7. Coherence resonance

7.1. What is coherence resonance?
(CR in excitable systems)

noise

destructive role (robustness)
- destroys determ. dynamics
- induces chaos
- induces irregularities

constructive role (counter-intuitive)
- induces oscillations
- stoch. resonance
- coh. resonance

One of the most prominent examples of the constructive role of noise is the phenomenon of coherence resonance (CR).

The best temporal regularity of the noise-induced oscillations occurs for an intermediate value of noise intensity.
- discovered by Haken et al. in 1993
- named coherence resonance by Pikovsky and Kurths in 1997

! counter-intuitive phenomenon

CR in excitable systems

FitzHugh-Nagumo system (model of a neuron) FHN

$$\begin{cases} \varepsilon \dot{u} = u - \frac{u^3}{3} - v, \\ \dot{v} = u + a + \sqrt{2D}\,\xi(t) \end{cases}$$

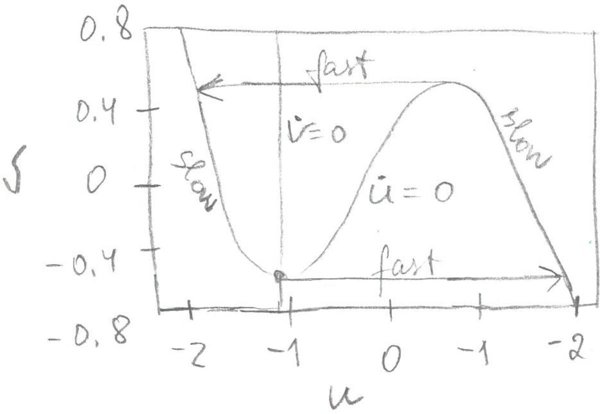
u - activator (fast)
v - inhibitor (slow)

This is an example of a relaxation oscillator. This is typical for spike generations in a neuron after stimulation by an external input.

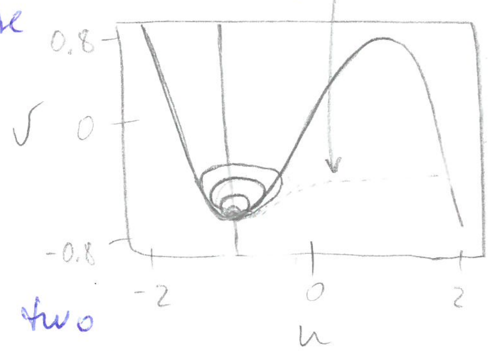
a short nonlinear elevation of membrane voltage u, diminished over time by a slower, linear recovery variable v.

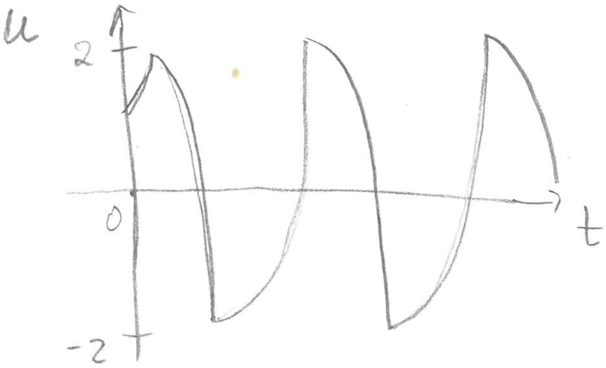
ε - time scale separation parameter ($\varepsilon = 0.01$) $\Rightarrow$ the motion along the limit cycle is highly nonuniform.

$|a| > 1 \rightarrow$ excitable regime ($a = 1.001$)



FHN system rests in a locally stable steady state (rest state) and upon excitation beyond a threshold emits a spike → performs a long excursion in phase space before returning to the rest state.



