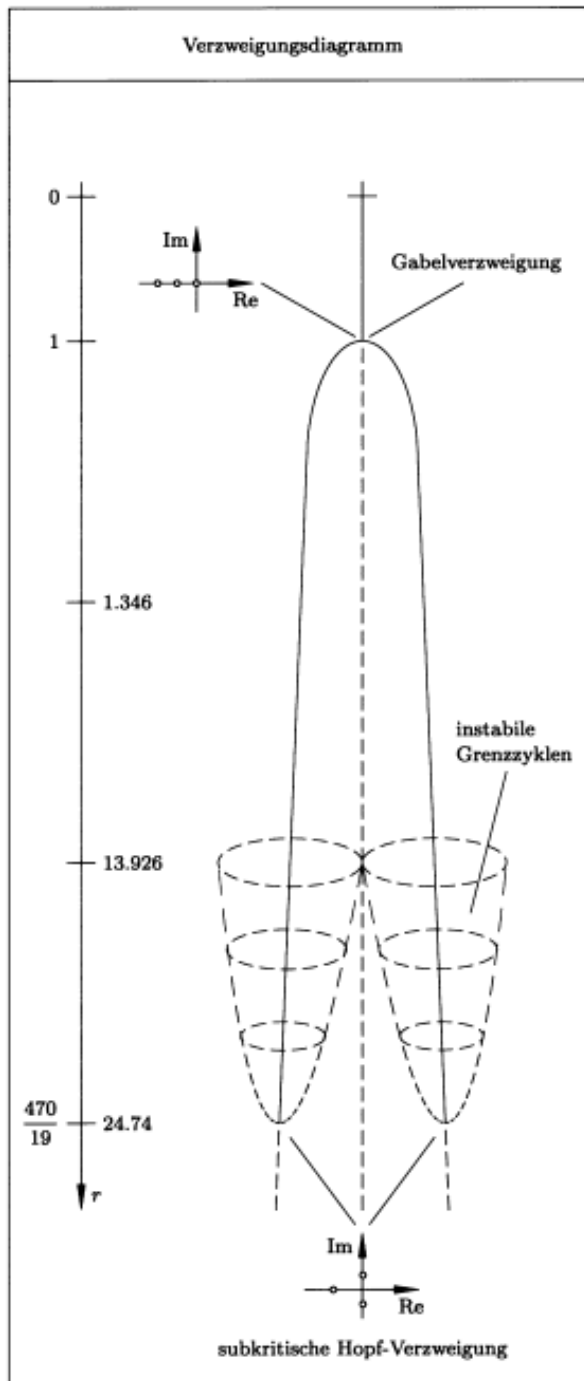
c)

Die Lorenz-Moden

- a) Strömungsmuster (X-Mode), b) Temperaturzellen (Y-Mode),
 c) Temperaturschichtung (Z-Mode)

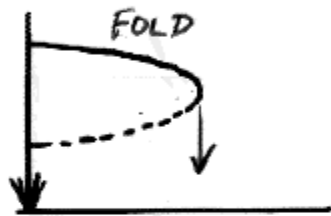


Lorenz-Attraktor für $r = 28$, $\sigma = 10$ und $b = 8/3$.
 Der Trajektorienbereich, den die Ebene $Z = r - 1 = 27$ verdeckt, ist punktiert
 (Lanford, 1977)



TWO CAUSAL FACTORS
 MULTIDIMENSIONAL EFFECT
 MINIMISING PRINCIPLE

⇒ { EQUILIBRIA FORM A
 SMOOTH SURFACE
 ONLY SINGULARITIES ARE
 FOLDS & CUSPS.



NUMBER OF CAUSAL FACTORS	1	2	3	4	5
NUMBER OF SINGULARITIES	1 FOLD	1 CUSP	3	2	4	...

