

English summary

3.2 Morris-Lecar model

$$C_m \dot{V} = \overline{I} - g_K w(V) (V - E_K) - \overbrace{g_{Ca} w_{Ca}(V) (V - E_{Ca})}^{\text{fast Ca dynamics}} - g_L (V - E_L)$$

$$\tau_w(V) \dot{w} = -w + w_{\infty}(V)$$

with the following functions:

$$w_{\infty}(V) = \frac{1}{2} \left[1 + \tanh \frac{V - V_1}{V_2} \right], \quad w_{Ca}(V) = \frac{1}{2} \left[1 + \tanh \frac{V - V_3}{V_4} \right], \quad \tau_w(V) = \frac{1}{\cosh \frac{V - V_5}{2V_6}}$$

⇒ homoclinic bifurcation

4 Interplay of noise and delay

4.1 Coherence resonance

- Constructive role of noise
- **Coherence resonance**: noise-induced oscillations become most regular for finite noise strength

- Quantification by the following measures:

(i) correlation time $\tau_{cor} = \frac{1}{\psi(\omega)} \int_0^{\infty} |\psi(\omega)| d\omega$

↑ autocorrelation function

(ii) normalized fluctuations of interspike intervals

$$R = \frac{\sqrt{\langle T_{ISI}^2 \rangle - \langle T_{ISI} \rangle^2}}{\langle T_{ISI} \rangle}$$

(iii) signal-to-noise ratio

$$\beta = h \frac{\omega_p}{\Delta \omega}$$

height ↑ ↑ peak ↑ width ($h, \omega_p = \frac{h}{\tau_p}$)

• example: FitzHugh-Nagumo system

$$C \dot{x} = x - \frac{x^3}{3} - y, \quad \dot{y} = x + a + D \xi(t)$$

4.1 Kohärenzresonanz (Fortsetzung)

Grundbegriffe zu stochastischen Prozessen:

- zeitunabhängig oder Zufallsvariable

Langevin-Gleichung: fluktuierende, stochastische Kraft $f(t)$ (Rauschen)

z.B. Brownsche Bewegung (1827)

