English summary

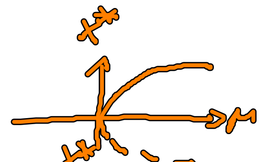
1.2 Bifurcations (continued)

- attractors in systems of dimension 1 (fixed points), 2 (limit cycles), 3 (tori, strange attractors)

classification:

(A) eigenvalue-zero bifurcation

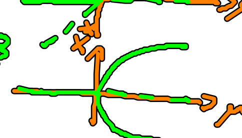
(A1) saddle-node bifurcation: $\dot{x} = \mu - x^2$



(A2) transcritical bifurcation: $\dot{x} = \mu x - x^2$

(A3) pitchfork bifurcation: $\dot{x} = \mu x - x^3$

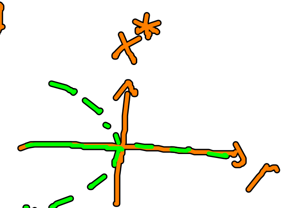
supercritical



change of sign

$\dot{x} = \mu x + x^3$

subcritical

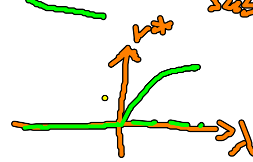


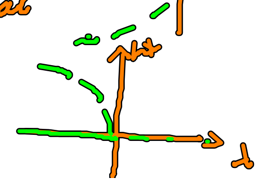
(B) Hopf bifurcation: $\dot{z} = (\lambda + i\omega \mp (1 - i\gamma)|z|^2)\, z$

supercritical

subcritical

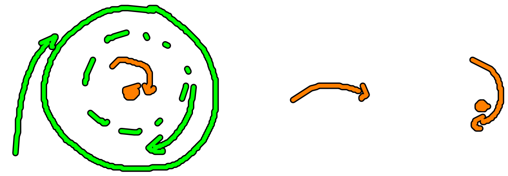
$z = re^{i\varphi} \Rightarrow \dot{r} = (\lambda \mp r^2)\, r, \quad \dot{\varphi} = \omega \mp \gamma r^2$





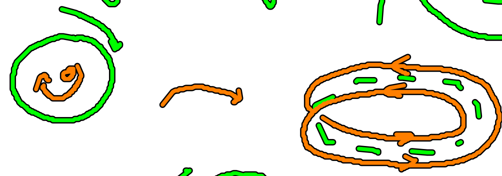
(C) local bifurcations of limit cycles

(C1) saddle-node bifurcation of limit cycles



stable+unstable limit cycle annihilate

(C2) flip bifurcation:



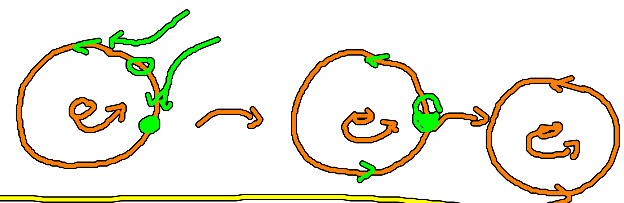
period doubling

(C3) Neimark-Sacker bifurcation: (secondary Hopf)

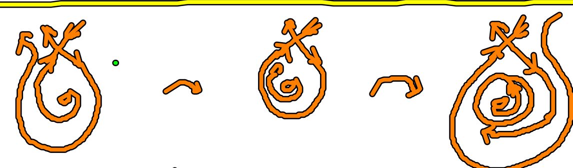
closed → limit cycle
not closed → dense on torus

(D) global bifurcations of limit cycles:

(D1) saddle-node bifurcation on limit cycle (saddle-node infinite period bifurcation: SNIPER)



(D2) homoklinic Bifurkation:



Sattel Kollision mit Grenzzyklus

(E) räumliche Bifurkationen: $\partial_t u(x,t) = F(u;p) + D\Delta u(x,t)$

$\Delta = \partial_{xx}$: 2. räumliche Ableitung

2. Phänomenologische Modelle

Mathematische Beschreibung neuronaler Dynamik

- Normal-form equations:
  - e.g. FitzHugh-Nagumo equations, SNIC/SNIPER equations
  - Main dynamical behavior (type of excitability)
  - Feasible for bifurcation analysis
  - Few equations, few parameters
  - Applicable to ensembles of many oscillators
  - Detailed description of single cell?
  - Physiological relevant processes?

- Physiological models: → siehe Kap. 3
  - e.g. Hodgkin-Huxley equations
  - Many physiological details and processes
  - Detailed description of single cell
  - Many equations, many parameters
  - Applicable to ensembles of many oscillators?
  - Feasible for bifurcation analysis?

