

**Time-delayed feedback controlled pattern formation
in globally coupled semiconductor systems**

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1 Objective

The objective of the project has been the numerical and theoretical analysis of self-organized spatiotemporal patterns in bistable and multistable semiconductor systems. Our aim has been to influence, control and select such patterns. Various semiconductor nanostructures like the resonant tunneling diode (DBRT = Double Barrier Resonant Tunneling Diode), the superlattice, as well as the HHED (= Heterostructure Hot Electron Diode), which have been studied by our group for a long time, have served as model systems. The modelling of the nonlinear electronic transport was done on the level of semiclassical electron density dynamics. This leads to reduced model equations, very similar to those of reaction-diffusion systems, which are studied intensely in the framework of other projects of Sfb 555. Therefore our research also has a fundamental, methodical character far beyond semiconductor nanostructures.

In the model systems of the resonant tunneling diode and the superlattice, the first step was to analyse the complex and chaotic spatiotemporal oscillatory scenarios, which occur through competing spatial and temporal instabilities. Patterns should then be selected through a time delayed feedback loop of the output signal. Thereby, methods of time delayed feedback control (*Autosynchronization*) which allow the stabilization of unstable periodic orbits should be applied to spatiotemporal patterns.

2 Results

Our group has had a long-time experience in nonlinear and chaotic spatiotemporal pattern formation in semiconductors [31, 32, 33, 34]. During the past proposal period, we mainly dealt with the control of such patterns by time delayed feedback. Thereby we have obtained numerous fundamental results of chaos control in spatially extended systems with global coupling, which are independent of the specific microscopic properties of the system. In particular we achieved to get a deeper understanding of how various control schemes (local, global and periodically modulated) work. Moreover we managed to obtain generic results for complex front dynamics. Three semiconductor systems served as concrete models: (i) a simple generic reaction-diffusion system with global coupling for transport in semiconductor heterostructures, like the HHED, (ii) the resonant tunneling diode (DBRT) and (iii) the semiconductor superlattice. The two latter are of particular interest with regard to applications in semiconductor physics. Besides collaborations within the Sfb, from which joint publications arose, we also had important collaborations with - guest scientist Dr. P. Rodin (Ioffe Physico-Technical Institute St. Petersburg), Dr. W. Just (Queen Mary and Westfield College, London), Dr. N. Janson (Lancaster University and Loughborough University, UK) and Dr. A. Balanov (Lancaster University, UK, and Saratov State University, Russia) as well as other scientists (J. Socolar, USA,

L. Bonilla, Madrid and others). Dr. Rodin's stay has been at large supported by an Alexander-von-Humboldt fellowship.

The work in progress is divided into three parts, according to the respective system under investigation.

(i) Control of the spatiotemporal dynamics in a generic reaction-diffusion system

Already in the first proposal period, we developed a generic activator-inhibitor reaction-diffusion model, which originally was derived for vertical charge transport through semiconductor structures like the HHED and which is typical for a large class of spatially extended systems in physics, chemistry and biology. Global coupling is achieved through the connected electric load circuit. This model exhibits not only front dynamics but complex and chaotic spatiotemporal scenarios as well [7]. During the report period we have obtained surprising results and have made substantial progress managing to control spatiotemporal patterns using time-delayed feedback methods (Pyragas-control [1, 40]). In dimensionless units, the model equations for the generic model have the form of a reaction-diffusion system of activator-inhibitor type, with global coupling. We have extended the model by adding the control forces F_a and F_u [15]:

$$\partial_t a(\vec{x}, t) = \frac{u - a}{(u - a)^2 + 1} - Ta + \Delta a - KF_a(\vec{x}, t), \quad (1)$$

$$\partial_t u(t) = \alpha [j_0 - (u - \langle a \rangle)] - KF_u(t). \quad (2)$$

where a is the space dependent activator variable (electron density distribution in the layer vertical to the transport direction) and u is the inhibitor variable (voltage across the device). The first equation represents the continuity equation for electrons that flow through the layer and the second equation is the Kirchhoff-equation for the total current $\sim j_0$, which causes a global coupling through the voltage drop across the load resistance. The mean spatial charge density distribution $\langle a \rangle$ enters into the current and represents the global coupling. α , j_0 and T are system parameters, which represent the time scale, the external control parameter or a parameter determining the bistability regime, respectively. The spatial variable in the layer $\vec{x}$ can be one- or two-dimensional depending on the problem. In the following we consider the one-dimensional case. The control forces F_a and F_u , each of which is multiplied by a control amplitude K , can be chosen arbitrarily. First let $K = 0$ (no control). Complex and chaotic behaviour is expected in a parameter range for which, at the same time, conditions for both a spatial instability (current filament) and a temporal one (Hopf bifurcation) are satisfied, as we have shown in the one-dimensional case [7]. The question arises under which circumstances similar behaviour for systems of two spatial dimensions is possible. In collaboration with guest scientist W. Just

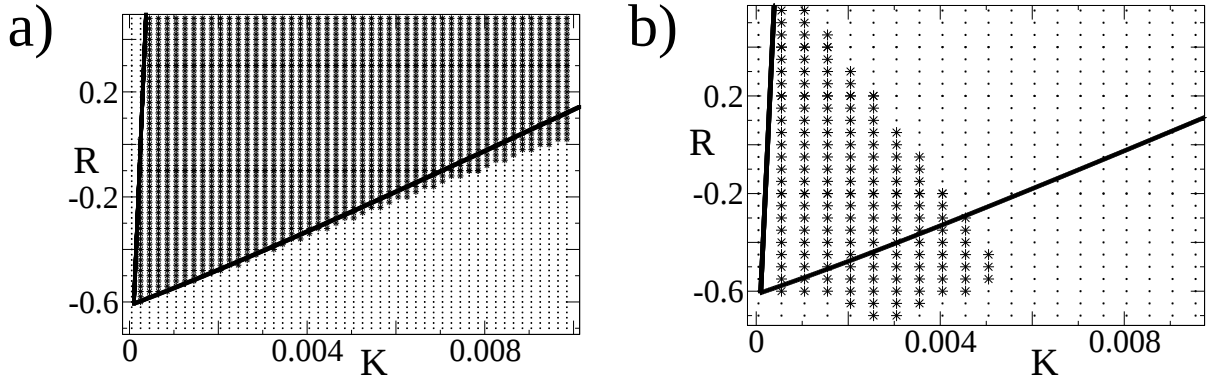


Figure 1: Control regimes of the time-delayed feedback in the $K - R$ plane for the generic HHED model with (a) diagonal and (b) local control without inhibitor control. Here $\star$ means successful and $\cdot$ non successful control. The solid lines denote the analytical solution for the boundaries of the control regime according to (5). See [15]