$$m\ddot{x} = -\eta\dot{x} + \xi(t)$$

Reibung

Rauschen

$\hat{=}$ Kraftdauer zufällig Stöße

Bewegungsgleichung eines Teilchens

Rauschen

stochastische Differentialgleichung: $dx_t = \theta(\mu - x_t)dt + \sigma dW_t$

kleine Änderung von x_t

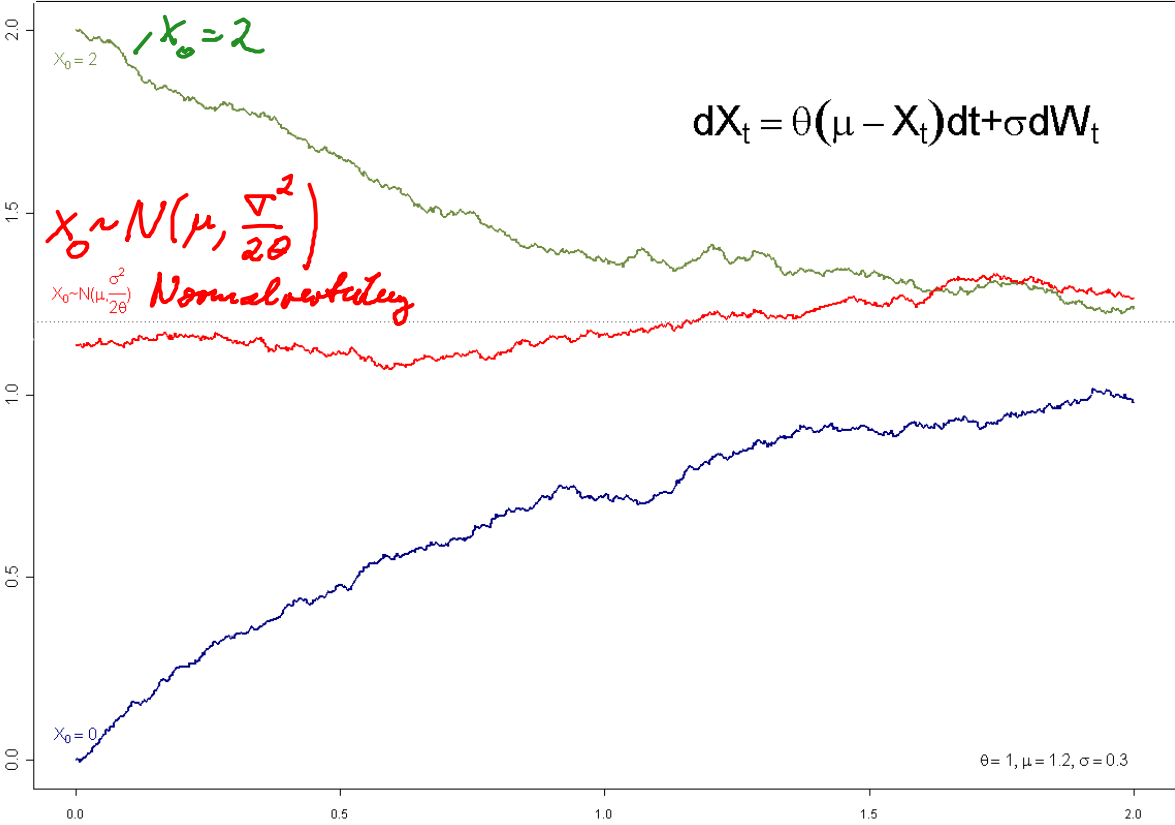
Rück-
stärke

Langfristige
Mittelwert

Volatilität

Ornstein-Uhlenbeck-Prozess

Ornstein - Uhlenbeck



W_t : Zufallsvariable eines Wiener-Prozesses

$\Rightarrow W_0 = 0$

$W_t - W_s \sim N(0, t-s)$

Normalverteilung mit Mittelwert $\mu=0$ und Varianz $t-s$

Zentriert der Grenzwertsatz: unkorrelierte Zufallsvariablen gehorchen einer Gauß-Verteilung

Gaußsches weißes Rauschen: $\langle \xi(t) \rangle = 0$ im zeitlichen Mittel keine gerichtete Kraft

$\langle \xi(t) \xi(t') \rangle = \delta(t-t')$ unkorreliert, keine Gedächtniseffekte

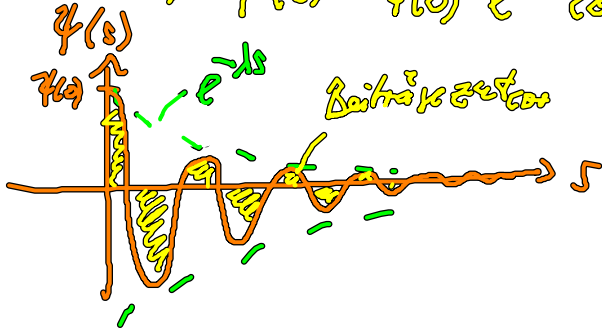
Verteilung der Zufallszahlen $\xi(t)$: Gauß-Verteilung

Autokorrelationsfunktion: $\psi(s) = \langle (x(t) - \langle x \rangle) (x(t+s) - \langle x \rangle) \rangle$ Mittelg über die Zeit

$\psi(s) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) x(t+s) dt$ um s verschobene Zeitserie x
Berechnung über endliches Zeitintervall
Annahme $\langle x \rangle = 0$

Bsp.: linearer stochastischer Prozess: $\dot{x} = -(\lambda + i\omega_0)x + \xi(t)$

$\rightarrow \psi(s) = \psi(0) e^{-\lambda s} \cos(\omega_0 s)$



$t_{\text{cor}} = \frac{\psi(0)}{\psi(0)} \int_0^{\infty} e^{-\lambda s} |\cos(\omega_0 s)| ds \approx \frac{2}{\pi \lambda}$

