

English summary

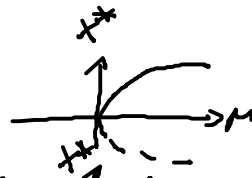
1.2 Bifurcations (continued)

• attractors in systems of dimension 1 (fixed points), 2 (limit cycles), 3 (tori, strange attractors)

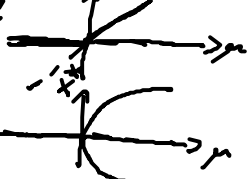
classification:

(A) eigenvalue-zero bifurcation

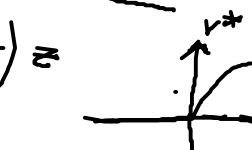
(A1) saddle-node bifurcation: $\dot{x} = \mu - x^2$



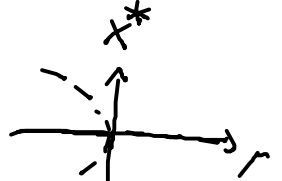
(A2) transcritical bifurcation: $\dot{x} = \mu x - x^2$



(A3) pitchfork bifurcation: $\dot{x} = \mu x - x^3$
 supercritical

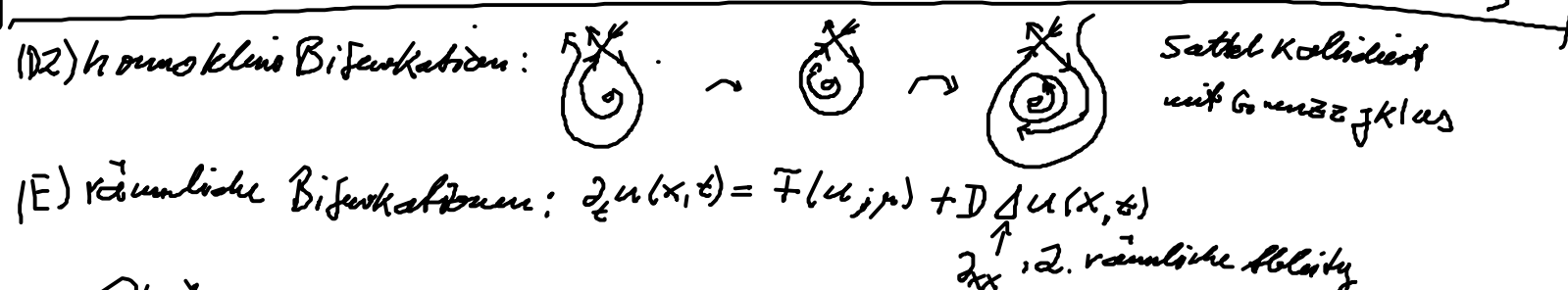
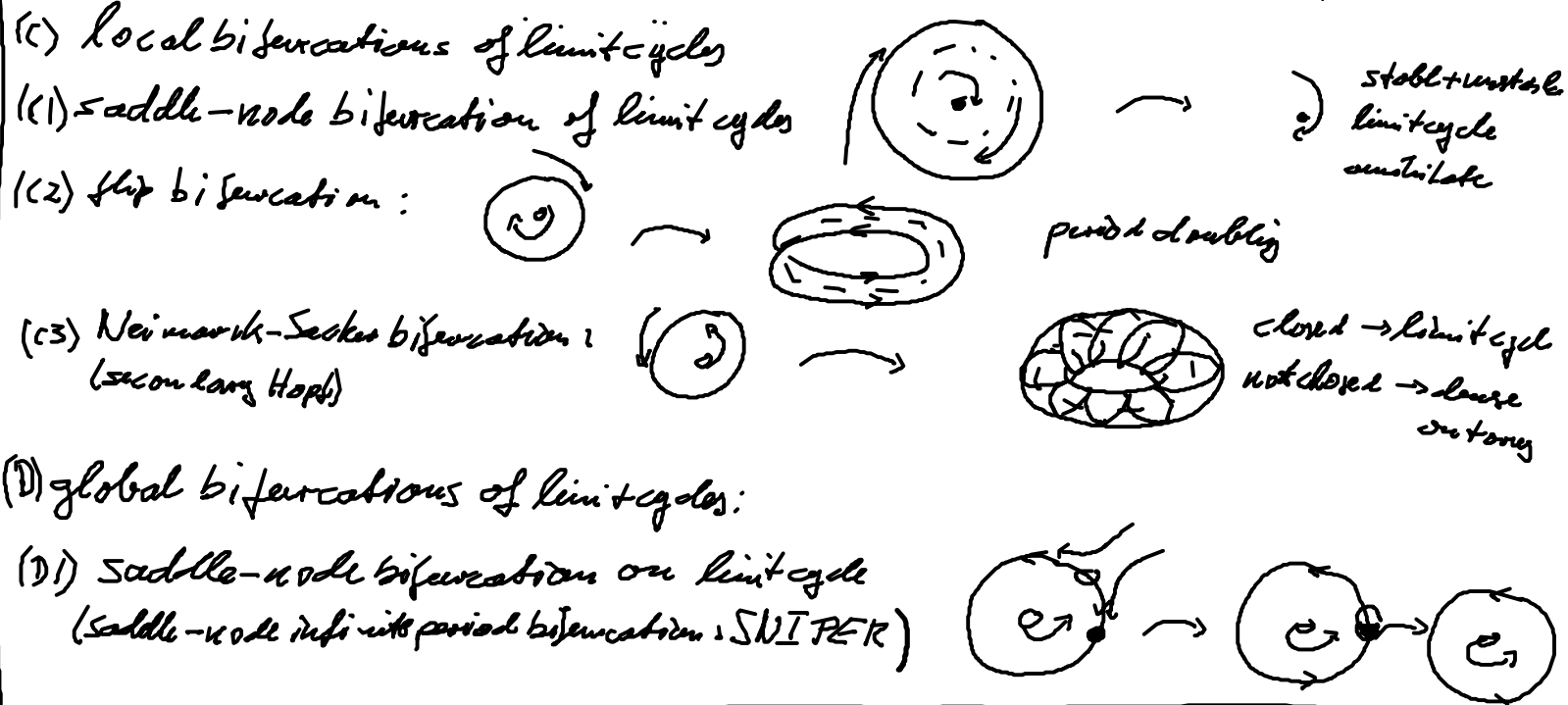


