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Cascading Failures on the Banking Network

Dynamics, Topology and Stability

Bachelorarbeit

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Abstract

Systemic risk – the risk of the collapse of the entire financial system – is a major public concern, the 2008 financial crisis led to a recession affecting the entire world economy. Research in the field of complex systems suggests, that increasing complexity might lead to increasing instability of the financial system.

Based on an empirical network of interbank-borrowing data among 204 US-American financial institutions this thesis studies cascades triggered by single bank defaults. The network is characterized using measures from complex network theory and definitions of local and global stability are given. The influence of network topology and node dynamics on the global stability of the system is investigated. Three different methods of identifying banks that pose a risk to global stability ("super-spreaders") based on their topological features are compared: backbone-reduction, eigenvector centrality and core numbers.

It is found, that the network consists of a small number of high-degree nodes with strong interconnectivity and a large number of low-degree nodes that are only sparsely connected. Similarly distributions of node-strengths, tier 1 capital and normalized liabilities can be characterized by a modified "90-10" Pareto principle. Cascade sizes of bank defaults follow a bimodal distribution, their average magnitude decreases as the dynamical threshold-paramter q_{th} is increased. Based on local stability arguments it is shown, that the network is globally stable for $q_{\text{th}} \geq 0.6$. Topological changes that randomly alter the network toplogy (random link and random weight exchange) decrease its global stability. Among the three methods of identifying super-spreaders it turns out that eigenvector-centrality gives the best predictions.

Zusammenfassung

Systemisches Risiko – das Risiko des Zusammenbruchs des gesamten Finanzsystems – hat große gesellschaftlicher Bedeutung: die Finanzkrise 2008 hat zu einer Rezession von weltweitem Ausmaß geführt. Forschung im Bereich komplexer Systeme legt nahe, dass zunehmende Komplexität zu zunehmender Instabilität des Finanzsystems führt.

Basierend auf einem empirischen Netzwerk der zwischenbanklichen Anleihen von 204 US-amerikanischen Finanzinstituten werden in dieser Arbeit die durch das Versagen einzelner Banken ausgelösen Kettenreaktionen studiert. Das Netzwerk wird mit Hilfe bekannter Größen aus der Theorie komplexer Netzwerke charakterisiert, lokale und globale Stabilität werden definiert. Der Einfluss der Netzwerk-Topologie und der Dynamik einzelner Knoten des Netzwerks auf die globale Stabilität des Systems wird untersucht. Drei verschiedene Methoden zur Identifikation von Banken die ein großes Risiko für die globale Stabilität darstellen ("super-spreaders") werden verglichen: Rückgrat-Reduktion (backbone-reduction), Eigenwert-Zentralität (eigenvalue-centrality) und Kern-Zahlen (core numbers).

Es stellt sich heraus, dass das Banken-Netzwerk aus einer geringen Anzahl untereinander gut vernetzer Knoten hohen Grades und einer großen Anzahl gering vernetzer Knoten niedrigen Grades besteht. Ebenso folgen Knoten-Stärken, Tier 1 Kapital, und normalisierte Anleihen in guter Näherung einem modifizierten 90-10 Pareto-Prinzip. Die Stärke von Kettenreaktionen folgt einer bimodalen Verteilung, die durchschnittliche Stärke nimmt ab wenn der dynamische Kontroll-Parameter q_{th} erhöht wird. Basierend auf lokaler Stabilität wird gezeigt, dass das Netzwerk für $q_{\text{th}} \geq 0.6$ global stabil ist. Veränderungen der Topologie durch zufälligen Austausch von Kanten und Gewichten führen zu einer Verringerung globaler Stabilität. Von den drei Methoden zur Identifizierung von "super-spreader"-Banken gibt Eigenvektor-Zentralität die besten Vorhersagen.

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Eigenständigkeitserklärung

Hiermit versichere ich, dass ich die vorliegende Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel verwendet habe.

Maximilian Thess

Contents

1. Introduction

In the course of the 2008 financial crisis it became apparent, that the default of a single bank can lead to the potential collapse of the whole financial system. Governments throughout the world spent billions of taxpayers' money to save banks that were "too interconnected to fail" or "too big to fail".

Interestingly, mainstream-economics which to a large extent relies on the assumption of stable equilibria in economic systems was unable to anticipate the crisis of 2008[1] and sophisticated financial instruments based on idealized assumptions are seen as a major cause of the events of 2008[2].

A number of notable economists, including nobel laureates[3] have suggested the development of new methods in economics and welcomed the recent contributions of physicists to the field.

And indeed physicists can contribute, as they have improved the understanding of other complex systems before: the question of whether large complex systems are stable (or inherently unstable) has been a topic of intense research in the field of ecological networks since pioneering work by theoretical physicist Robert May in the seventies. Using rigorous mathematical analysis he proved that ecosystems become indeed less stable as they increase in diversity and complexity, a result which ran completely against the belief of ecologists at that time. Recently these insights from the study of ecosystems have been applied to the financial system [26], [34]. The question is the same: "Are large complex economic systems unstable?" [52]

The study of complex systems, among which networks are the most intensely studied is a largely interdisciplinary endeavor: researchers from biology, engineering, economics, physicics and other disciplines work together to find unifying principles underlying such diverse systems as the human brain, transportation networks, cities or the financial system.