$\Rightarrow \psi(\omega) = \psi(\omega) e^{-\frac{2}{\pi} \frac{s}{t_{\text{cor}}}} \cos(\omega_0 s)$

Korrelationszeit = Abfallrate der Enveloppe der Autokorrelationsfunktion

spektrale Leistungsdichte: $S(\omega) = \lim_{T \rightarrow \infty} \frac{\pi}{T} |\tilde{x}(\omega, T)|^2$

$\Rightarrow S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle x(t) x(t+s) \rangle e^{i\omega s} ds$ $-\frac{1}{2\pi} \int_{-T}^T e^{i\omega t} x(t) dt$ Fourier-Transformierte

S ist die Fourier-Transformierte von $\psi(s) \hat{=}$ Wiener-Khinchin-Theorem

4.2 Zeitverzögerte Rückkopplung

time-delayed feedback control = "Pyragas control"

Continuous control of chaos by self-controlling feedback

K. Pyragas^{1,2}

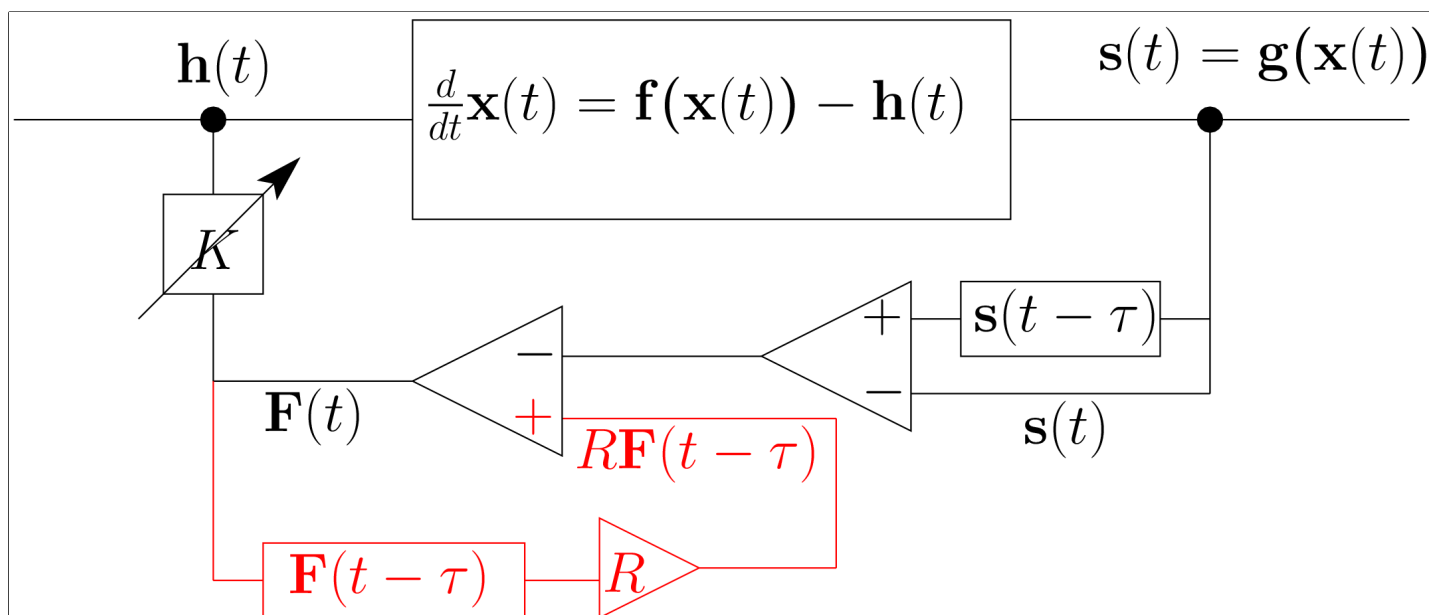
Institute for Theoretical Physics, Technical University of Berlin, W-1000 Berlin, Germany

Received 17 August 1992; accepted for publication 16 September 1992

Communicated by J. Flouquet

ursprünglich: Kontrolle instabiler periodischer Orbits/Grenzzyklen in selbstregulierten Attraktoren chaotischer Systeme