change of sign
 $\dot{x} = \mu x \oplus x^3$
 subcritical



(B) Hopf bifurcation: $\dot{z} = (\lambda + i\omega \mp (1 - i\mu)|z|^2)z$
 $z = r e^{i\varphi} \Rightarrow \dot{r} = (\lambda \mp \omega^2 r^2)r, \dot{\varphi} = \omega \mp \mu r^2$
 subcritical





2. Phänomenologische Modelle

Mathematische Beschreibung neuronaler Dynamik

- ▶ Normal-form equations:
 - ▶ e.g. FitzHugh-Nagumo equations, SNIC/SNIPER equations
 - ▶ Main dynamical behavior (type of excitability)
 - ▶ Feasible for bifurcation analysis
 - ▶ Few equations, few parameters
 - ▶ Applicable to ensembles of many oscillators
 - ▶ Detailed description of single cell?
 - ▶ Physiological relevant processes?

► Physiological models: → *siehe Kap. 3*

- e.g. Hodgkin-Huxley equations
- Many physiological details and processes
- Detailed description of single cell
- Many equations, many parameters
- Applicable to ensembles of many oscillators?
- Feasible for bifurcation analysis?

2.1 FitzHugh-Nagumo - Modell

2.2 SNIPER - Modell

2.3 Hindmarsh-Rose - Modell

2.1 FitzHugh-Nagumo - Modell

Bifurkationsparameter

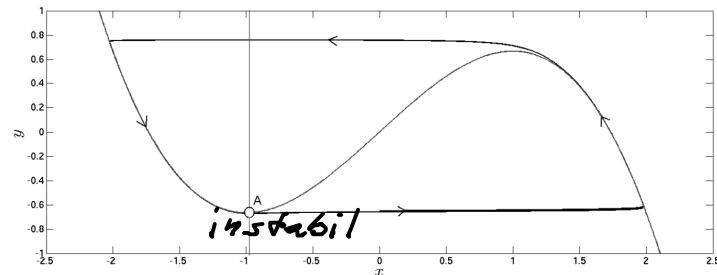
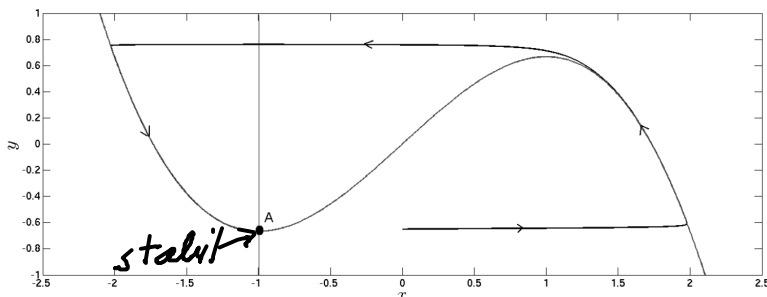
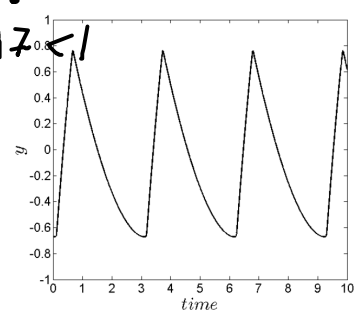
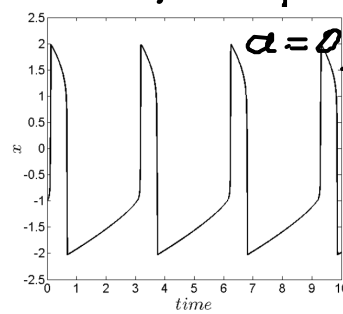
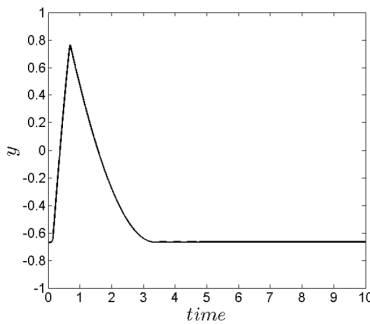
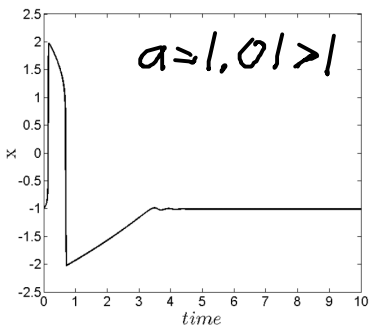
dynamische Gleichungen: $\epsilon \dot{x} = x - \frac{x^3}{3} - y$, $\dot{y} = x + a$

Zeitskalen-
trennung ($\epsilon \ll 1$)

Aktivator
(schnell)

Inhibitor
(langsam)