(London) as well as with project B6 we managed to show, using an amplitude expansion of the supercritical codimension-two-bifurcation, that one should not in general expect a coexistence of Turing- and Hopf instabilities in two dimensions in a (through a second diffusive coupling) locally coupled system unless the system size of the two directions is so small that a quasi-onedimensional dynamics arises. [10].

The generic structure of equations (1) and (2) is underlying for other projects as well. A very interesting collaboration arose with project B4, in which an electrochemical model for pattern formation in electrode surfaces was investigated: a detailed comparison to our globally coupled reaction-diffusion model revealed astonishing similarities in the scenarios of complex spatiotemporal dynamics [11]. Extending the globally coupled two-component reaction-diffusion system by a third diffusive component, one can describe apart from stationary, breathing or spiking current filaments, also moving filaments (or domains, respectively) [27].

Now we consider the case $K \neq 0$ in one spatial dimension. The system parameters are chosen such that for $K = 0$ chaotic spatiotemporal spiking arises. Our aim is to stabilize an unstable periodic spatiotemporal orbit, which is characterized by the period τ and the Floquet exponents λ . Furthermore the control forces F_a and F_u should vanish for successful control (*noninvasive control*).

This can be achieved using variants of time-delayed autosynchronization (Pyragas-control) [1, 2, 3]. Our goal here was to extend this method to spatiotemporal patterns. As starting point we use a delay feedback loop of the form $F_a = F_{\text{loc}}$, $F_u = F_{\text{vf}}$ (*diagonal coupling*), with

$$F_{\text{loc}}(x, t) = a(x, t) - a(x, t - \tau) + RF_{\text{loc}}(x, t - \tau), \quad (3)$$

$$F_{\text{vf}}(t) = u(t) - u(t - \tau) + RF_{\text{vf}}(t - \tau), \quad (4)$$

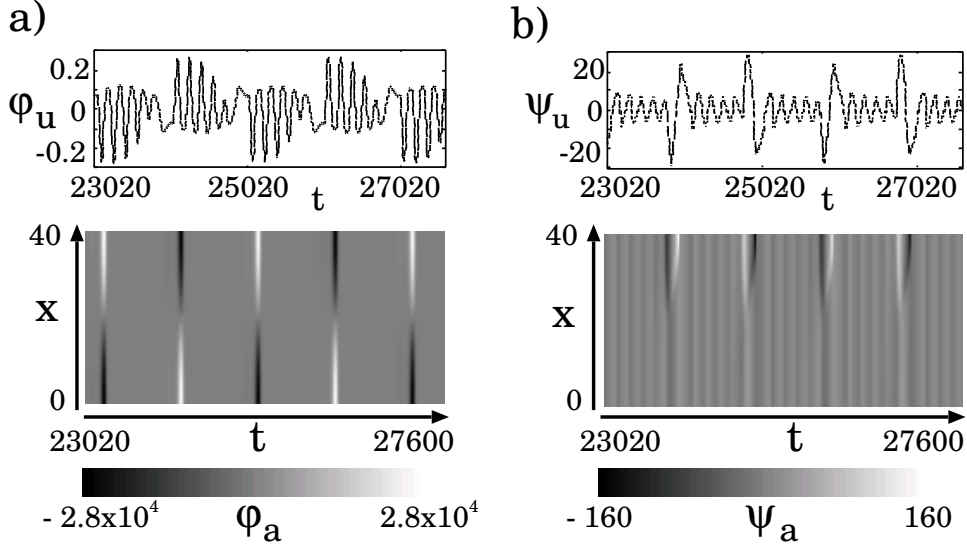


Figure 2: (a) Floquet-left eigenmodes $\phi_u(t)$ and $\phi_a(x, t)$ for the largest Floquet-exponents of a periodic orbit, as well as the corresponding Floquet-right eigenmode (b) $\psi_u(t)$ and $\psi_a(x, t)$. See [14]

where R is a memory parameter. For the diagonal control the Floquet-exponent Λ of the controlled orbit satisfies the exact implicit equation [4],

$$\Lambda + K \frac{1 - e^{-\Lambda\tau}}{1 - R e^{-\Lambda\tau}} = \lambda. \quad (5)$$

As shown in Fig. 1(a), the resulting control domain in the K-R plane is numerically reproduced with high accuracy for the generic model [15]. The control regime is bounded by a flip bifurcation for small K values, and a Hopf bifurcation for large K values. Now it is interesting to examine how the control range deforms when applying other control schemes. One example is *local* control without inhibitor control, which arises for $F_u = 0$, $F_a = F_{\text{loc}}$. In Fig. 1(b) one can recognize that thereby new control boundaries arise. A systematic comparison of different local and global control schemes has been performed by collaboration with W. Just (London) and J. Socolar (Duke University, USA). There we could show, for instance by calculating the Floquet spectra, that in Fig. 1(b) for large R and K , the control regime is bounded by a subcritical flip bifurcation, and that for global control the control regime gets even bigger if the inhibitor control is omitted.