Vgl. SS 14, 15, 16: nichtlineare Dynamik und Kontrolle



Bewegungsgleichung der Form: $\dot{x} = f(x(t)) - k [s(t) - s(t - \tau)]$

zeitverzögerte Rückkopplung

hier: Anwendung auf neuronale Dynamik

Delay control of coherence resonance in type-I excitable dynamics

R. Aust¹, P. Hövel¹, J. Hizanidis^{1,2}, and E. Schöll¹

¹ Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstraße 36, 10623 Berlin, Germany

² Optical Communications Laboratory, Department of Informatics and Telecommunications, University of Athens, Athens 15784, Greece

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$$\dot{x} = x(1 - x^2 - y^2) + y(x - b) + D\xi + K(x_\tau - x) \quad (1a)$$

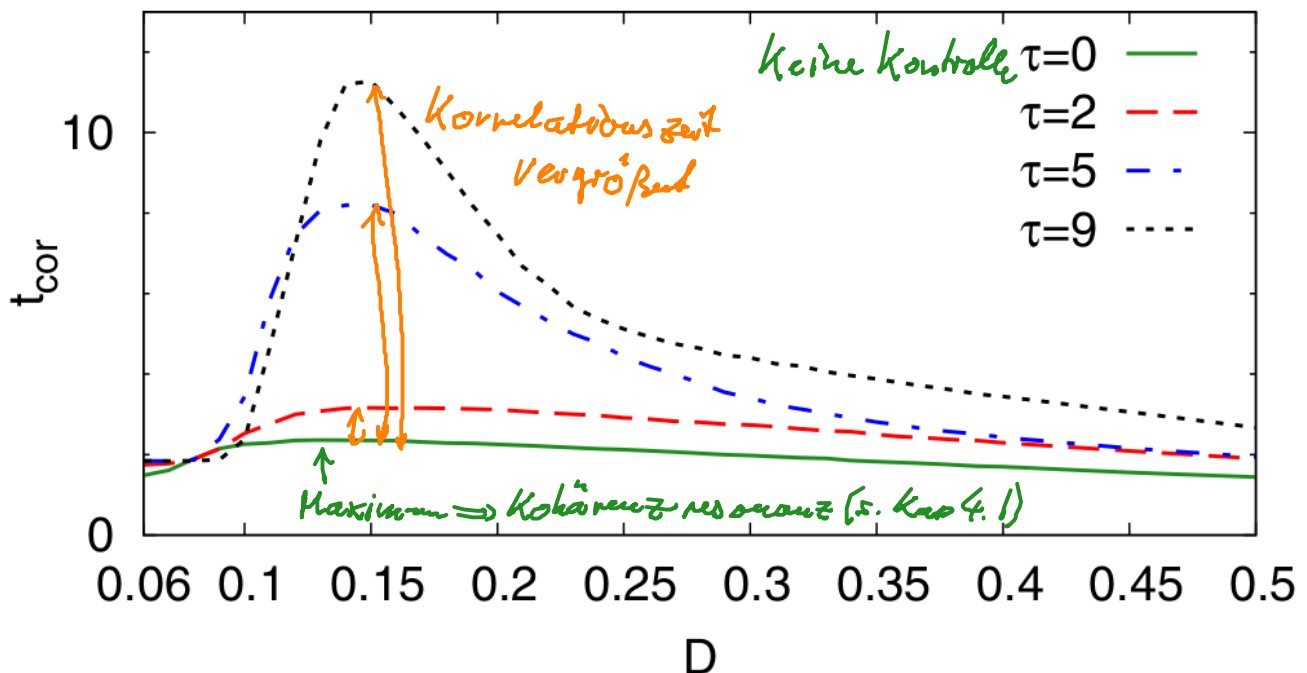
$$\dot{y} = y(1 - x^2 - y^2) - x(x - b) + D\xi + K(y_\tau - y). \quad (1b)$$