2.1 FitzHugh-Nagumo-Modell
2.2 SNIPER-Modell
2.3 Hindmarsh-Rose-Modell

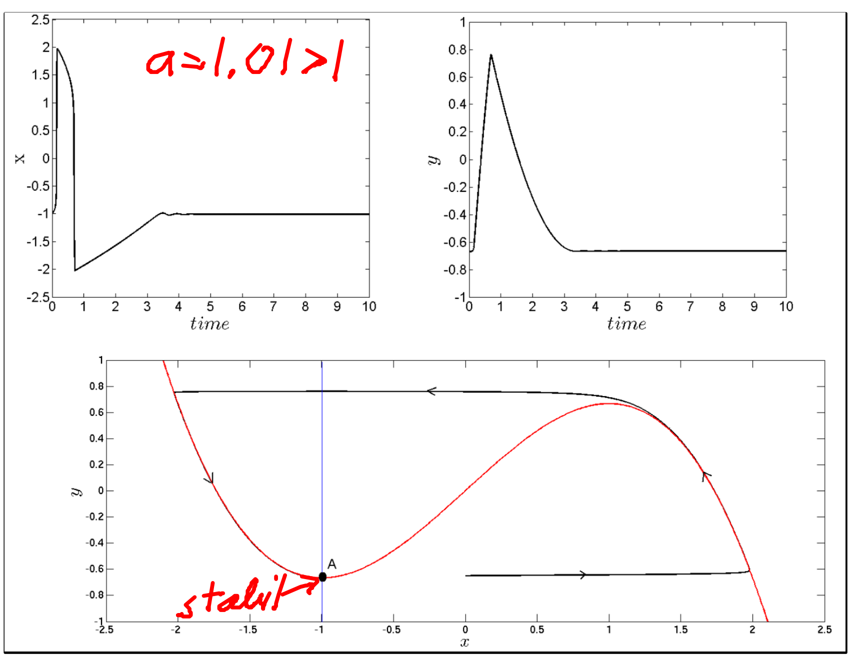
2.1 FitzHugh-Nagumo-Modell

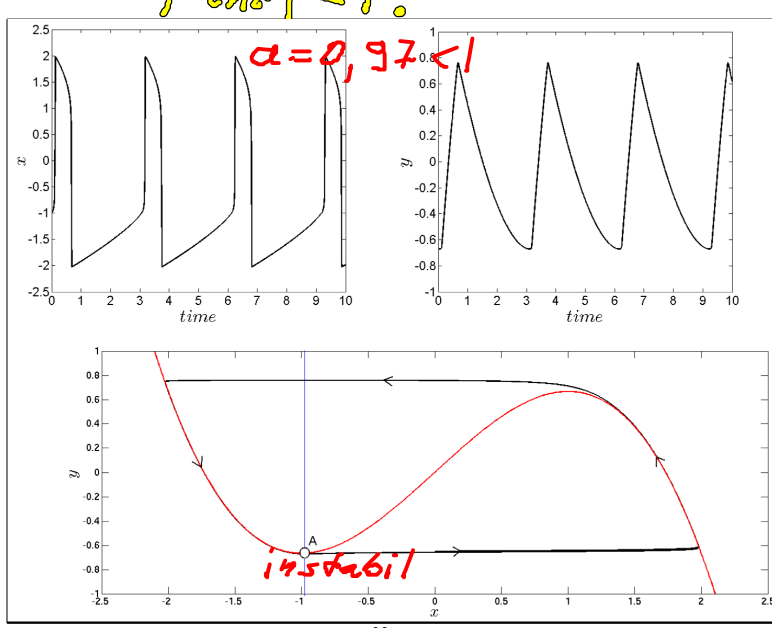
Bifurkationsparameter

dynamische Gleichungen: $\varepsilon \dot{x} = x - \frac{x^3}{3} - y \,, \quad \dot{y} = x + a$

Zeitskalentrennung ($\varepsilon \ll 1$) — Aktivator (schnell) — Inhibitor (langsam)

Fazit (s. Kap. 1.2): Der Fixpunkt ($x^* = -a$, $y^* = -a + \frac{a^3}{3}$) verliert seine Stabilität in einer Hopf-Bifurkation bei $|a_{crit}| \gtrsim 1$.

