Lecture 13

25.01.2017

constructive role of noise

7. Coherence resonance (CR)

8. Stochastic resonance (SR)

nonlinear dynamics

noise

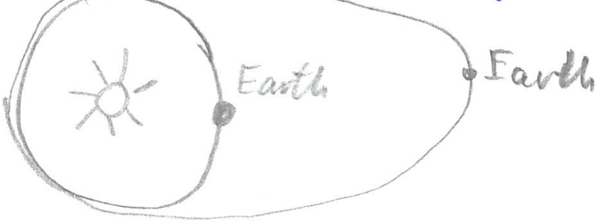
8. Stochastic resonance.

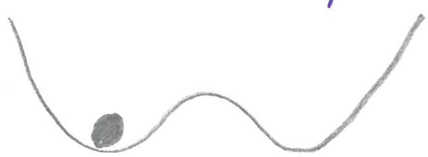
It was traditionally considered that the presence of noise can only make the operation of any system worse. In contrast, it has been recently shown that noisy sources in nonlinear dynamical systems can induce completely new regimes that can not be realized without noise. Moreover, increasing noise can induce more ordered behavior in nonlinear systems. Unexpectedly it can lead to the formation of more regular temporal dynamics, increase the degree of coherence. The examples are CR and SR.

The term SR was introduced in 1981–1982

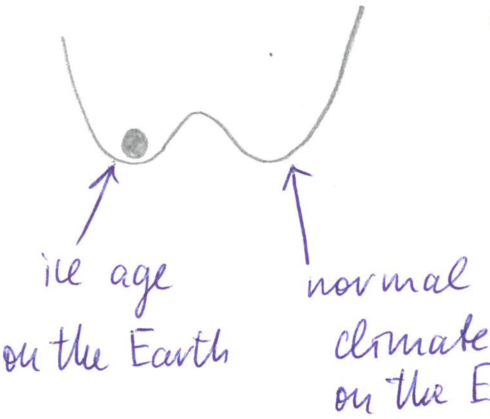
[Benzi et al. J Phys. A: Math. Gen. 14, L453 (1981)
Benzi et al. Tellus 34, 10 (1982)
Nicolis, Tellus 34, 1 (1982)]

Study on the periodicity of the ice age on the Earth: a model to explain the almost periodic recurrences (repetitions) of the Earth's ice ages. The temporal evolution of Earth's climate was modeled as the motion of a particle in a symmetric double-well potential





driven by a periodic force. Both states are stable: ice period and warm climate. The periodic force refers to tiny oscillations of the eccentricity of Earth's orbit (a period is about 10^5 years): the orbital change of from the (nearly) circular to more elliptical, it takes 100.000 years to go from nearly circular to elliptical form and back. The estimations have shown that the actual amplitude of this periodic force is not enough (too small) to cause the drastic climate change. The possibility of switchings was achieved by introducing additional random forcing into the model arising from fluctuations of the atmosphere. Atmospheric fluctuations induce transitions in the climate from stable cold to warm periods and vise versa.



The fundamental result: temporally ordered transitions; the climate almost followed the vanishingly small external periodic perturbation for some finite noise strength of the atmosphere.

Stochastic resonance
- nonlinear system
- periodic force
- noise

→ is a mechanism by which a nonlinear dynamical system in a noisy environment gets an enhanced sensitivity towards small external periodic force for an interm./optimal noise intensity.

General scheme of stochastic resonator

$$s(t) \longrightarrow \boxed{dx/dt = f(x, s(t), D)} \xrightarrow{x(t)}$$

The response of the nonlinear system to the weak input signal $s(t)$ is significantly increased for an optimal non-zero noise intensity.

Physical background of SR.

The physical mechanism of SR is rather simple. The system has two time scales

intrawell or local dynamics: fluctuations within the well around the stable fixed points; in the case of deep wells and weak noise this time scale is independent of noise

global dynamics: mean time of barrier crossings; corresponds to the mean velocity (or frequency) of the escape from a stable state – Kramers rate; depends on noise

The pecularity of SR is that one of the time scales depends on noise. In the case of bistable overdamped dynamics one can control the switching events between the two states by tuning noise intensity.

Let us consider qualitatively the motion of a Brownian particle in a system with a symmetric double-well potential $U(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$ under the influence of a weak periodic perturbation. The motion of the particle in the potential field $U(y)$ in the presence of damping is described by:

$$\underbrace{\ddot{y}+\gamma\dot{y}}_{\text{damping}} = -\frac{\partial U(y)}{\partial y}$$

In the case of large damping ($\gamma \gg 1$) one can neglect the second derivative and consider a first-order dif. equation:

$$\dot{y} = -\frac{1}{\gamma}\frac{\partial U(y)}{\partial y} \quad \text{or} \quad \boxed{\dot{x} = -\frac{\partial U(x)}{\partial x}}$$

Under the conditions described above the equation of the overdamped bistable oscillator with noise and under the influence of the periodic force is as follows:

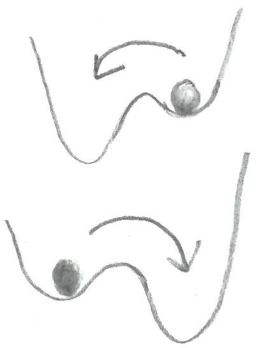
$$\dot{x} = x - x^3 + A\cos(\Omega t + \varphi) + \sqrt{2D}\,\xi(t).$$