Here x and y are the variables at time t , while x_τ and y_τ denote the respective variables at a delayed time $t - \tau$. The bifurcation parameter $b \in \mathbb{R}$ is a real constant. K denotes the control strength and τ is the delay time. Random input ξ is realized as Gaussian white noise with mean $\langle \xi(t) \rangle = 0$, variance $\langle \xi(t)\xi(t') \rangle = \delta(t - t')$, and noise intensity D . In polar coordinates the system equations for $D = 0$ and $K = 0$ are given by [22]:

$$\dot{r} = r(1 - r^2) \quad (2a)$$

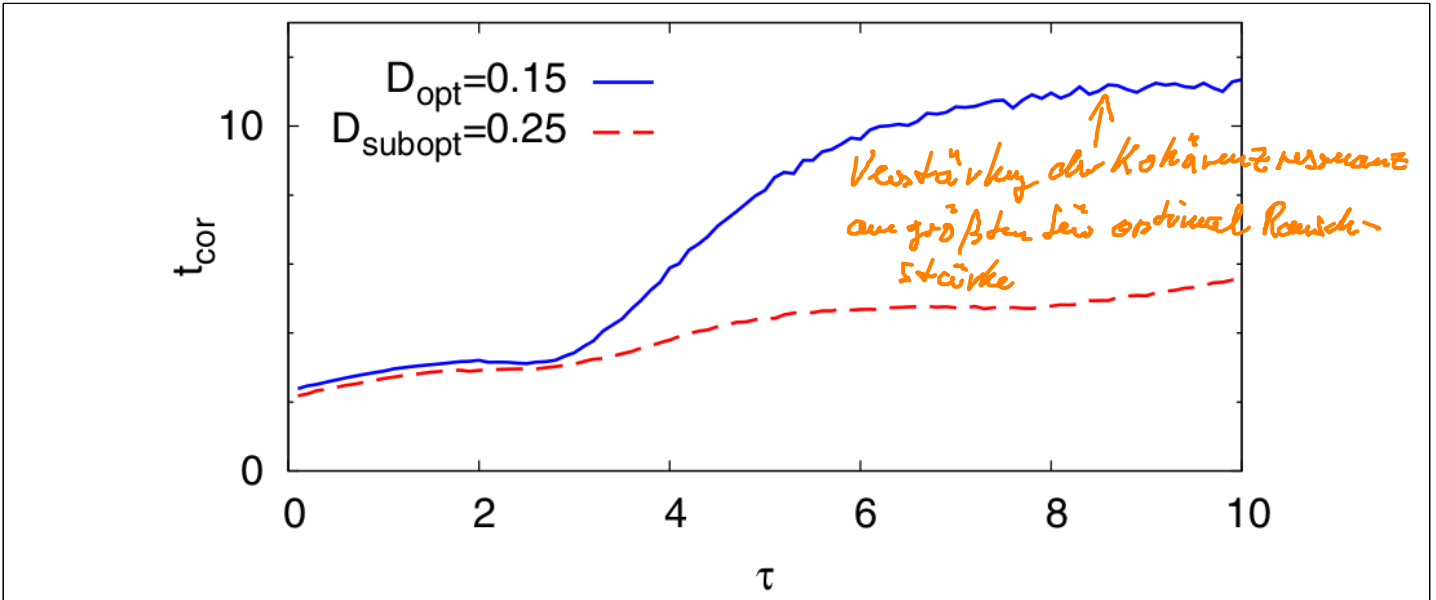
$$\dot{\varphi} = b - r \cos \varphi. \quad (2b)$$

Ziel: Kontrolle (Verstärkung der Regelmäßigkeit) rausdifferenzierter Oszillationen