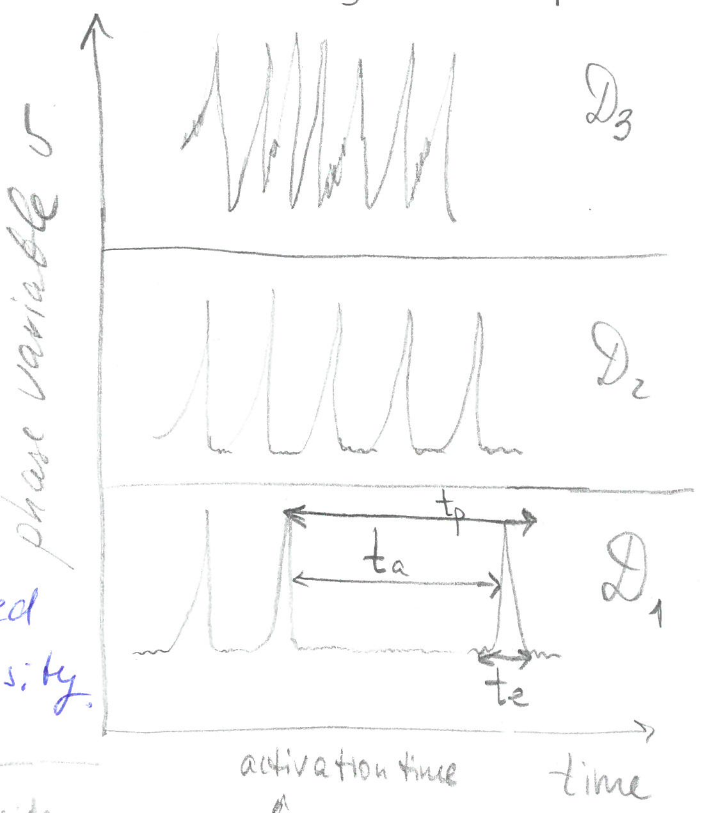


> Type II excitability: the stable fixed point loses its stability in a Hopf bifurcation.

The presence of two competing time scales with opposite dependence upon noise intensity is crucial in producing the nonmonotonic dependence of the coherence upon noise intensity ⇒ this leads to optimum coherence of the noise-induced oscillations at non-zero noise intensity.

$D_3 > D_2 > D_1$

With increasing noise the excitation across threshold occurs more frequently ⇒ the interspike intervals become more regular. On the other hand, with increasing noise the deterministic spiking dynamics becomes smeared out. This counteracts the former effect. The best temporal regularity of the noise-induced spiking is observed for the intermediate noise intensity.



t_a rapidly decreases with noise intensity;
t_e weakly depends on noise intensity

activation time

$t_p = t_a + t_e$

t_p → pulse duration; t_e → excursion time

7.2. Measures of coherence

- normalized fluctuations of the pulse durations (interspike interval)

$$R = \frac{\sqrt{\langle t_p^2 \rangle - \langle t_p \rangle^2}}{\langle t_p \rangle} = \frac{\sqrt{Var(t_p)}}{\langle t_p \rangle}$$

[Pikovsky, Kurths, PRL 78, 775 (1997)]

This measure can be applied in the case of spiking behaviour (not used for non-excitable systems since there are no spikes).

- correlation time [Stratonovich, 1963]

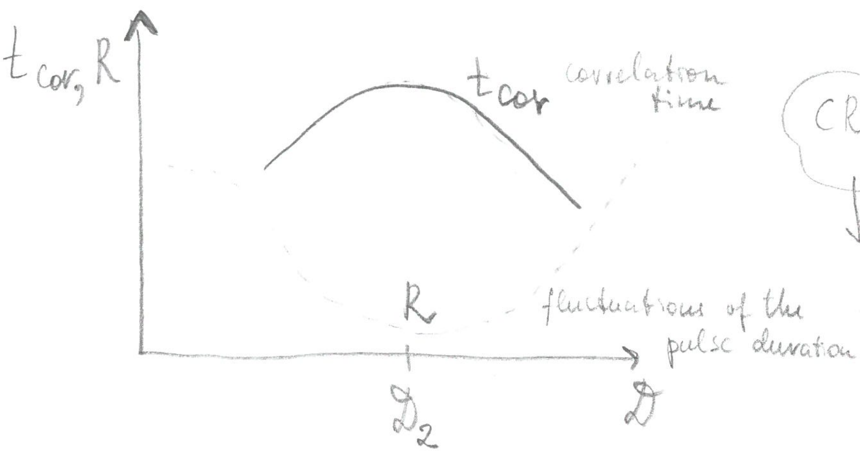
$$t_{cor} = \frac{1}{\Psi(0)} \int_0^\infty |\Psi(s)|\, ds$$