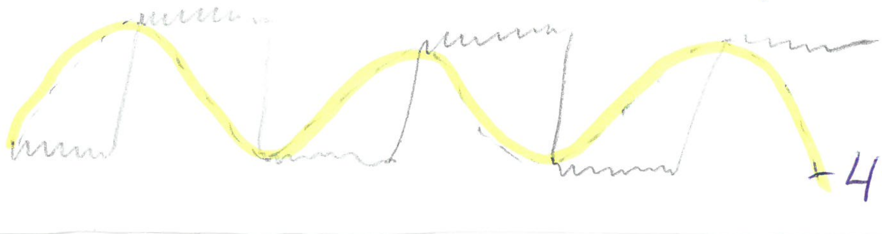
One can show that for white noise the Kramers rate r_K is defined by noise intensity D:

$$r_K = \nu \exp\left(-\Delta U_0 / D\right),$$

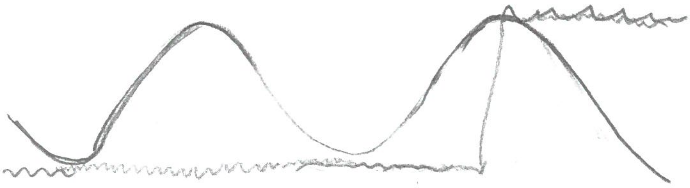
ν is defined by the curvature of the potential wells

The amplitude A of the periodic force is assumed to be small → without noise the transitions between the wells are not possible. However, the barrier height is slightly modulated by the periodic force. When D is too small → the switching events are rather rare. When D is too large → too many switching events. For optimal noise:

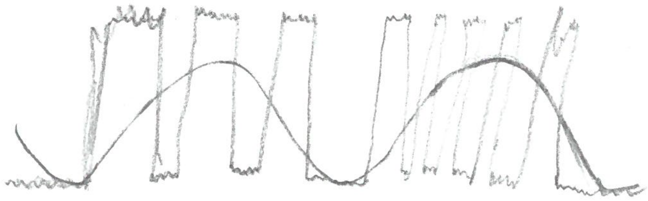




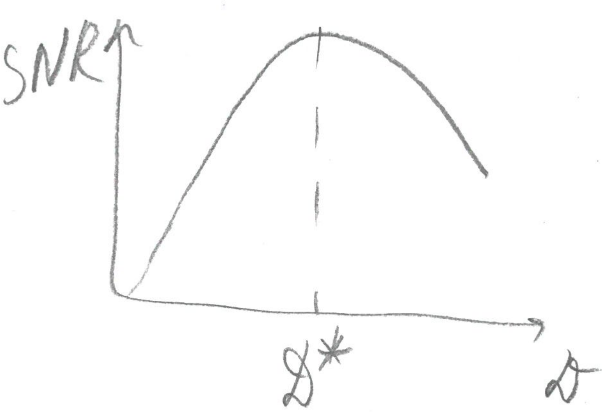
The frequency of the external forcing and the frequency of switching become very close for an optimal noise intensity D^*



D is too small: the escape time is longer than the period of the per. forcing



D is too large: during one period the system switches many times.



Examples: Brownian particles, lasers, tunnel diodes, quantum systems, chemical systems, sociological and biological systems (music as noise for mental activity).

Example: Paddlefish → in North America the rivers are not clear → the fish has developed an antenna with electrical receptors >10.000 electro-receptors → antenna is used instead of eyes to catch the food (plancton).
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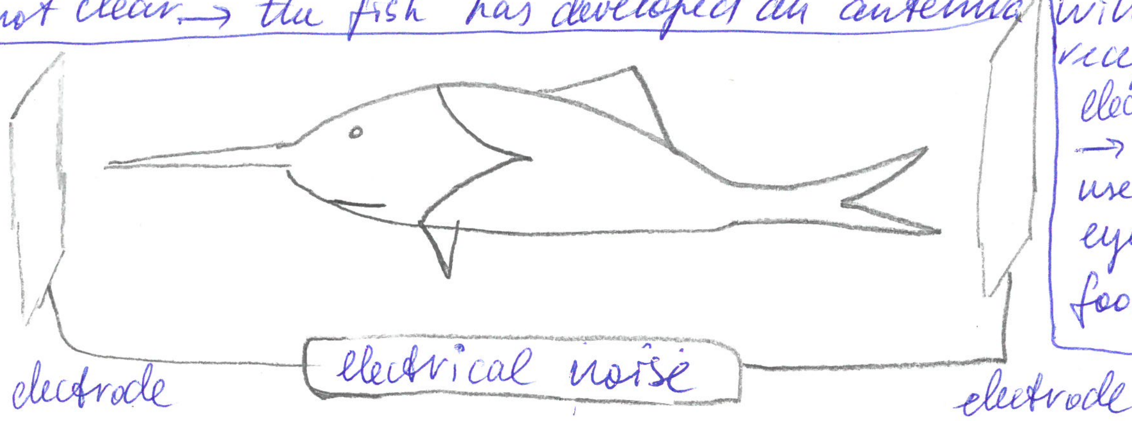
emits signal of low frequency.



[Russel, Wilkens and Moss Nature 1999]

In the experiment external random electrical field was applied and the amount of plancton was measured.

low noise: a certain amount of food

intermediate noise: the largest amount of food

large noise: can not find plancton