For applications, particularly interesting are control schemes which work with the smallest possible amplitude factor K . In this regard, a new control scheme which we developed in collaboration with W. Just [14], has proven surprisingly efficient. For this control scheme, the Floquet-left eigenmode $\phi_{u/a}$ and the, associated with the adjoint problem, Floquet-right eigenmode $\psi_{u/a}$ are calculated for the largest Floquet-exponent of the orbit to be stabilized. A concrete example of such modes is depicted in Fig. 2. The control forces are

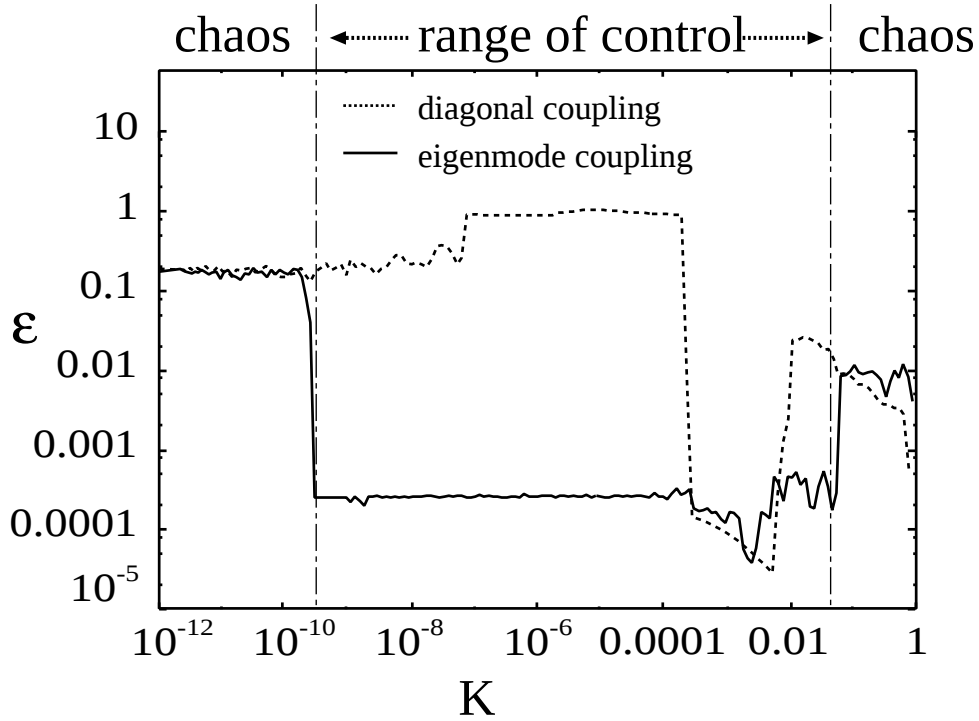


Figure 3: Comparison of Floquet-eigenmode control (solid line) and diagonal control (dashed line). The spatiotemporal average $\epsilon = \langle |a(x, t) - a(x, t - \tau)| + |u(t) - u(t - \tau)| \rangle_{x,t}$ versus the control amplitude is plotted K . See [14]

then constructed as follows:

$$F_u(t) = \psi_u(t)s(t), \quad F_a(x, t) = \psi_a(x, t)s(t), \quad (6)$$

$$\text{mit } s(t) = \int_0^L \phi_a(x', t) [a(x', t) - a(x', t - \tau)] dx' + \phi_u(t) [u(t) - u(t - \tau)] \quad (7)$$

By applying this control force to the generic model, we discovered that control works even for extremely small K -values, i.e. the control threshold decreases by six orders of magnitude, as demonstrated in Fig. 3 in comparison to diagonal control. Investigating this phenomenon in more detail, we determined that a phase shift δ of the controlled orbit over the phase of the Floquet modes plays an important role which can be treated by perturbation theory [14]. We also applied this new type of Floquet mode control to the Rössler-Model [24]. In this low dimensional system we succeeded in analysing the dependence of the minimal control amplitudes of the phase shift δ far beyond the perturbation theory Ansatz.

With a suitable expansion of the Floquet mode control in two unstable modes, we succeeded for the first time to place localized spatiotemporal patterns (spikes) aimed at a chosen position of the system [14]. Whereas normally, stable or unstable spatiotemporal spikes in a globally coupled reaction-diffusion system with Neumann boundary conditions are pinned on the boundary of the system, we could stabilize the spikes for vanishing con-

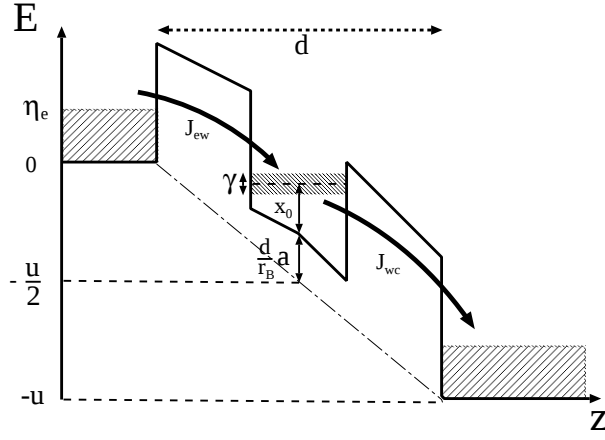


Figure 4: Schematic energy band structure of the resonant tunneling diode (DBRT). See [25]

trol force, with tended Floquet mode control, in the centre of the system. This involves the control of an unstable orbit on the repeller.