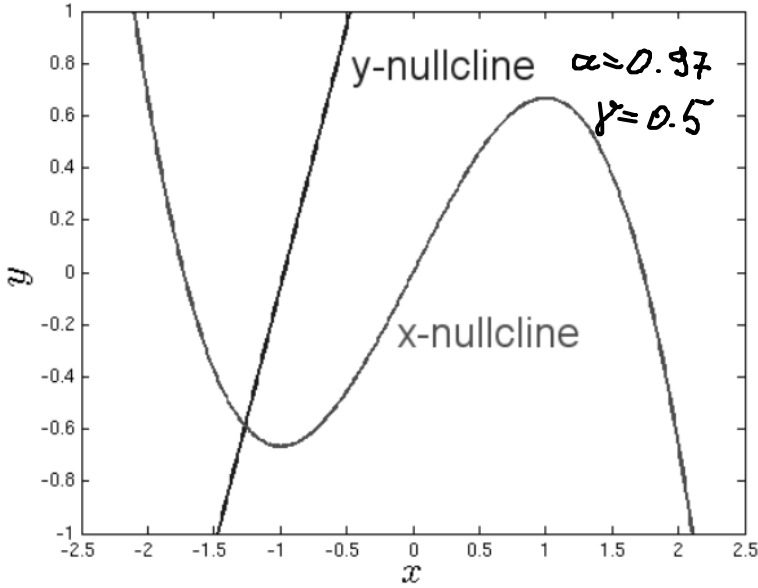
Fazit (s. Kap. 1.2): Der Fixpunkt $(x^* = -a, y^* = -a + \frac{a^2}{3})$ verliert seine Stabilität in einer Hopf-Bifurkation bei $|a_{crit}| \geq 1$.



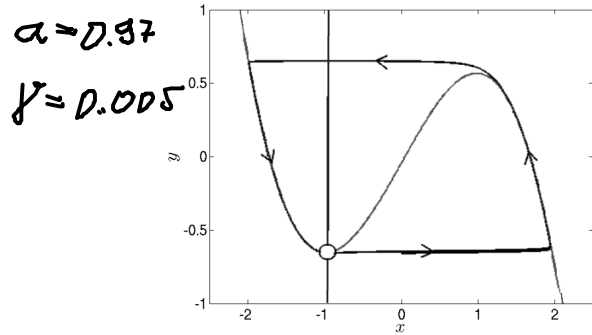
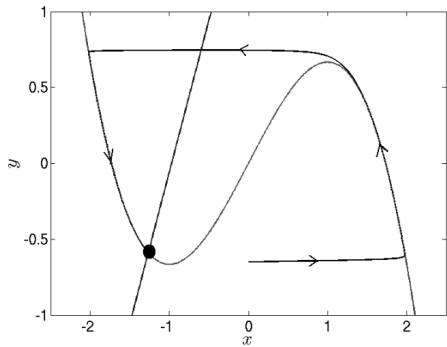
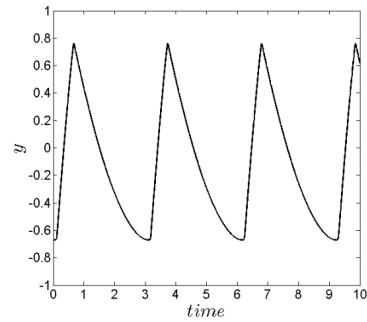
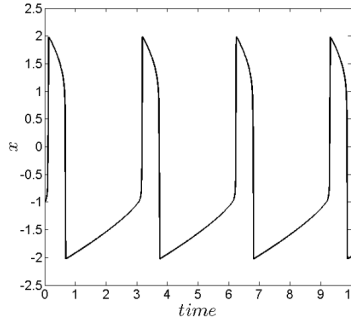
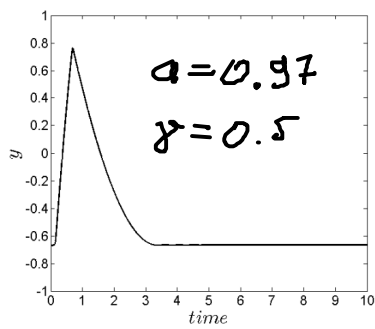
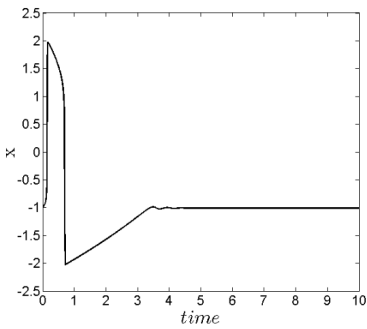
oszillierend

α > 1 möglich

Ursprüngliches Modell: $\dot{x} = x - \frac{x^3}{3} - y$, $\dot{y} = x + \alpha - \gamma y$



Y-Nullcline ist nicht mehr senkrecht für $\gamma \neq 0$.