Economists often remark[4] that unlike to physics their science does not allow experiments on the object of study to verify theories. But with the availability of data on the US interbank-borrowing network the answer of whether the financial system is stable can now be tackled by means of simulation, the next best thing to a real experiment.

The organization of this thesis is as follows:

In the first chapter basic terminology from the theory of complex networks will be introduced followed by a brief overview of the field of cascading failures which are well known from black-outs in electric power grids. Power-laws, their relevance to complex networks and their detection in empirical data are explained. After a rather simplified explaination of the way banks function in the financial system a model is proposed that allows to describe and simulate the propagation of bank-defaults in a network of banks.

Chapter two constitutes the main part of this thesis: First the *tools* used for the analysis, visualization and simulation of the network are explained. Then the *structure of the empirical network is characterized* using measures from the theory of complex networks, attention is paid to properties that relate to the stability of the network. To study

[1] [53] p 2
[2] [26] p 351-352
[3] Akerlof [1], Krugman [31] and others [28]
[4] [32] chapter 2

the influence of dynamical and topological properties on the stability of the network a few concepts are introduced that characterize and quantify *local and global stability*. Then *simulations* are carried out to establish quantitative results. Lastly three different ways of identifying "dangerous" banks based on their topological properties are compared.

Eventually the goal of this thesis is to give tentative answers to the following three questions:

Description What is the banking-network like?

Simulation How do the dynamics of the nodes and the topology of the network influence the stability of the system?

Prediction Can one identify "super-spreaders" based on their topological features?

2. Basic Concepts

2.1. Networks

Beginning with a seminal paper by Watts and Strogatz in 1998 [59] in which they pointed out a similar structure (small-world structure) underlying biological, technological and social networks and another paper by Strogatz in 2001 [56] in which he explored the influence of network topology on dynamics of networks, physicists have surged into the field of complex networks. Some instances of complex networks that were studied in recent years include:

- the internet and WWW[5], [37]
- social networks, e.g. the network of actors that played in the same movie, [37]
- the Indian rail network, [14]
- food-networks in ecological systems [50]
- protein interaction networks, [45]
- neuronal networks, e.g. the brain of the worm c. elegans, [59]

Since 1998 the field of complex network has grown rapidly, the following review articles give an overview of the field:

- Albert and Barabasi

  Statistical Mechanics of Complex Networks, 2001, 51 pages, [2]
- Newman

  The Structure and Function of Complex Networks, 2003, 58 pages, [39]
- Boccaletti et al

  Complex networks: Structure and dynamics, 2006, 134 pages, [10]
- Costa et al

  Analyzing and Modeling Real-World Phenomena with Complex Networks: A Survey of Applications, 2007, 83 pages, [17]
- Dorogotsev et al

  Critical phenomena in complex networks, 2008, 79 pages, [19]

[5]The internet is the physical structure of computers and cables, the WWW is one particular service that is run on this structure.

2.1.1. Topology

What is a network? In mathematics networks are called *graphs* and are defined as an ordered pair $G = (V, E)$ that consists of a set of N vertices, also called nodes $V = \{v_i\}$ and a set of E edges (also called links) $E = \{e_i\}$.

An *adjacency matrix* $A_{i,j}$ of dimension $N \times N$ is the most convenient form of representing a network. An entry (i, j) in that matrix indicates a directed connection between the ith and jth node in the network. Networks can be *undirected* $A = A^T$, *directed* $A \neq A^T$, *binary*, also called unweighted ($A_{i,j} \in \{0, 1\}$) or *weighted* (arbitrary values of $A_{i,j}$). Sometimes the edges of directed networks are called *arcs*.

The *degree k* of a node in the network is the number of incoming (*in-degree*) or outgoing (*out-degree*) links. Given an adjacency matrix A these quantities can be calculated:

$$k_i^{\text{in}} = \sum_j |sgn\,(A_{j,i})| \tag{1}$$

$$k_i^{\text{out}} = \sum_j |sgn\,(A_{i,j})| \tag{2}$$

The in-degree of the ith node is the number nonzero elements in the ith column of the adjacency matrix, the out-degree the number of nonzero elements in the ith row. The *degree distribution* $P(k)$ is a discrete probability distribution that gives the probability of a randomly selected node having degree k and can often give valuable insights into the structure of the network.

For weighted networks the *node strength* s_i is defined as

$$s_i = \sum_j A_{i,j}. \tag{3}$$

If the network is directed one can also define in- and out-strengths s_i^{in}, s_i^{out} that are defined in accordance with the in- and out-degree mentioned above.

Another important property of networks is the *clustering* of its nodes: “In many networks it is found that if vertex A is connected to vertex B and vertex B to vertex C, then there is a heightened probability that vertex A will also be connected to vertex C. In the language of social networks, the friend of your friend is likely also to be your friend. In terms of network topology, transitivity means the presence of a heightened number of triangles in the networklsets of three vertices each of which is connected to each of the others.”[6] Clustering is quantified by a *clustering coefficient* that can be defined in several ways, for details see [39].

For any two nodes i and j in the network that can be connected through a path, a shortest distance $l_{i,j}$ can be defined. While in undirected networks $l_{i,j} = l_{i,j}$ in directed networks generally $l_{i,j} \neq l_{i,j}$ From this the average shortest distance $\langle l \rangle$ can be computed. Surprisingly even in large networks like the WWW with more than 200 million pages on average two pages are only ≈ 11 clicks away from each other. This is related to the *small world phenomenon* [39].