(ii) Transverse spatiotemporal dynamics in a resonant tunneling diode

The schematic energy band structure of the resonant tunneling diode (DBRT) is shown in Fig. 2. The electrons tunnel from the emitter contact through the left barrier into the quantum well and from there through the right barrier to the collector. The dynamical variables in this case are the space-dependent electron density $a(x, t)$ in the quantum well (activator), as well as the voltage applied to the tunneling diode $u(t)$ (inhibitor) (each in dimensionless units), where x is the transverse spatial coordinate vertical to the current transport direction. Based on our previous work on transverse dynamics [6, 8], the next task was the microscopic calculation of the tunneling currents J_{ew} and J_{ec} [16]. Expanding the model by adding control forces F_a , F_u , one obtains in appropriate units a system of equations of the form

$$\frac{\partial a}{\partial t} = \frac{\partial}{\partial x} \left(D(a) \frac{\partial a}{\partial x} \right) + f(a, u) - K F_a(x, t), \quad (8)$$

$$\frac{du}{dt} = \frac{1}{\epsilon} (U_0 - u - r \langle j \rangle) - K F_u(t). \quad (9)$$

Here, the nonlinear function $f(a, u)$ characterizes the difference between the inflow and the outflow of the tunneling currents J_{ew} , J_{ec} and $D(a)$ is an effective diffusion coefficient. Equation (9) describes the global coupling of the system through an applied circuit with a resistance r at an external voltage U_0 . ϵ is a time scale parameter and $\langle j \rangle$ is the spatially averaged current density. This reaction-diffusion system is structurally like the

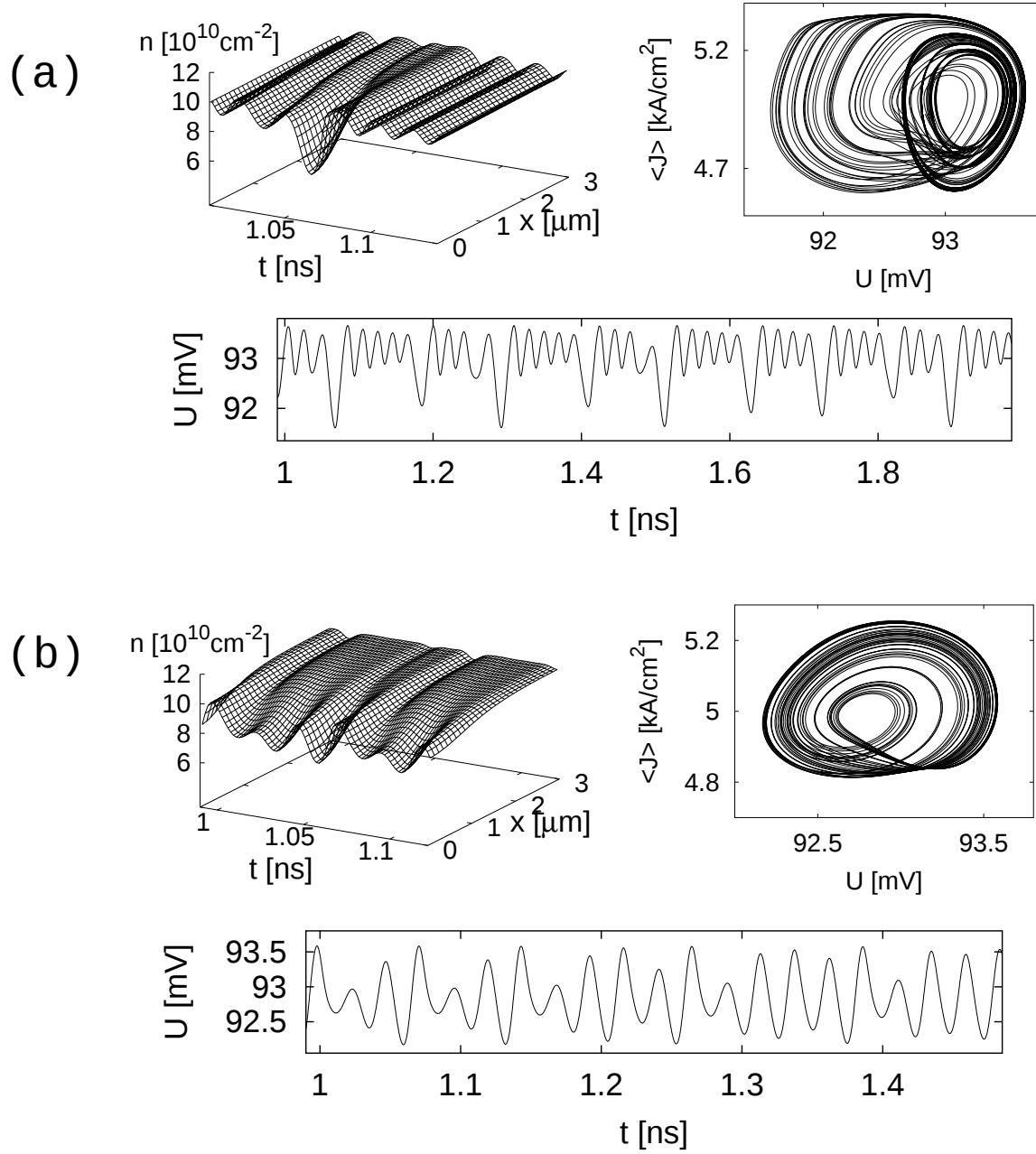


Figure 5: Chaotic spiking (a) and breathing (b) of the DBRT current density patterns. For each, the spatiotemporal pattern of the electron density, the projection of the phase portraits on the global current-voltage plane and the time series of the voltage U are shown. Parameter: $\epsilon = 16.5$ (a) and $\epsilon = 9.1$ (b). See [37].