3 CAUSAL FACTORS



SWALLOWTAIL

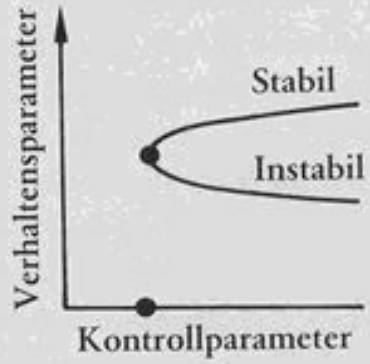


ELLIPTIC UMBILIC

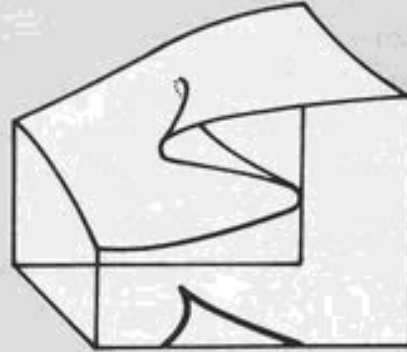


5

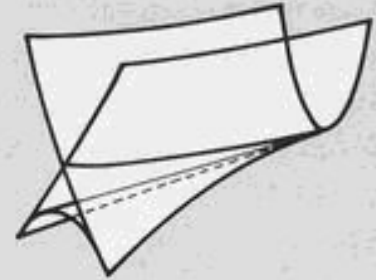
Faltung



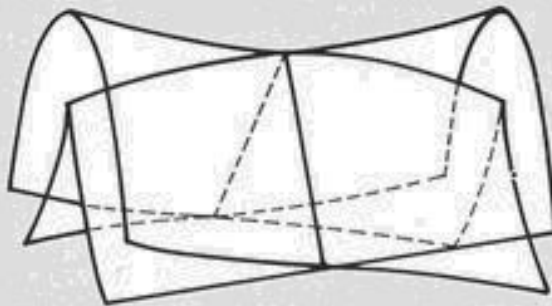
Spitze



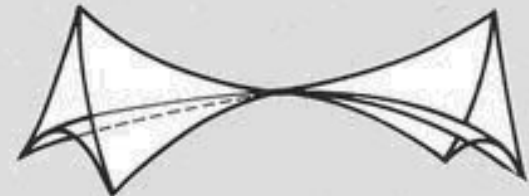
Schwalbenschwanz



Hyperbolischer Sattel



Elliptischer Sattel



Universelle Entfaltungen für $k \leq 4$

k	n	singularity	universal unfolding	name
1	1	$x^3/3$	$x^3/3 + \mu_1 x$	fold (saddle-node)
2	1	$x^4/4$	$x^4/4 + \mu_1 x^2/2 + \mu_2 x$	cuspidal cusp
3	1	$x^5/5$	$x^5/5 + \mu_1 x^3/3 + \mu_2 x^2/2 + \mu_3 x$	swallowtail
4	2	$x^3/3 + y^3/3$	$x^3/3 + y^3/3 + \mu_1 xy - \mu_2 x - \mu_3 y$	hyperbolic umbilic
5	2	$x^3/3 - xy^2$	$x^3/3 - xy^2 + \mu_1 (x^2 + y^2) - \mu_2 x - \mu_3 y$	elliptic umbilic
4	1	$x^6/6$	$x^6/6 + \mu_1 x^4/4 + \mu_2 x^3/3 + \mu_3 x^2/2 + \mu_4 x$	butterfly
4	2	$x^2y + y^4/4$	$x^2y + y^4/4 + \mu_1 x^2 + \mu_2 y^2 - \mu_3 x - \mu_4 y$	parabolic umbilic