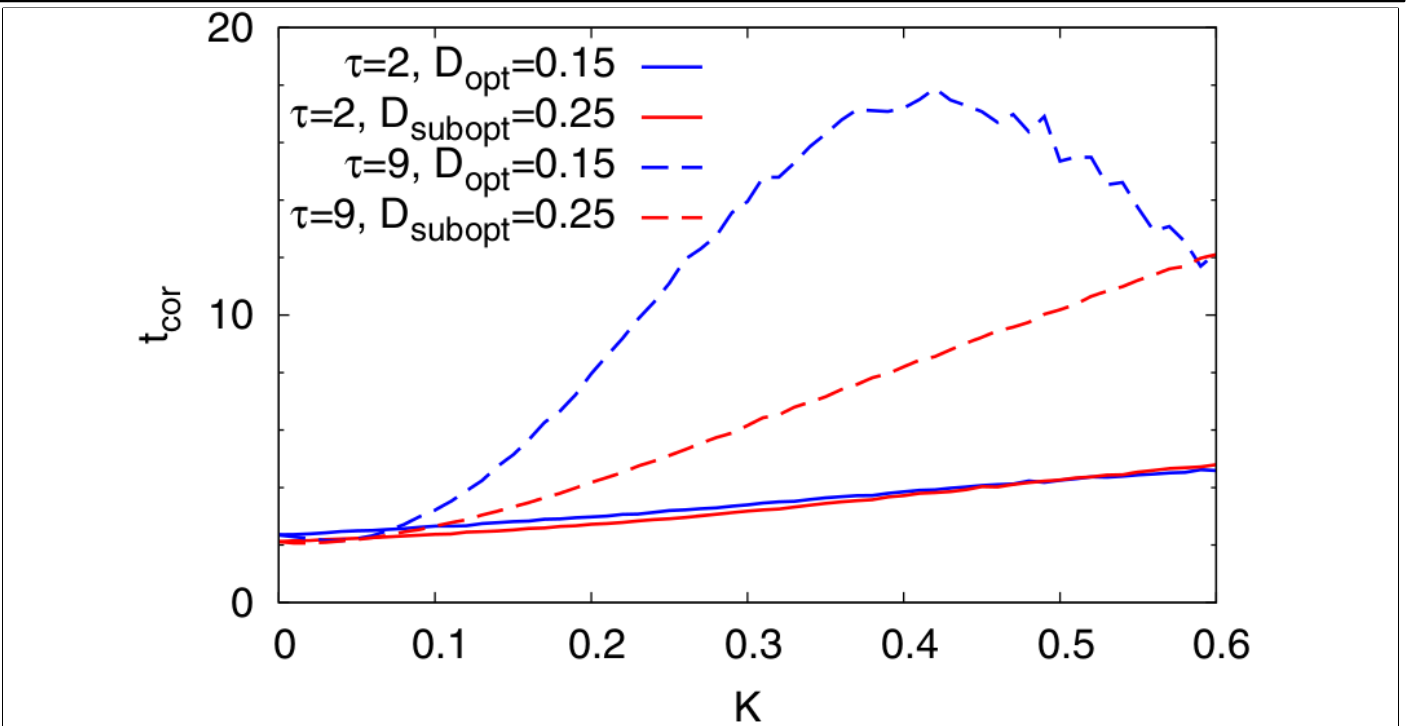


Correlation time in dependence on the noise intensity D for different time delays τ . The solid (green) curve corresponds to the uncontrolled system ($\tau = 0$). The dashed (red)

delays τ . The solid (green) curve corresponds to the uncontrolled system ($\tau = 0$). The dashed (red), dash-dotted (blue), and dotted (black) curves refer to values of $\tau = 2, 5,$ and $9,$ respectively. Other parameters: $b = 0.95$ and $K = 0.25$.



Correlation time t_{cor} in dependence on the time delay τ for two values of the noise intensity D . The dashed (red) curve corresponds to $D_{subopt} = 0.25$ and the solid (blue) curve refers to $D_{opt} = 0.15$. Other parameters: $b = 0.95$ and $K = 0.25$.



Correlation time t_{cor} in dependence on the control strength K for two values of the noise intensity D and two values of the delay time τ . The gray (red) and black (blue) curves depict the cases of D_{subopt} and D_{opt} , respectively. The solid and dashed lines correspond to $\tau = 2$ and $\tau = 9$, respectively. Other parameter: $b = 0.95$.