WS 2016/2017 Nonlinear dynamics and stochastic processes.

Anna Zakharova

We $10^{15} - 11^{45}$ EW 733 (2 SWS)

Introduction

Webpage → all the info is regularly updated

Seminar "Control of complex systems and networks"

- synchronization
- bifurcation ——— ?

nonlinear dynamics ⟷ theory of stochastic processes

- deterministic

complex systems (not complicated)

- stochastic
- noisy
- random fluctuations

- deterministic and stochastic nonlinear systems
- bifurcations in the presence of random fluctuations
- noisy periodic and deterministic chaotic dynamics
- synchronization of stochastic systems
- Fokker-Planck and Langevin equations
- stochastic resonance
- coherence resonance
- control by means of noise
- noise effects in complex networks (chimera states)

Dynamics as an approach to analyze complex systems.

Viewed from the perspective of dynamics the subjects can be placed in a common framework.

Nonlinear
- linear systems = sum of its parts
- nonlinear systems ≠ sum of its parts
  the parts interfere, cooperate or compete.

Most of everyday life is nonlinear => the principle of superposition fails.

Example: if you listen to two your favourite songs at the same time you do not get double pleasure.

- operation of laser
- formation of turbulence in a fluid
- superconductivity of materials

Nonlinear dynamics is an approach to analyze nonlinear systems!

What is a dynamical system?

(physics): any system that evolves in time

(math.): an object for which

1) the state is uniquely defined at a given time
2) there is a law (operator) which defines the evolution of this state in time

if we know the state at $t = t_0$ =>
=> we can uniquely define the state of the system at $t > t_0$ (in the future)

$$\boxed{\dot{\vec{x}} = \vec{F}(\vec{x}(t), t)} \quad (*)$$

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_N, t), \quad i = 1 \ldots N$$

$$\dot{x}_i = f_i(x_1, x_2, \ldots, x_N, t), \quad i = 1 \ldots N$$

$x_1, \ldots x_N$ → populations of different species in an ecosystem

→ concentrations of different chemicals in a reactor

→ positions and velocities of the planets in a solar system

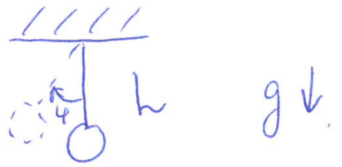
$\vec{F}$ is a vector function of dimension N

$(*)$ defines a velocity vector field

$\vec{F}$ is the law (operator) which defines the evolution of the system

$$\begin{cases} \dot{x}_1 = \dfrac{dx_1}{dt} = f_1(x_1, x_2, \ldots, x_N, t) \\ \dot{x}_2 = \dfrac{dx_2}{dt} = f_2(x_1, x_2, \ldots x_N, t) \\ \vdots \quad - - - - - - - \\ \dot{x}_N = \dfrac{dx_N}{dt} = f_N(x_1, x_2, \ldots, x_N, t) \end{cases} \Longleftrightarrow \dot{\vec{x}} = \vec{F}(\vec{x}(t), t)$$

Example Swinging of a pendulum



φ – the angle

L – length of the pendulum

g – acceleration due to gravity

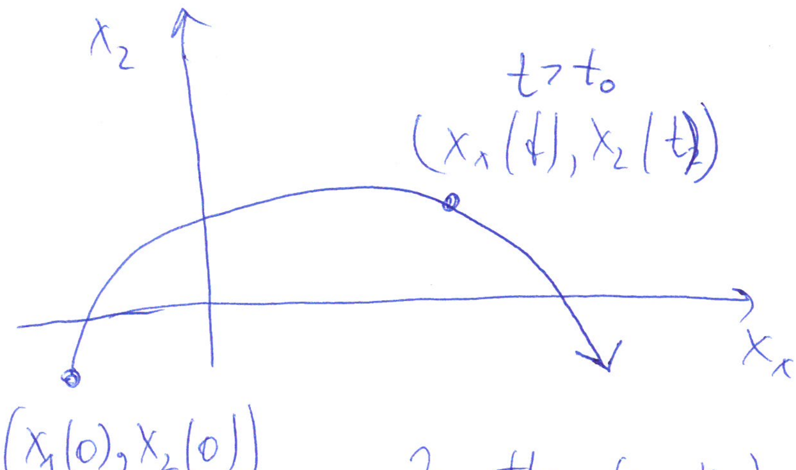
$$\boxed{\ddot{\varphi} + \frac{g}{L}\sin\varphi = 0} \Rightarrow \boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 \end{aligned}} \quad (**)$$

$N = 2$ nonlinearity

x_1 is the position

x_2 is the velocity

We construct an abstract space with coordinates (x_1, x_2)



$t = t_0$

Suppose we know a solution for $(**)$ for a particular initial condition. The solution would be a pair of functions: $x_1(t)$ and $x_2(t)$.

In the (x_1, x_2)-plane the solution corresponds to a point (phase point) $(x_1(t), x_2(t))$ moving along a curve in this space.

The curve is called trajectory and the space is called phase space for the system.

We need the initial values of both x_1 and x_2 to define the solution uniquely.