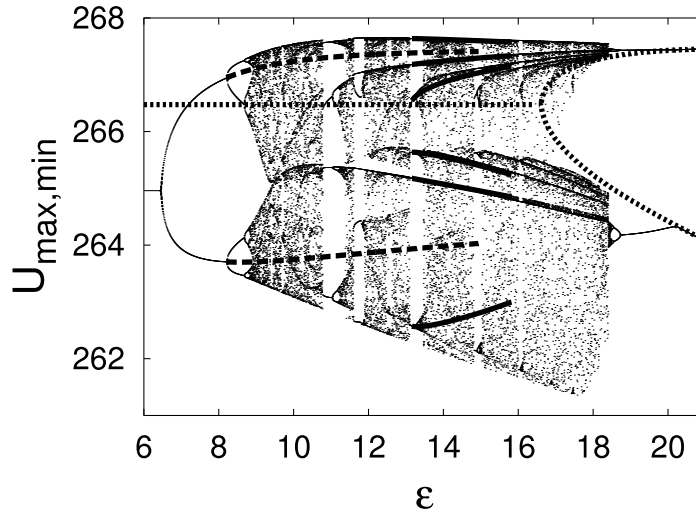


Figure 6: Bifurcation diagram of maxima and minima of the voltage U vs the timescale parameter ϵ . Thick dotted lines: spatially homogeneous solution, thick dashed lines: periodic breathing, thick solid lines: periodic spiking. See [25]

one examined in (1), but in contrast to that one, leads to a Z -shaped current-voltage characteristic rather than to an S -shaped one.

Without control, $K = 0$, we found transverse trigger fronts in the bistable regime [26], stochastic pulse trains in the excitable regime [25] and breathing current filaments and spatiotemporal spiking in the oscillatory regime [16].

Our research in collaboration with P. Rodin (St. Petersburg) showed that the dynamical behaviour of the DBRT can be chaotic when an electric circuit acts on the device [37]. Formally this can be achieved by choosing a negative r in Eq. (9). Thus we could prove both breathing and spiking chaotic behaviour as shown in Fig. 2.5. The complete bifurcation diagram in Fig. 2.6 shows a complex bifurcation scenario, which was further examined in [25].

Now we switch on the control, i.e. $K \neq 0$ [25]. The aim here was to compare the effectiveness of various control methods. Our starting point is the theoretically well understood diagonal control $F_u = F_{vf}$, $F_a = F_{loc}$, where F_{vf} and F_{loc} are calculated similarly to (3) and (4). As already done for the generic model in (i), we can numerically reproduce analytical conditions for successful control also for the DBRT model (5) [25].

For local control without voltage feedback, i.e. $F_u = 0$, $F_a = F_{loc}$, the control regime deforms as in Fig. 7(a). Floquet diagrams are essential for the bifurcation analysis as shown in Fig. 7(b). In this case it follows from the Floquet diagram, that the left boundary of the control regime is associated with a flip bifurcation whereas the lower and right boundary are each associated with Hopf bifurcations.

The most suitable control scheme for practical applications is a pure voltage feedback,

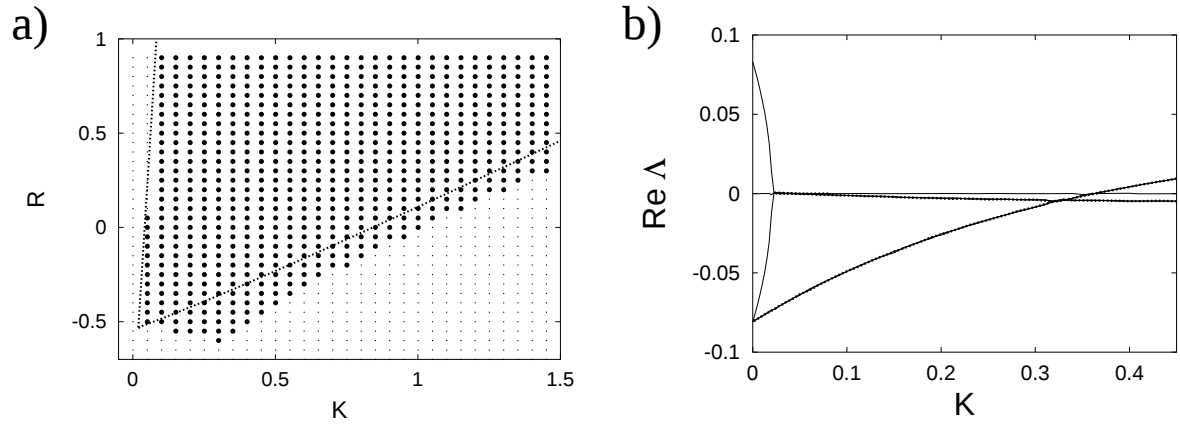


Figure 7: Control of an unstable periodic orbit using a local control scheme without voltage feedback for $\epsilon = 9.1$. (a) Control regime in the K - R plane. $\bullet$ means successful control, $\cdot$ no control, solid lines: analytical results. See (5). (b) The largest real part Δ of the Floquet spectrum vs K ($R = -0.55$). Dotted lines mean complex conjugate pairs of eigenvalues. See [25]

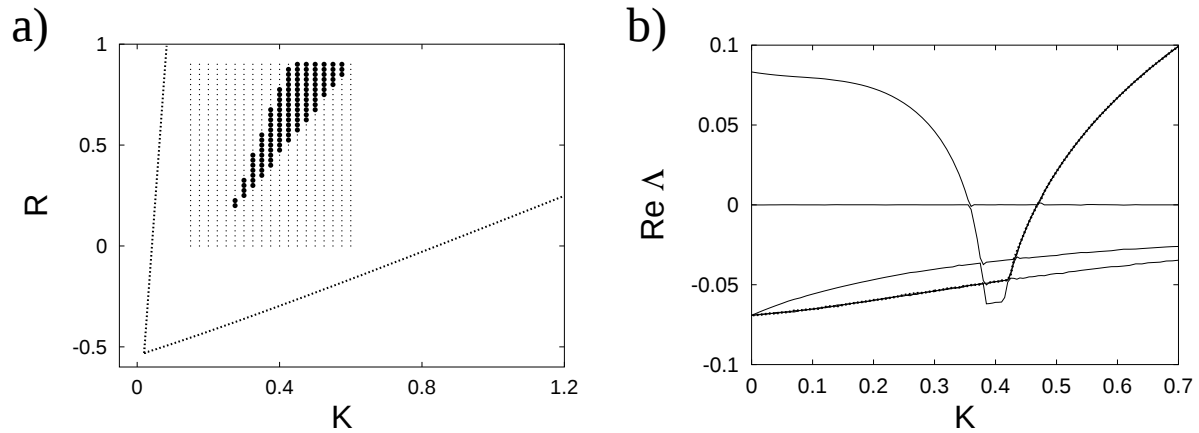


Figure 8: Like in Fig. 7, but with a pure voltage control. See [25].