It is observed that in some networks nodes with a high degree tend to preferentially connect to other nodes with a low degree. Those networks are called *disassortative*,

[6]From [39] p.11

most biological and technological networks are found to have a disassortative structure. Networks in which highly connected nodes are more often seen to connect to other highly connected nodes are called *assortative*. Social networks often have this property.

It was found that many networks in disparate fields have similar properties. This lead to the development of a number of network-models. One of the earliest models, which is still used to determine whether the structure of empirical networks deviates from a completely random structure is the *Erdös-Renyi* random graph model in which connections among any two nodes are made with a probability p. More recently the *small world model* combines a high degree of local clustering with small average distances (compared to a random network). *Scale free networks* exhibit a degree-distribution that follows a power law and show many remarkable properties [2]. *Generalized random graph models* [43] can be used to derive a graph ensemble that on average reproduces a set of observations of a real network. *Modular networks* are another example of networks that are currently studied. It should be noted that this classification is not overlap-free, e.g. modular networks might or might not show the small world property etc.

Current research on complex networks is concerned with

- finding suitable measures to quantify network-properties
- the identification network structure in empirical data
- mechanisms that can lead to observed network structures (e.g. preferential attachment for scale-free networks)
- the interaction between topology and dynamics

2.1.2. Dynamics

Networks change: power stations and transmission lines in grids break down due to overload, viruses spread along patterns of human travel, people make new friends and thereby add a new node to their social network. There are two ways of looking on dynamics in connection with networks [23]:

- dynamics *on* networks – the state of *nodes* evolves over time.
- dynamics *of* networks – the *topology* of the network evolves over time, e.g. new nodes or links are added

In models of complex networks the dynamics of nodes can be quite different: in neural networks neurons might exhibit complicated nonlinear, even chaotic dynamics while in SIR-models of disease spreading nodes can take only one of three different states (susceptible, infected, recovered). Nevertheless in both cases the individual node-dynamics are influenced by the wiring of the network. Stability (which is yet to be defined) as a dynamical property is therefore influenced by the topology of the network.

2.1.3. Stability and Cascading Failures

Stability The networks studied by physicists usually perform some function, be it power-grids, the internet, the WWW, ecological networks of plants and animals or the network of international trade (ITN) among countries.

With this in mind the following definition of *stability* can encompass the diverse mathematical models that exist [10]:

> **Stability.** Stability of a network describes its ability to perform its function in the presence of disturbances.

These disturbances can be of different kinds and stability is accordingly classified into *static stability* and *dynamical stability*.

Static stability is understood as the ability of a network to retain its connectivity properties after nodes have been removed [10]. Depending on the choice of nodes that are removed networks can exhibit *error tolerance* if the nodes have been selected at random or *attack tolerance* if nodes have been selected on the basis of some of their topological propeties. The difference between random and intentional removal of nodes can be quite drastic [10]: In the scale-free network structure that describes e.g. the internet, a random removal of nodes has only a small effect on the connectivity and average pathlength while a targeted attack on the most highly connected nodes can quickly disconnect the network into separate clusters [3]. This property is sometimes called the "robust-yet-fragile" nature of (a particular class of) complex networks.

Dynamical stability takes into account the time-evolution of the network. In the theory of dynamical systems a system is said to be stable, if small perturbations from an equilibrium decay with time. Pioneering work on the stability of networks of interacting species in ecosystems was carried out by Robert May [33]. He showed, that contrary to common belief among ecologists of that time ecological networks would become less stable as their complexity (number of interacting species and their interactions) increases. His work was criticised for the assumption of a fixed point equilibrium and random interaction among species, but more recently [49], [51] it was shown that his results might hold as well for non-equilibrium dynamics and other topologies of species-interaction. Following the 2008 financial crisis parallels between systemic risk in the financial sector and the stability of ecosystems spawned a new interest in the influence of *diversity and complexity* of networks on their *stability* [34], [26], [52].

Cascading failures A particularly impressive instance of dynamical instability are *cascading failures* in which an initially small perturbation can propagate through the network and eventually lead to the default of a large fraction of its nodes. Examples include the blackouts in power-grids [36], congestion of communication networks, rupturing of fiber bundles [18], avalanches[7] on sand-pile models [42] and bank-defaults in financial networks [22].

Often these large-scale, catastrophic events can be triggered by comparatively small disturbances: The blackout of August 10, 1996 that left 4 million people in eight West

[7] See appendix for the BTW-model of self-organized criticality. The term avalanches often describes cascacing failures in the sand-pile model.

Coast states without electricity was caused by a tree cutting a single power-line in Oregon[8]. The largest blackout US history on August 14, 2003 started from Ohio to and cascaded all the way to Michigan and New York, eventually covering the whole north-east of the United States [12] affecting 55 million people. As Watts [58] points out, these complex networks "routinely display great stability in the presence of continual small failures and shocks that are at least as large as the shocks that ultimately generate a cascade (...) and may appear stable for long periods of time". It is found [36], [10] that heterogeneity of the network plays an important role in stability.