Weiterführende Literatur

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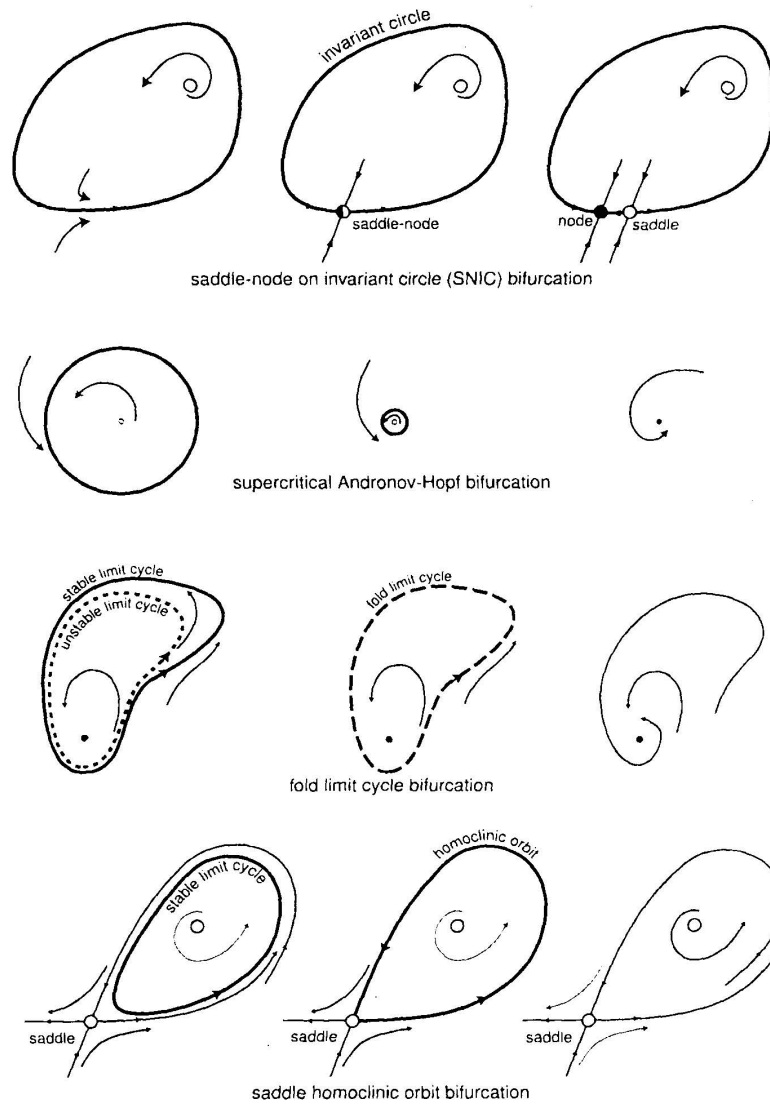


Figure 6.47: Summary of all codimension-1 bifurcations of a stable limit cycle (tonic spiking state) on a plane.

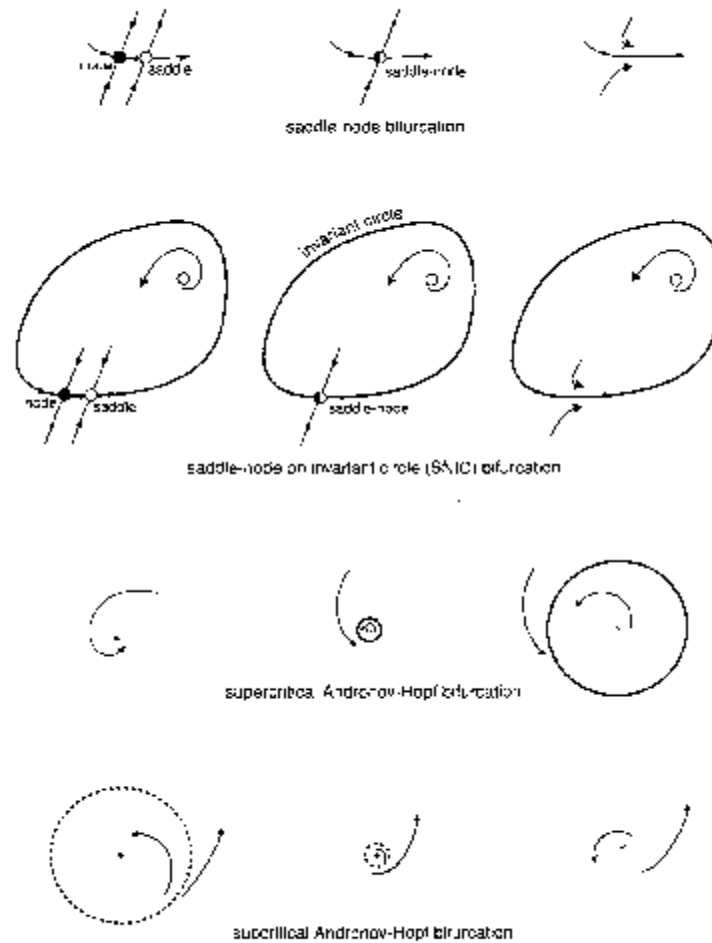


Figure 6.40: Summary of all codimension-1 bifurcations of a stable equilibrium (resting state).