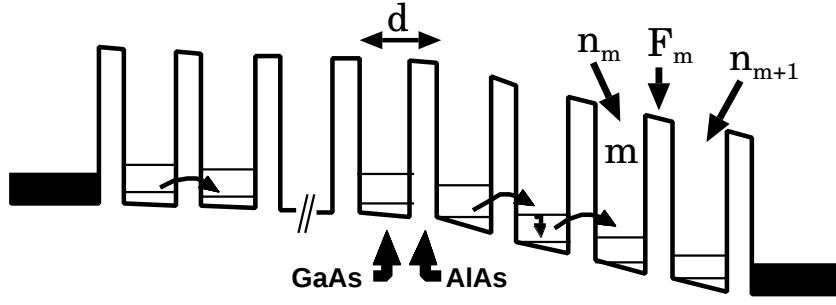


Figure 9: Schematic energy band structure of a superlattice.

$F_u = F_{vf}$, $F_a = 0$, where the physically easily accessible voltage variable is used in the control. We managed with this simple method to stabilize an unstable periodic spatiotemporal orbit for the DBRT. [25]. The control regime (cf. Fig.8(a)) is in comparison to diagonal control obviously smaller, which can also be seen in the corresponding Floquet diagram (Fig.8). Further interesting control schemes arise by choosing a spatially averaged control force $F_a(x, t) = \langle F_{loc}(x', t) \rangle_{x'}$ [25].

(iii) Pattern formation and selection in semiconductor superlattices

Semiconductor superlattices consist of an alternating layer sequence of two different materials. At sufficient barrier thickness electrons are assumed to be localized in the individual wells. The resulting schematic energy band structure is shown in Fig. 9. Furthermore it is assumed, that electrons in one well are in local equilibrium with the majority of them occupying the lowest energy level. Electrons can tunnel from the ground state of a well to a free state of the next well, where a possible difference between the state energies can be compensated by the electric field acting between neighbouring wells. The current density $j_{m \rightarrow m+1}(F_m, n_m, n_{m+1})$ from well m to $m+1$ is thus a nonlinear function of the electric field F_m between the two wells as well as of the electron densities n_m and n_{m+1} in the involved wells. For concrete microscopic calculations of $j_{m \rightarrow m+1}$ we have used the sequential tunneling model which has been developed in our group, cf. review paper by A. Wacker [17]. For the contact currents at the emitter $j_{0 \rightarrow 1}$ and the collector $j_{N \rightarrow N+1}$ we simply assume Ohmic boundary conditions, which are characterized by a contact conductivity σ . N is the number of quantum wells in the superlattice.

Therefore the following equations of motion for the electron densities arise:

$$e\dot{n}_m = j_{m-1 \rightarrow m} - j_{m \rightarrow m+1} \quad \text{für } m = 1, \dots, N, \quad (10)$$

$$\epsilon_r \epsilon_0 (F_m - F_{m-1}) = e(n_m - N_D) \quad \text{für } m = 1, \dots, N, \quad (11)$$

$$U_0 = - \sum_{m=0}^N F_m d, \quad (12)$$

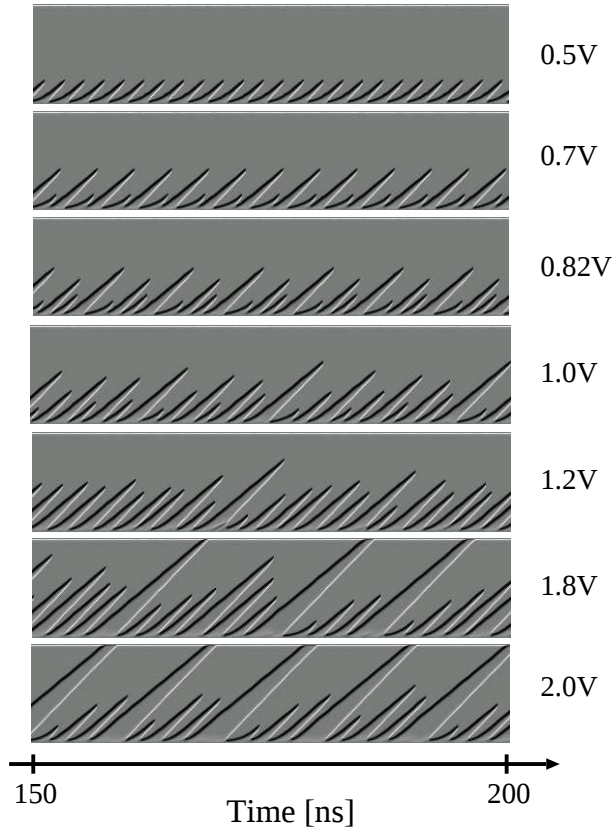


Figure 10: Chaotic scenario of the dynamics of the electron densities in a superlattice for various voltages (space-time plots). Light regions correspond to electron accumulation, dark regions to electron depletion. See [18].

with electron charge $e < 0$, ϵ_r and ϵ_0 the relative and absolute permittivities respectively, N_D the doping, U_0 the external voltage and d the period of the superlattice. Eq. (12) describes a global constraint due to the total voltage. The total current through the superlattice is given by $j = \sum_m j_{m \rightarrow m+1} / (N + 1)$ [34].