IMPULSES AND PHYSIOLOGICAL STATES IN THEORETICAL MODELS OF NERVE MEMBRANE

Biophys. J. 1, 445 (1961)

RICHARD FITZHUGH

From the National Institutes of Health, Bethesda

The following linear differential equation describes an oscillating quantity x with damping constant k (the dots represent differentiation with respect to time t):

$$\ddot{x} + k\dot{x} + x = 0$$

Van der Pol (1926) replaced the damping constant by a damping coefficient which depends quadratically on x :

$$\ddot{x} + c(x^2 - 1)\dot{x} + x = 0$$

where c is a positive constant. It is convenient to use Liénard's transformation (Liénard, 1928; Minorsky, 1947):

$$y = \dot{x}/c + x^3/3 - x$$

and obtain the following pair of differential equations:

$$\dot{x} = c(y + x - x^3/3)$$

$$\dot{y} = -x/c$$

The BVP model is obtained by adding terms to these equations as follows:—

$$\dot{x} = c(y + x - x^3/3 + z) \tag{1}$$

$$\dot{y} = -(x - a + by)/c \tag{2}$$

An Active Pulse Transmission Line Simulating Nerve Axon*

J. NAGUMO†, MEMBER, IRE, S. ARIMOTO†, AND S. YOSHIZAWA†

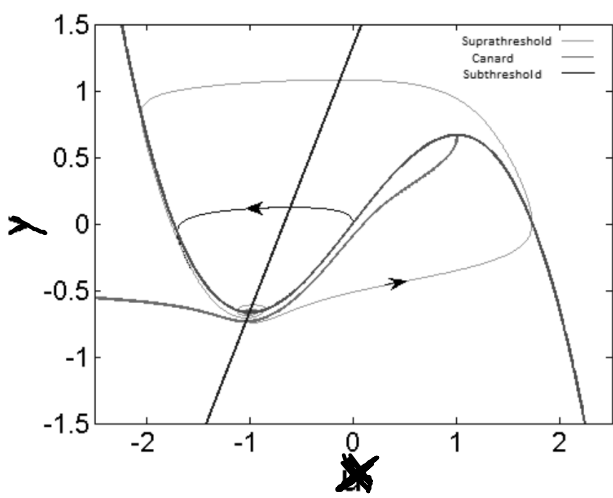
Recently, FitzHugh ingeniously simplified the H-H equations in case of a "space clamp," making use of an analog computer, and proposed the following BVP model (Bonhoeffer-van der Pol model).⁹

$$\begin{cases} J = \frac{1}{c} \frac{du}{dt} - w - \left(u - \frac{u^3}{3} \right), \\ c \frac{dw}{dt} + bw = a - u, \end{cases} \tag{2}$$

Lineare Stabilitätsanalyse: $\dot{x} = 0, \dot{y} = 0 \Rightarrow 0 = -\frac{a}{c} + \left(1 - \frac{1}{c}\right)x^ - \frac{(x^*)^3}{3}$*
 $y^* = x^* - \frac{(x^*)^3}{3}, y^* = \frac{1}{c}x^* + \frac{a}{c}$ ↑ Nullstelle einer kubischen Gl.