In the numerous models of cascading failures the following concepts reoccur and can be seen as the ingredients of a system that can exhibit cascading failures:

- *load*, e.g. amount of sand in the BTW model
- *threshold & failure*, e.g. maximum number of grains on one site
- *redistribution under conservation*

2.2. Power-laws

In physics and other scientific disciplines one often encounters **power laws** of the form

$$p(x) = Cx^{-\alpha}.$$

describing the relationship between two quantities p and x. The range of phenomena that are described by power-laws is vast:

- the distribution of the number of inhabitants in cities, [37]
- the distribution of the population of countries, [63]
- numbers of copies of bestselling books sold in the US between 1895 and 1965, [37]
- wealth-distributions in a society, [53]
- length and degree of connectivity for the road network of US-cities, [30]
- relationship between population density in a city and annual car gasoline consumption, [40]
- magnitude of earthquakes (Guttenberg-Richter law) [4]
- fluctuation in the water levels of the Nile and the time-intervals between the extinction of species during evolution [4]

Applying the logarithm to both sides of the equation 2.2 one can see that a quantity p that follows such an equation will appear as a linear function of x in a double-logarithmic (often called log-log) plot.

[8] `http://www.nwcouncil.org/history/blackout.asp`

The occurrence of power laws in such a variety of seemingly disparate fields spurred physicists to look for general mechanisms that can generate such powerlaws. Mark Newman's 2005 review [37] of power laws and their origin contains a number of mechanisms which can give rise to a power-law relationship between two quantities. Most notably

- the *Yule process* – a rich-get-richer principle, also called the Gibrat principle, the Matthew effect, the effect of cumulative advantage or preferential attachment
- *phase transitions* and *critical phenomena*[9]
- *random walks* which are known to exhibit power-law distributions in certain properties (e.g. such as the number of steps in between two crossings of the origin)

Probability distributions that follow a power law are called *scale-free distributions*. Scale-free networks for example obtained their name because of their power-law degree-distribution. Cumulative distributions which exhibit a power-law behavior are sometimes also said to follow a *Zipf's law* or *Pareto distribution*.

Newman [37] explains how to identify such relationships in empirical data. The simplest way is to plot the two quantities $p(x)$ and x in a double-logaritmic diagram. This method, however leads to noisy tails of the distribution because of the decreasing number of data-points with higher values x. Linear fits in these double-logarithmic plots give systematically biased estimations of the power-law exponent. The problem of noisy tails can be overcome by logarithmic binning of the sample-values x, however the choice of the bin-size remains arbitrary and information is lost in the binning process. The best way of identifying powerlaws and obtaining their exponent is to plot the *complementary cumulative distribution function* (CCDF) in a double-logarithmic diagram: the CCDF of a scale-free distribution follows a power law, but with an exponent $\alpha - 1$. Building on this idea Clauset, Shalizi and Newman [16] developed a number of tools[10] that can be used to obtain power-law exponents through maximum likelihood estimation.

2.3. Banks: Function and Failure

Function Originally banks had a very simple role: they kept money for people who would not want to store it themselves. By giving loans banks could earn interest that they would share with the depositors. The balance-sheet of a bank still gives a rather simple overview of its activities, though behind the comparatively few quantities that are included in the balance sheet complex and potentially volatile financial instruments (like the now notorious credit default swaps) can be hidden.

The balance sheet is divided in two parts: *assets* and *liabilities*. In a simplified model of a bank [26] a banks activities can be grouped into one of four categories: interbank loans l_i and external assets e_i (e.g. real estate held by the bank) which together constitute the assets and interbank borrowing b_i and deposits d_i constituting the liabilities. Solvency for a bank requires that its "net worth" $\gamma_I \equiv (e_i + l_i) - (d_i + b_i) \geq 0$. Figure 1 illustrates this concept for one banks in the interbank system that has a number of

[9] See appendix A.1 for more details.
[10] Also see section 3.1.2

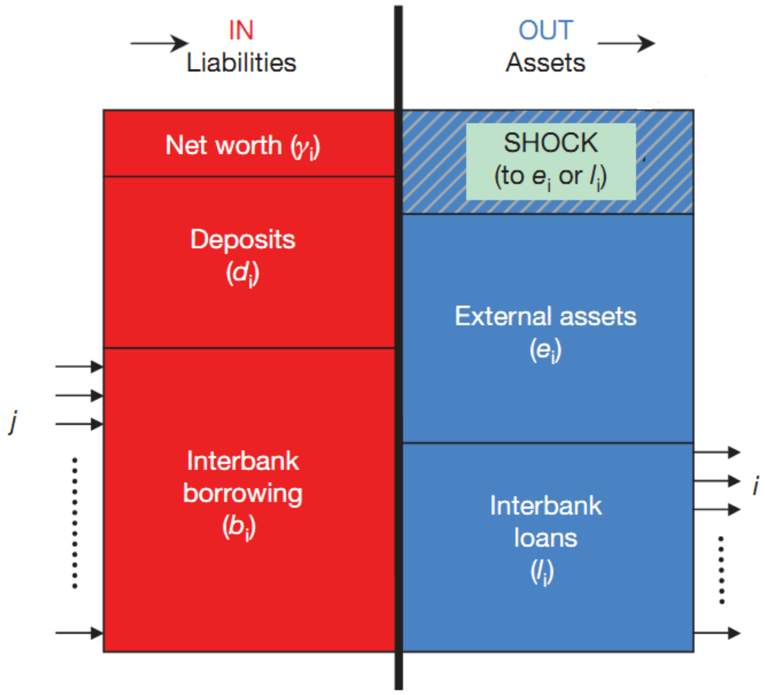


Figure 1: Graphical representation of a bank in the interbanking system and a shock to its assets. From [26].

incoming and outgoing connections that represent its interconnection to other financial institutions.