Depending on the physical parameters (especially on σ and N_D) the system of equations (10), (11) and (12) at a constant U_0 has either stationary or oscillatory spatially inhomogeneous solutions (field domains bounded by electron accumulation and depletion). In the stationary case the system is in general multistable, i.e. for one value of the voltage there are many stable branches, which differ for instance, in the resulting current. In collaboration with L. Bonilla (Madrid), the next question to be treated was which of these branches would be selected by the system after an abrupt or continuous change of the external voltage [9, 36]. At this point we determined that the final state of the system can depend very sensitively on the difference between the initial and final voltage. This property could then be used to select operating points. The majority of the partially surprising effects could be explained by the fact that at the emitter, pairs of electron accumulation and depletion fronts (dipole) were generated. Our theoretical predictions

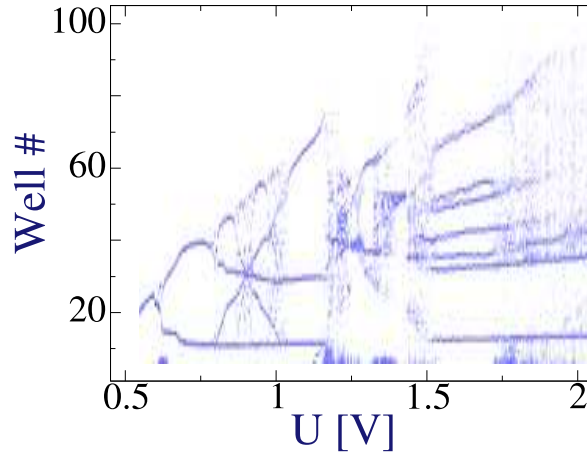


Figure 11: Bifurcation diagram of the collision positions (quantum well index) of accumulation and depletion fronts for various voltage values U . See [18].

on switching dynamics between multistable states were later quantitatively confirmed by experiments performed at the Paul-Drude-Institute in Berlin [5].

By closer investigations on the front generation process at the emitter, as well as of the motion of the fronts inside the device, we successfully generated complex self-oscillations like tripole modes [19]. It was shown that the front generation at the emitter depends substantially on the contact conductivity σ and the total current j . In particular, chaotic front dynamics in a non driven superlattice were proven for the first time [18]. A typical bifurcation scenario is shown in the electron density plots in Fig. 10. We can see that with increasing voltage the superlattice exhibits both periodic and chaotic behaviour. The full bifurcation diagram (Fig. 2.11) exhibits an alternating sequence of chaotic and periodic regions as well as a striking cobweb structure, whose center lies at $U_0 = 0.9V$.

Our further objective was to reduce the front model to a simple elementary basis. In collaboration with the group of U. Parlitz (Göttingen) we found a surprising analogy to a tank model, which is normally used in a totally different context, for describing industrial production processes [28]. Consider a system of a given number of tanks. A switching server fills one of the tanks and at the same time all nonempty tanks drain. The server then switches to a new tank, as soon as it is empty, under the condition that the tank which it is currently filling has reached the minimum filling height p_h . The relation between the inflow- and outflow rates is chosen such, that the total amount of the water L_h stays constant. The filling heights of the tanks correspond to the length of the high-field domain (in the superlattice system) between a depletion and an accumulation front or between the first depletion front and the emitter for the tank that is actually being filled. The switching of the server in the tank system corresponds to the generation of a dipole front at the emitter in the superlattice system.

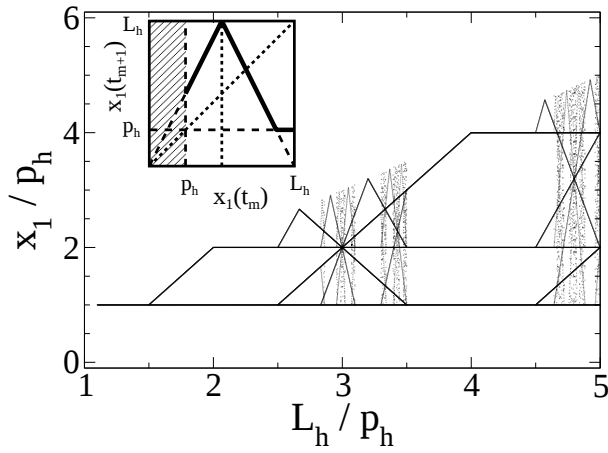


Figure 12: Bifurcation scenario shown in the modified tent map in the inset. The parameters x_1 and L_h correspond to the front position and the applied voltage respectively. See [28].

For three tanks the resulting dynamics is described by a one dimensional piecewise linear iterated map as in the inset of Fig. 12. This modified map has only one bifurcation parameter L_h/p_h . The corresponding bifurcation diagram in Fig. 12 agrees in detail with the microscopically calculated bifurcation diagram in Fig. 11. In particular the cobweb structure is reproduced in detail. We can therefore show that the front dynamics in the superlattice can be explained on a very fundamental basis using iterated maps [28]. Because the microscopic properties of the superlattice do not come up, it must be assumed, that a similar reduction may also be possible for complex front systems with global coupling in many other disciplines, and that our reduced model may describe a universal bifurcation scenario.

From a technological point of view, oscillatory superlattices are interesting as GigaHertz-generators. In collaboration with the experimental group of E. Schomburg and K. Renk (Regensburg) we analyzed the high frequency impedance of the superlattices, as well as the behaviour of the superlattices in a resonator under the influence of an external AC voltage [20, 21, 38]. The front dynamics are controlled by a periodic AC voltage and exhibits typical behaviour like Arnold tongues, devil's staircase and phase synchronization. Apart from that we discovered that applying a suitable external circuit with capacitive and inductive elements to the superlattice, may change its oscillation mode in which front motion is suppressed (quenched mode) and which leads to an eigenfrequency more than twice as big as the nominal frequency of the superlattice. In this context we developed together with the Regensburg group concrete proposals for experimental realization of electronic high frequency oscillators [22].