ACF

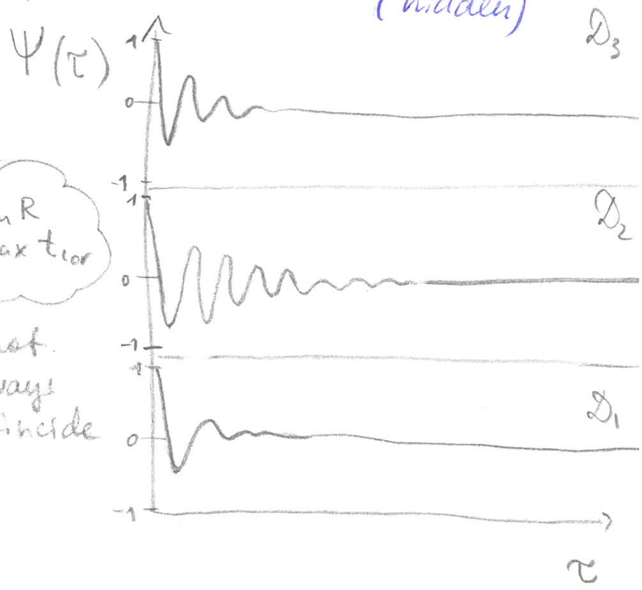
it shows how fast the correlation decays in a stochastic process

ACF → see lecture 6

Autocorrelation is the correlation of a signal with itself at different points in time. It is a math. tool for finding repeating patterns, for example, the presence of a periodic signal obscured (hidden) by noise.

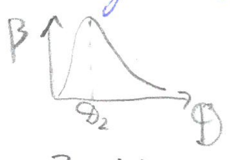


CR - min R
- max t_{cor}
do not always coincide

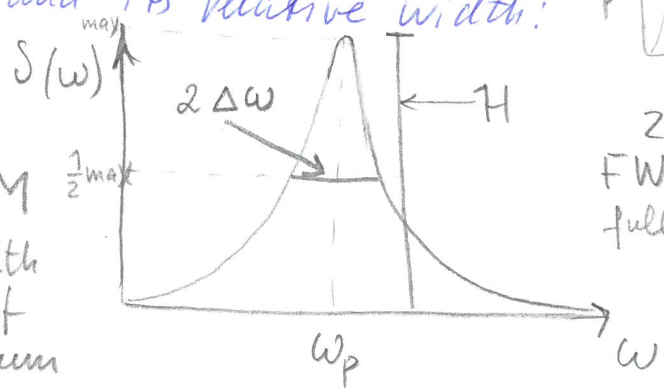


- signal-to-noise ratio (SNR) of the spectral peak hight H and its relative width: [Haken, 1993]

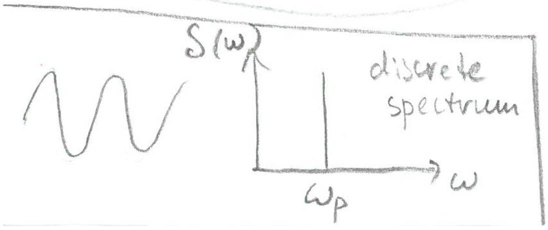
$$\beta = \frac{H}{\Delta\omega / \omega_p}$$



$\Delta\omega$
HWHM
half width at half maximum



$2\Delta\omega$
FWHM
full width at half maximum



Wiener - Khinchin theorem connects spectral characteristics of a random process with its correlational characteristics.

$$\boxed{\begin{aligned} S(\omega) &= \int_{-\infty}^{\infty} \Psi(\tau) e^{-i\omega\tau} d\tau \\ \Psi(\tau) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega\tau} d\omega \end{aligned}}$$

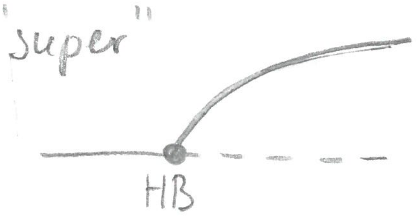
valid only for stationary stoch. processes! (wide-sense stationary).

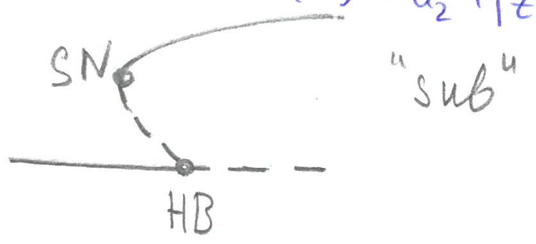
7.3. CR in non-excitable systems

CR has been shown for dif. types of excitable systems: excitability type I (SNIPER model) Haken 1993, excitability type II (FHN model) Rik & Kurths 1997. Recently it has also been shown for non-excitable systems (no spiking behaviour) with a subcritical Hopf bifurcation [Ushakov et al. PRL 95, 2005]:

$$\dot{z} = -i\omega_0 z + z F(z) + \sqrt{2D}\, \xi(t),$$ z is a complex variable $z = x + iy$.

Stuart-Landau oscillator: supercritical $F(z) = a_1 - |z|^2$
subcritical $F(z) = a_2 + |z|^2 - |z|^4$





Coherence resonance occurs in a strict sense only in the subcritical case.