Failure In this simple model a bank fails, when the amount lost in a shock exceeds its *capital reserves*. Shocks can originate either from *interbank loans* that are not being paid back if a debtor fails or from a decrease in the value of its *external assets*, e.g. decreasing prices of houses following the burst of a real estate bubble.

In 1988 central banks from around the world agreed upon a set of minimal capital requirements for banks [60] called Basel I, now largely viewed as outdated. In 2004 Basel II and in the aftermath of the 2008 economic crisis Basel III proposed new regulatory standards. One of the most important quantities introduced by the Basel comittee to measure a bank's financial strength is the *tier 1* capital. This capital should protect banks against unexpected losses.

To measure interbank lending and borrowing the quantities *gross negative fair value* (GNFV) and *gross positive fair value* (GPFV) are commonly used. GNFV is the amount that a bank's counterparies (creditors) would lose if the bank defaults. GPFV is the amount a bank would lose if its counterparties (debtor) would default.[11] For both measures it is assumed that there is no netting of contracts. In the following the term *liability* is used for both the amount lent out and the amount borrowed by a bank.

[11] See `www.investopedia.com` for more detailed explainations of the terms.

2.4. The Dynamical Model

Our available data[12] contains the tier 1 capital, GPFV and GNFV for 204 US-American financial institutions. To capture the effect of bank failures we propose a simple dynamical model in which each bank can be in a "healthy" (solvent) $s_i = 1$ or "sick" (defaulted) $s_i = 0$ state.

A matrix of netted interbank liabilities J describes the directed creditor-debtor relationship. An entry $J_{i,j}$ denotes a link from bank i to bank j with the weigth $J_{i,j}$ indicating the amount that bank i owes to bank j.

In the event of a bank default the banks creditors lose all the money they lent out and default themselves if their total losses exceed a critical percentage q_{th} of their tier 1 capital t_i.

The update rule

$$s_i^{t+1} = 1 - \Theta\left(\sum_j (1 - s_j^t) J_{j,i} + q_{\text{th}} \cdot t_i\right) \tag{4}$$

governs the time evolution of the state of each bank. The state-vector $\vec{s}$ with elements $s_1, ..., s_i, ..., s_N$ describes the state of the entire banking network considered. $\Theta(...)$ stands for the Heaviside step-function which is defined as[13] $\Theta(x) = 1$ if $x \geq 0$ and $\Theta(x) = 0$ otherwise. This mathematical form represents an *update rule* for a *two-state cellular automaton* on a directed network. Cellular automata on networks were recently studied by Yang and Yang [62].

[12]For more details see next section 3.1.1

[13]Contrary to the `heaviside()` function in `MATLAB` for which $\Theta(0) = 1/2$

3. Cascading Failures on the Banking Network

3.1. Methods and Materials

3.1.1. Available Data

Banking network The following statistics and simulations are based on a dataset of the American banking-sector that contains

- *202 banks* and their interbank-liabilities,
- *insurances and other financial institutions* together with its liabilities,
- *other non-US financial institutions* that were not contained in the above set as one aggregated institution
- *tier 1 capital* for each institution

The liabilities among the banks, insurances and other institutions were stored in a 204×204 matrix. An entry in that matrix indicates the sum that the "row"-institution borrowed from the "column"-institution, the units being billions USD. In the following, insurances and other financial institutions are excluded from the analysis. The data[14] was imported into `MATLAB`, saved as a 204×204 matrix, the 203rd and 204th row and column corresponding to insurances and other financial institutions were removed.

The remaining matrix, referred to as $B_{i,j}$ or interbank-liability matrix is an asymmetric 202×202 matrix. Diagonal elements are zero, since no bank borrows from itself.

The B-matrix describes a bi-directional weighted network: for a given pair of banks i and j it contains both $B_{i,j}$, the amount that i borrowed from j and $B_{j,i}$, the amount that j borrowed from i (see figure 2 a).

However, in the event of a bank-default only the net amount that i borrowed from j is relevant. Therefore we construct a network J in which entries $J_{i,j}$ denote the net amount of debt settlement from i (row bank) to j (column bank). For this network it follows, that if $J_{i,j} \neq 0$, then $J_{j,i} = 0$.

This directed network $J_{i,j}$ (net liability matrix) has to be calculated from $B_{i,j}$. The upper 2×2 elements of B are used to illustrate the calculation:

$$\begin{bmatrix} 0 & 222.91 & \dots \\ 221.42 & 0 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

The intermediate matrix $\hat{J}$ (figure 2 b) is obtained through $\hat{J} = B - B^T$. This makes $\hat{J}$ antisymmetric: $\hat{J}^T = -\hat{J}$, which still describes a bidirectional network. Discarding matrix elements $\hat{J} < 0$ leads to the final adjacency matrix J of the directed network (figure 2 c):

$$\begin{bmatrix} 0 & 1.49 & \dots \\ 0 & 0 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

[14]See file B_tier1.mat

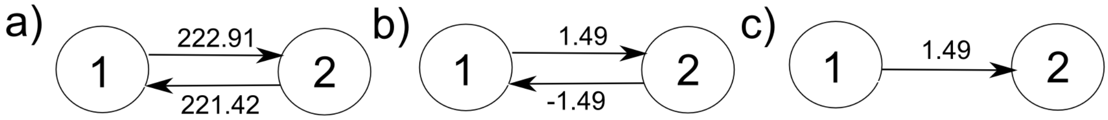


Figure 2: Netting of interbank liabilities. Arrows indicate flow of debt settlement. a) Original bidirectional network, b) Netted bidirectional network, c) Uni-directional network. Node 1 is the debtor, node 2 the creditor

.