Jacobi-Matrix: $\begin{pmatrix} \delta \dot{x} \\ \delta \dot{y} \end{pmatrix} = \begin{pmatrix} \frac{1-(x^)^2}{c} & -\frac{1}{c} \\ 1 & b \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix} \Rightarrow \lambda_{1,2} = \frac{b \pm \sqrt{(b-A)^2 - 4 \det A}}{2}$*

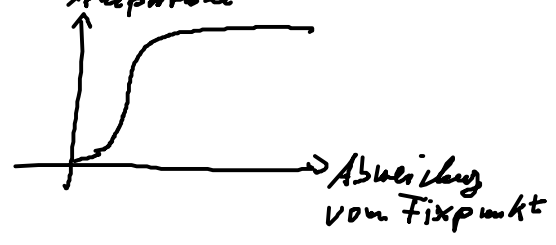
$$\underline{DF}|_{x^*} = \underline{A}$$



$$\gamma = 0.5$$

$$a = 0.67$$

Auftreten einer Canard-Trajektorie
Amplitude



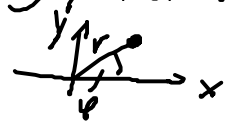
2.2 SNIPER-Modell

Saddle-Node Infinite PERIOD Bifurcation

oder auch SNIC: Saddle-node bifurcation on an invariant cycle

$$\begin{cases} \dot{x} = x(1-x^2-y^2) + y(x-b) \\ \dot{y} = y(1-x^2-y^2) - x(x-b) \end{cases} \begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases}$$

$$\begin{cases} \dot{r} = r(1-r^2) \\ \dot{\varphi} = b - r \cos \varphi \end{cases}$$



PHYSICAL REVIEW LETTERS

Stochastic Resonance without External Periodic Force

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(Received 21 December 1992)

Bestimmung der Fixpunkte:

$$\left. \begin{aligned} 0 &= x(1-x^2-y^2) + y(x-b) \\ 0 &= y(1-x^2-y^2) - x(x-b) \end{aligned} \right\} \text{trivialer Fixpunkt bei } (x_A^*, y_A^*) = (0, 0)$$

weitere Fixpunkte für $x = +b$ und $1-x^2-y^2=0 \xrightarrow{x=-b} y = \pm \sqrt{1-b^2}$


$$\left. \begin{aligned} \Rightarrow (x_B^*, y_B^*) &= (+b, \sqrt{1-b^2}) \\ (x_C^*, y_C^*) &= (+b, -\sqrt{1-b^2}) \end{aligned} \right\} \text{Existenz nur für } |b| < 1$$

Linearstabilitätsanalyse:

$$\underline{DF}|_{x_i^*, y_i^*} = \begin{pmatrix} 1-3x^2-y^2+y & -2xy+x-b \\ -2xy-2x+b & 1-x^2-3y^2 \end{pmatrix} \Big|_{x_i^*, y_i^*}$$

1. Fall: $(x_A^*, y_A^*) = (0, 0) \rightarrow \underline{DF}|_A = \begin{pmatrix} 1 & -b \\ +b & 1 \end{pmatrix} \Rightarrow \lambda_{1,2} = \frac{2 \pm \sqrt{4 - 4(1+b^2)}}{2} = 1 \pm ib$

\Rightarrow Fixpunkt A ist ein Fokus



2. Fall: $(x_B^*, y_B^*) = (+b, +\sqrt{1-b^2})$

$$\underline{DF}|_B = \begin{pmatrix} -2b^2 + \sqrt{1-b^2} & -2b\sqrt{1-b^2} \\ -2b\sqrt{1-b^2} - b & -2 + 2b^2 \end{pmatrix}$$

Eigenwerte als Lösung der charakteristischen Gleichung:

$$0 = \det(\underline{DF}|_B - \lambda \underline{1}) = \dots = \underbrace{(\lambda + 2)}_{\lambda = -2} \underbrace{(\lambda - \sqrt{1-b^2})}_{\lambda = \sqrt{1-b^2} > 0 \text{ für } |b| < 1}$$

Fortsetzung folgt...