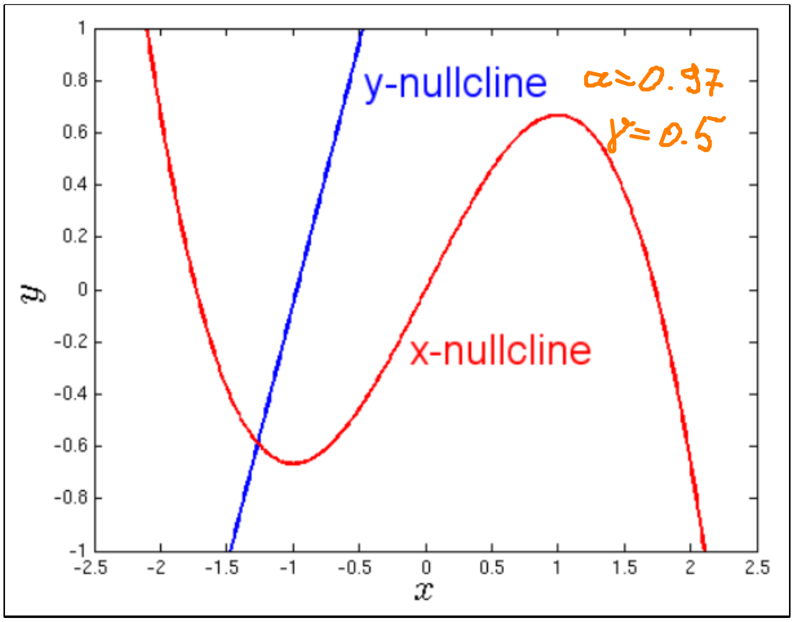




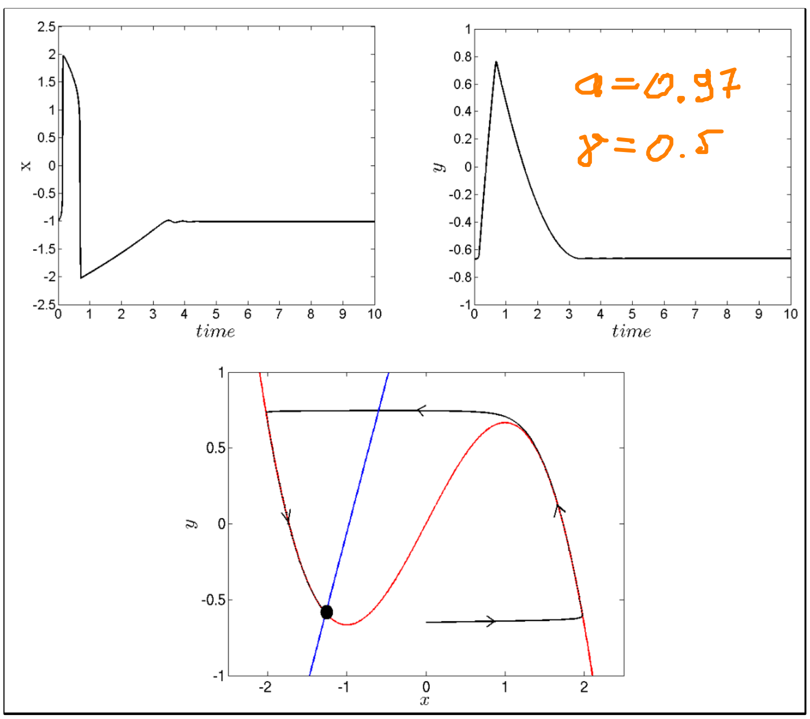
oszillierend

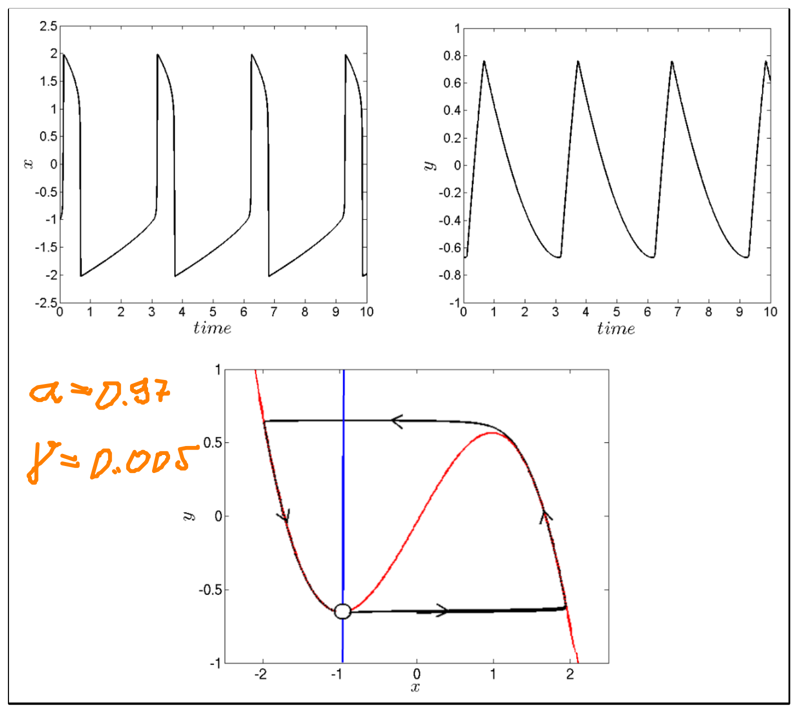
anregbar

Ursprüngliches Modell: $\epsilon \dot{x} = x - \frac{x^3}{3} - y \quad , \dot{y} = x + a - \gamma y$



y-Nullkline ist nicht mehr senkrecht für $\gamma \neq 0$.





IMPULSES AND PHYSIOLOGICAL STATES IN THEORETICAL MODELS OF NERVE MEMBRANE

Biophys. J. 1, 445 (1961)

RICHARD FITZHUGH

From the National Institutes of Health, Bethesda

The following linear differential equation describes an oscillating quantity x with damping constant k (the dots represent differentiation with respect to time t):

$$\ddot{x} + k\dot{x} + x = 0$$

Van der Pol (1926) replaced the damping constant by a damping coefficient which depends quadratically on x:

$$\ddot{x} + c(x^2 - 1)\dot{x} + x = 0$$

where c is a positive constant. It is convenient to use Liénard's transformation (Liénard, 1928; Minorsky, 1947):

$$y = \dot{x}/c + x^3/3 - x$$

and obtain the following pair of differential equations:

$$\dot{x} = c(y + x - x^3/3)$$

$$\dot{y} = -x/c$$

The BVP model is obtained by adding terms to these equations as follows:—

$$\dot{x} = c(y + x - x^3/3 + z) \tag{1}$$

$$\dot{y} = -(x - a + by)/c \tag{2}$$

1962 PROCEEDINGS OF THE IRE 2061

An Active Pulse Transmission Line Simulating Nerve Axon*

J. NAGUMO†, MEMBER, IRE, S. ARIMOTO†, AND S. YOSHIZAWA†

Recently, FitzHugh ingeniously simplified the H-H equations in case of a "space clamp," making use of an analog computer, and proposed the following BVP model (Bonhoeffer-van der Pol model).[9]

$$\begin{cases} J = \dfrac{1}{c}\dfrac{du}{dt} - w - \left(u - \dfrac{u^3}{3}\right), \\ c\dfrac{dw}{dt} + bw = a - u, \end{cases} \tag{2}$$

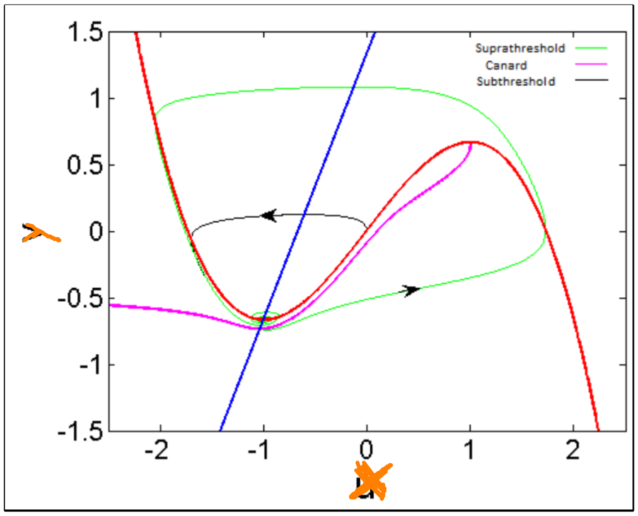
Lineare Stabilitätsanalyse: $\dot{x} = 0,\ \dot{y} = 0 \Rightarrow 0 = -\frac{a}{\gamma} + \left(1 - \frac{1}{\gamma}\right)x^* - \frac{(x^*)^3}{3}$

$y^* = x^* - \frac{(x^*)^3}{3},\quad y^* = \frac{1}{\gamma}x^* + \frac{a}{\gamma}$

Nullstelle einer kubischen Fkt.