There were points to note about the matrices B and J:

- Fifteen banks in the B-matrix were completely isolated, they have no incoming or outgoing connections. Their row/column indices in the matrix are: $160, 161, 163, 167,$ $174, 175, 176, 177, 180, 181, 185, 187, 194, 195, 199$
- The tier 1 capital for all banks is nonzero.
- After calculating the J matrix bank #23 lost all of its incoming/outgoing connections.

The isolated banks, including bank #23 were removed from the B and J-matrix and their tier 1 capital was excluded from the dataset.

The final B-matrix of bidirectional interbank liabilities had 186×186 entries out of which 576 were nonzero. It is a highly sparse matrix with only 1.65 % nonzero entries.

The final J-matrix of netted interbank liabilities had 186×186 entries out of which 424 (1.21 %) were nonzero. Since there are a number of symmetric entries in the B-matrix that cancel out when calculating $J = B - B^T$ the J-matrix has fewer nonzero entries than the B-matrix.

3.1.2. Software used

MATLAB Commercial software, used for matrix manipulations, statistics and visualization.

Pajek Pajek is a freeware graph visualization software that can also be used for network analysis of large datasets. It is being developed by Slovenian computer scientists Vladimir Batagelj and Andrej Mrvar.

MatlabBGL MatlabBGL is a freeware toolbox of network-analysis tools for `MATLAB` and was developed by David Gleich of Stanford University. It contains functions to calculate network-properties like clustering coefficients, average pathlengths, core numbers, etc.

Brain Connectivity Toolbox This is a freeware toolbox developed for the analysis of neural networks and contains a large selection of complex network measures implemented in `MATLAB`.[15]

[15] It can be found at `http://www.brain-connectivity-toolbox.net`, a good overview of complex network measures and their application to neuroscience is given in [46].

adj2pajek Is a MATLAB function developed by Gergana Bounova of MIT to export adjacency matrices of networks to the Pajek file format for visualization. It was adaped to support the coloring of nodes (e.g. the nodes that are part of the backbone).

plfit and plplot Is part of a number of MATLAB functions developed by Clauset, Shalizi and Newman [16] to identify powerlaws in empirical data.[16]

3.1.3. Backbone-Reduction, Eigenvector Centrality and k-Core

Backbone-Reduction Research on complex networks has been fuelled in the last decade by the increased availability of large amounts of data (e.g. social networks) and the increase in computing power, which enabled researchers to sift through this data. Especially when visualizing large, complex networks one wishes to display only the most "relevant" part of the network.

Serrano and colleagues [47] analyzed the network of international trade (ITN) among 192 countries of the world and employed a backbone-reduction algorithm to visualize the most vital part of this network. In a more recent publication [48] Serrano and colleagues extended this approach to arbitrary networks.

The goal of the backbone-reduction algorithm is to reduce the number of links in the network, retaining only the most "important" connections. Their approach has the advantage that links at all scales of the weight distribution (extremely strong as well as extremely weak links) can in principle be retained.

The notion of "importance" of a link is quantified by comparing the original network with a null-model that is generated by a broken-stick process. In this process $k-1$ points corresponding to cracks in a "stick" are distributed with uniform probability in the interval $[0,1]$. The probability α for one of the k subintervals having length p_i can be calculated analytically:

$$\alpha = 1 - (k-1) \int_0^{p_i} (1-x)^{k-2} \, \mathrm{dx} \tag{5}$$

The value p_i can be obtained for each edge of the empirical network by normalizing the weight w_i of an edge with the strength $s_i = \sum_j J_{i,j}$ of the node it connects to: $p_i = w_i/s_i$ with $s_i = \sum_j J_{i,j}$. The probability of the link occuring in the random network α is then compared to a threshold α_{th}, also called the significance level. The link is only retained if it is statistically significant: $\alpha < \alpha_{\text{th}}$ for at least one of the two nodes it connects. This process can be applied to directed as well as undirected networks. In the case of directed networks incoming and outgoing link weights w_i^{in} and w_i^{out} have to be normalized by node in- and out strengths s_i^{in} and s_i^{out} respectively.

The resulting network, called the backbone (BB) has less links then the original network (NW). Some statistics are introduced to see how the properties of the backbone depend on the choice of the significance level α_{th}. These statistics are[17]:

[16]It can be found at http://tuvalu.santafe.edu/~aaronc/powerlaws/

[17]These statistics were introduced for the ITN by Serrano et al [47].