Furthermore, regarding applications of the superlattice as a high frequency oscillator, it is also important to create a stable periodic output signal and suppress potential chaotic

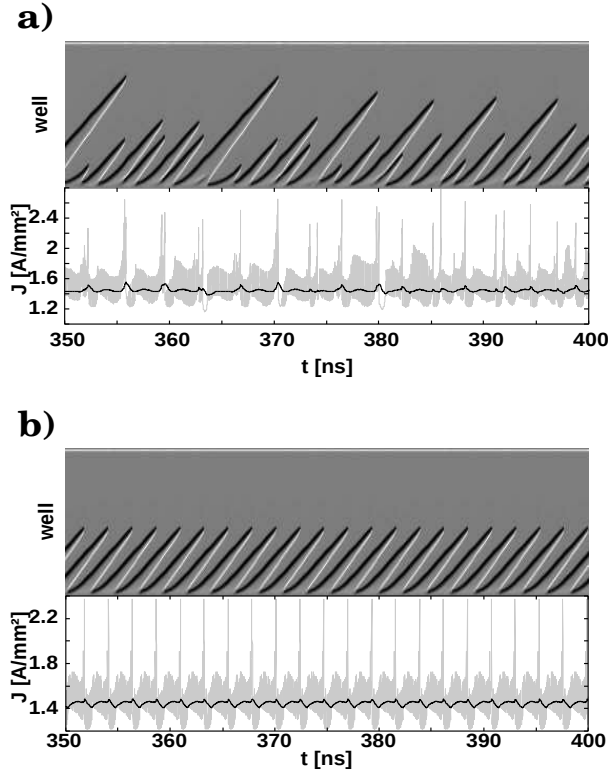


Figure 13: Successful stabilization of a chaotic front pattern (a) applying time-delayed feedback between voltage and current (b). See [29]

oscillations. With that in mind we investigated the chaotic front dynamics under various feedback schemes. We could show for the first time that a control scheme with global time-delayed feedback, simple to realize, is successful. [29, 30]. For this purpose we substitute in (12) U_0 by $U_0 + U_c(t)$, with a control voltage

$$U_c(t) = -K (\bar{J}(t) - \bar{J}(t - \tau)) + RU_c(t - \tau), \quad (13)$$

with

$$\bar{J}(t) = \alpha A \int_0^t j(t') e^{-\alpha(t-t')} dt', \quad (14)$$

where A is the cross section of the device and α is a damping constant. We have shown that it is necessary to modify the conventional Pyragas method by a low-pass filter (14) due to the discrete structure of the superlattice. Successful control using this method is demonstrated in Fig. 13.

3 Ph. D. theses, Master theses

Name	Theme	Date
Schwarz, Georg	Current Filamentation in Doped GaAs Corbino Disks (Dissertation)	Jan. 2001
Amann, Andreas	Nonlinear and chaotic front dynamics in semiconductor superlattices (Dissertation)	Dez. 2003
Beck, Oliver	Zeitverzögerte Autosynchronisation in Reaktions-Diffusions-Systemen mit unterschiedlicher Kopplung (Diplomarbeit)	Jan. 2001
Baba, Nilüfer	Stabilisierung instabiler räumlicher Muster durch zeitverzögerte Rückkopplung mit räumlichen Filtern (Diplomarbeit)	Feb. 2001
Murawski, Jens	Nichtlineare Oszillationen von Stromfilamenten in Punktkontaktgeometrien (Diplomarbeit)	Apr. 2001
Rudolf, Marc	Raum-zeitliche laterale Instabilitäten in einer resonanten Tunneldiode (Diplomarbeit)	Juli 2001
Jappsen, Anne-Katharina	Modellierung von Halbleiterübergittern im äußeren Stromkreis (Diplomarbeit)	Aug. 2001
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Name	Subject	Date
Unkelbach, Jan	Rückkopplungsgesteuerte transversale Dynamik der resonanten Tunnel-diode (Diplomarbeit)	Mai 2002
Schlesner, Jan Henning	Nichtlineare Oszillationen in Halbleiterübergittern unter zeitverzögerter Rückkopplung (Diplomarbeit)	Juni 2002
Kilic, Nursel	Chaotische Dynamik in Halbleiterübergittern und ihre Kontrolle (Diplomarbeit)	Sep. 2003

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List of the group's publications

a) Original publications in reviewed journals

- [6] M. Meixner, P. Rodin, E. Schöll, and A. Wacker: *Lateral current density fronts in globally coupled bistable semiconductors with S- or Z-shaped current voltage characteristic*, Eur. Phys. J. B **13**, 157 (2000).
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Visiting Scientists

Dr. Pavel Rodin vom Ioffe Physico-Technical Institute der Russian Academy of Sciences, St. Petersburg, Russland (Juli 2001 - August 2003, Alexander-von-Humboldt-Stipendiat)

Wissenschaftliche Zusammenarbeit über komplexe raum-zeitliche Dynamik in Reaktions-Diffusionssystemen zur Beschreibung von Halbleiternanostrukturen und resonanten Tunnelndioden mit globaler Kopplung.

Dr. Wolfram Just vom Queen Mary College, London (September 2001 Oktober/November 2002, Mai + Juli/August 2003)

Wissenschaftliche Zusammenarbeit über Chaoskontrolle durch zeitverzögerte Autosynchronisation in global gekoppelten Reaktions-Diffusions-Systemen.

Dr. Natalia Janson von Lancaster University und Loughborough University, GB (Oktober/November 2002, Mai + Juli/August 2003)

Wissenschaftliche Zusammenarbeit über zeitverzögerte Rückkopplungskontrolle von deterministischem Chaos in Halbleiterübergittern sowie von rauschinduzierten Oszillationen.

Dr. Alexander Balanov von der Saratov State University, Russland (Mai + Juli/August + Oktober/November 2003)

Wissenschaftliche Zusammenarbeit über zeitverzögerte Rückkopplungskontrolle von rauschinduzierten Oszillationen.