Jacobi-Matrix: $\begin{pmatrix} \delta\dot{x} \\ \delta\dot{y} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1-(x^*)^2}{\varepsilon} & -\frac{1}{\varepsilon} \\ 1 & -\gamma \end{pmatrix}}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} \Rightarrow \Lambda_{1,2} = \frac{\operatorname{tr}\underline{\underline{A}} \pm \sqrt{(\operatorname{tr}\underline{\underline{A}})^2 - 4\det\underline{\underline{A}}}}{2}$

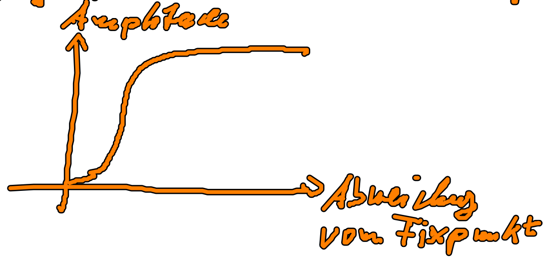
$\underline{\underline{DF}}\big|_{x^*} = \underline{\underline{A}}$



$\gamma = 0.5$

$a = 0.67$

Auftreten der Canard-Trajektorie



2.2 SNIPER-Modell

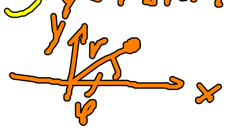
Saddle-Node Infinite PERiod bifurcation

oder auch SNIC : Saddle-node bifurcation on an invariant cycle

$$\left.\begin{aligned}\dot{x} &= x(1-x^2-y^2) + y(x-b)\\ \dot{y} &= y(1-x^2-y^2) - x(x-b)\end{aligned}\right\} \begin{aligned}&x = r\cos\varphi\\ &\Longleftrightarrow\\ &y = r\sin\varphi\end{aligned}$$

$$\dot{r} = r(1-r^2)$$

$$\dot{\varphi} = b - r\cos\varphi$$



PHYSICAL REVIEW LETTERS

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Stochastic Resonance without External Periodic Force

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(Received 21 December 1992)

Bestimmung der Fixpunkte:

$$\left.\begin{aligned} 0 &= x(1-x^2-y^2) + y(x-b) \\ 0 &= y(1-x^2-y^2) - x(x-b) \end{aligned}\right\} \text{ trivialer Fixpunkt bei } (x_A^*, y_A^*) = (0,0)$$

$x=-b$

weitere Fixpunkte für $x = +b$ und $1-x^2-y^2 = 0 \Rightarrow y = \pm\sqrt{1-b^2}$

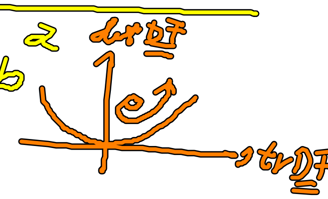
$$\left.\begin{aligned} \Rightarrow (x_B^*, y_B^*) &= (+b, \sqrt{1-b^2}) \\ (x_C^*, y_C^*) &= (+b, -\sqrt{1-b^2}) \end{aligned}\right\} \text{ existieren nur für } |b|<1$$

Lineare Stabilitätsanalyse:

$$\underline{\underline{DF}}\Big|_{x^*,y^*} = \begin{pmatrix} 1-3x^2-y^2+y & -2xy+x-b \\ -2xy-2x+b & 1-x^2-3y^2 \end{pmatrix}\Bigg|_{x^*,y^*}$$

1. Fall: $(x_A^*, y_A^*) = (0,0) \rightarrow \underline{\underline{DF}}\Big|_A = \begin{pmatrix} 1 & -b \\ +b & 1 \end{pmatrix} \Rightarrow \lambda_{1,2} = \dfrac{2 \pm \sqrt{4 - 4(1+b^2)}}{2}$

$= 1 \pm ib$



$\Rightarrow$ Fixpunkt A ist ein Fokus

2. Fall: $(x_B^*, y_B^*) = (+b, +\sqrt{1-b^2})$

$$\underline{\underline{DF}}\Big|_B = \begin{pmatrix} -2b^2+\sqrt{1-b^2} & -2b\sqrt{1-b^2} \\ -2b\sqrt{1-b^2} - b & -2+2b^2 \end{pmatrix}$$

Eigenwerte als Lösung der charakteristischen Gleichung:

$$0 = \det\left(\underline{\underline{DF}}\Big|_B - \lambda \mathbb{1}\right) = \dots = \underbrace{(\lambda+2)}_{\lambda=-2}\underbrace{(\lambda - \sqrt{1-b^2})}_{\lambda=\sqrt{1-b^2} > 0 \text{ für } |b|<1}$$

Fortsetzung folgt ...