- Relative sum of weights W contained in the backbone:

$$W = \frac{\sum_i \sum_j \mathrm{BB}_{i,j}}{\sum_i \sum_j J_{i,j}} \tag{6}$$

- Relative number of nodes N in the backbone

- Relative number of links E in the backbone:

$$E = \frac{\sum_i \sum_j \mathrm{sgn}(\mathrm{BB}_{i,j})}{\sum_i \sum_j \mathrm{sgn}(J_{i,j})} \tag{7}$$

  sgn(...) denotes the signum function.

If it is made sure that the BB-adjacency matrix as well as J, the adjacency matrix for the directed network contain only positive entries, the above definitions are sufficient. Absolute values of matrix entries have to be used if the matrix also has negative entries. The `MATLAB` function `bb_reduction` that was implemented for the backbone-reduction algorithm is described in the appendix.

To check whether the implemented algorithm works correctly, it has been tested on the 1960 and 2000 data of the ITN, since the above statistics are contained in [47] and can be compared easily, the results are shown in table 1.

α_{th}	0.2	0.1	0.05	0.01
W				
Calculated	0.88	0.827	0.779	0.689
Serrano et al	0.88	0.83	0.79	0.69
N				
Calculated	0.965	0.965	0.956	0.869
Serrano et al	1	1	0.99	0.92
E				
Calculated	0.253	0.192	0.151	0.091
Serrano et al	0.25	0.19	0.15	0.09

Table 1: Comparison of statistics of backbone for the international trade network of 1960, obtained from calculations and from Serrano et al [47]

While relative weigths W and edges E match the results obtained by Serrano and colleagues quite accurately, the relative number of links N differs up to 6 percentage points. Very likely the reason for this a different interpretation of equation 5 for degree $k = 1$. Strictly following equation 5 those nodes would be assigned $\alpha = 1$ and therefore be excluded from the network for any $\alpha_{\text{th}} < 1$.

Table 2 shows the dependence of the three statistics W, E and N on the significance level α_{th} for the the banking network and the 1960 ITN. We notice that the relative number of nodes N drops quite drastically as α_{th} is decreased from 1 to values slightly

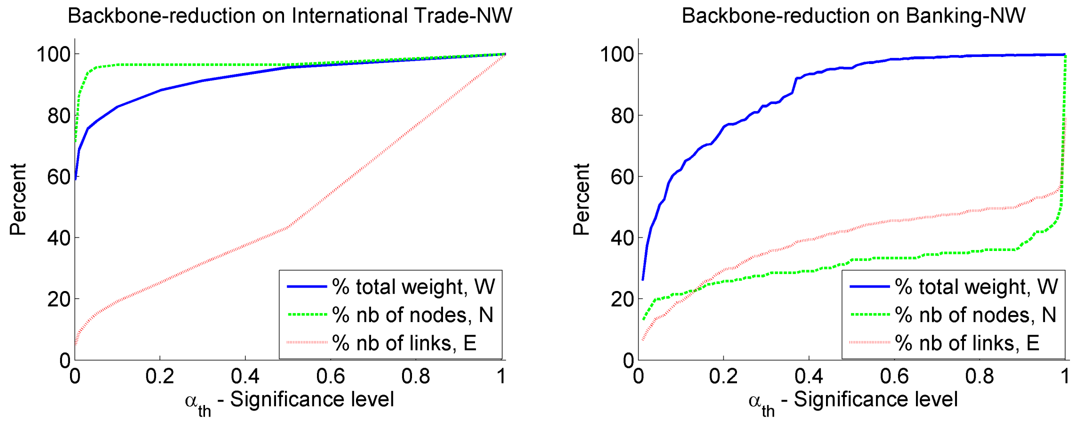


Table 2: Relative link weight W (blue), relative number of nodes N (green) and relative number of links E (red) in the backbone of the network as a function of the significance-threshold α_{th}.

below 1 for the banking network but does not change much for the ITN. The reason is, that in the ITN most countries have more than one trading partner (node degree > 1) while in the banking network (as we will see in chapter 3.2) many banks have a degree $k = 1$. These banks are removed from the backbone even for high values of α_{th}.

Eigenvector Centrality Besides the backbone-reduction developed by Serrano et al there are a number of other concepts that assign a value to the nodes of a network that corresponds to its importance in some sense.

In this thesis the *eigenvector centrality* and the *core number* of nodes in the network shall be examined in addition to the backbone-reduction. By discarding nodes with a centrality or core number below a certain threshold one can obtain two other kinds of "backbones" of the network.

There are several concepts of *node-centrality* in networks [10]. A node can exhibit a high centrality based on its degree, betweenness, closeness or the entry in the eigenvector of the adjacency matrix of a node. Here we shall only consider the eigenvector centrality. The eigenvector centrality (which is also employed in Google's PageRank algorithm) is based on the idea that the centrality x_i of a node should be proportional to the sum of the centralities of its neighbors[18]

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{i,j} x_j. \tag{8}$$

where λ is a constant. The vector of centralities $\mathbf{x}$ contains the centrality of all nodes of the network and can be calculated easily by solving the eigenvalue-problem

$$\lambda \mathbf{x} = A\mathbf{x}. \tag{9}$$

Since one is only interested in positive centrality-values one selects the eigenvector $\mathbf{x}$ corresponding to the largest eigenvalue, because the Perron-Frobenius theorem guarantees for a nonnegative matrix A that the eigenvector of the largest eigenvalue has only positive components.

[18]For a more detailed explaination see [38].

For a given adjacency matrix A one can easily compute the eigenvalues using `MATLAB`'s `eig(...)` function which returns all eigenvalues and eigenvectors.

k-Core The *k-core* of a graph is the largest subgraph that contains only nodes of degree equal or greater than k. The *core number* of a node is the largest k-value for which the node is still part of the k-core.[19] The k-core can be obtained computationally by iteratively removing all nodes with increasing degree number $\hat{k} \leq k$. This process has to be applied repeatedly, because after the removal of all nodes with degree $\hat{k}$ other nodes can obtain the degree $\hat{k}$ when they lose their neighbors. An efficient $O(n)$ algorithm is presented in [6], in this thesis the function `core_numbers(...)` which is part of the `Matlab_BGL` package is used. It assigns a core number to each node of the graph, the k-core is then obtained by selecting all nodes with a core number $\geq k$.

3.1.4. Randomized Networks

Often one wants to see how some properties of a network depend on the underlying topological structure. One example is the rich-club-coefficient that is used to measure how much rich countries tend to cluster in the ITN [9]. In order to do that, it is common to randomize certain topological properties of the network (e.g. the weights, the direction of edges, the nodes which the edges connect) while at the same time keeping other properties constant (e.g. the in- and out-degrees of nodes, their strength). In the case of the ITN randomization of links in the undirected, unweighted network reveals that the rich-club-coefficient depends only on the degree sequence, not the actual wiring of the network [24]. For the weighted, undirected ITN Bhattacharya et al [9] carried out the same randomization but applied a self-consistent iteration procedure that preserves the strengths $\{s_i\}$ of the links.

This approach has an interesting connection to statistical mechanics: The original network can be viewed as a member of a microcanonical ensemble of graphs. Instead of the energy which is constant in systems usually studied in statistical mechanics, here the degree sequence of the graph is held constant while through randomization other members of the ensemble are generated. The dependancy of the property of interest (e.g. rich-club-coefficient) can then be obtained by averaging over the ensemble of random graphs and comparing the result with the original network. Recently this approach was generalized for arbitrary network-properties by Squartini and Garlaschelli [55].

To see how the stability of the banking system depends on the topology of the underlying network, the network is randomized in a number of different ways:

random weight exchange Two links (entries in the J-matrix) are chosen at random and their weights are exchanged.

This will leave the topology[20] as well as in- and out-degree of the two nodes unaltered, but affect their in- and out-strengths.

random link exchange Two directed links (arcs) are chosen at random and the nodes at their ends are exchanged.

[19] The study of subgraphs, k-cores and other mathematical objects is a subfield of graph theory.

[20] Location of nonzero elements in the adjacency matrix

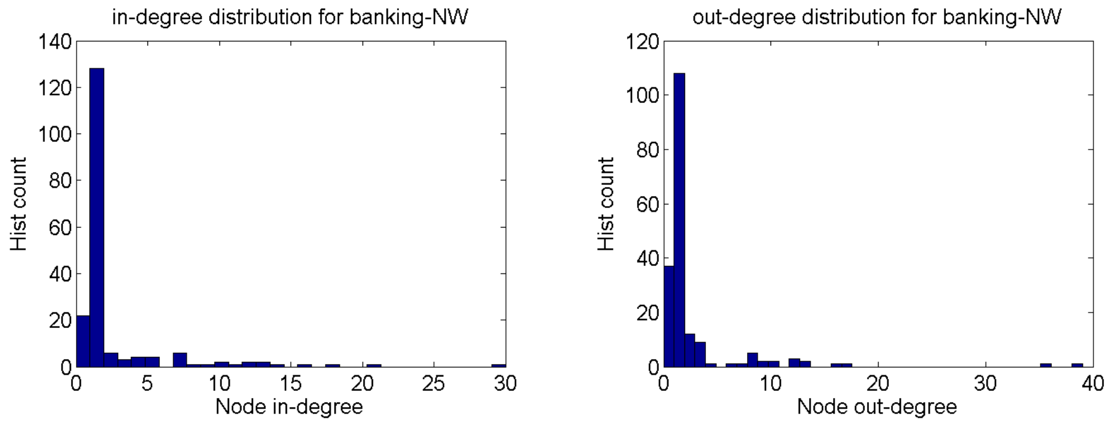


Table 3: Histogram of in- (left) and out-degree (right) of banks. Note the different x-axis limits of the in- and out-degree histogram.

This leaves in- and out-degree of the two nodes unaltered but affects their in- and out-strengths as well as the network topology.

random link exchange, preserving node-strength The random link exchange is carried out on the network, afterwards an iteration procedure is carried out to restore the in- and out-strength $\{s_i^{\text{in}}\}$, $\{s_i^{\text{out}}\}$ of the nodes.

Random weight exchange and random link exchange on a small network are illustrated in figure 8. The first two options are implemented in the `MATLAB` function `randomize_NW` for which details can be found in the appendix. The third possibility – strength-preserving link exchange – has not been implemented yet. While the original strength distribution for the undirected network can be restored using methods from [9], the problem in directed case is that adjusting the in-strength $\{s_{\text{in}}\}$ will change $\{s_{\text{out}}\}$. Therefore one has to switch back and forth between adjusting those two network-properties while convergence of this process is not guaranteed.

3.2. Description of Network-Structure

In this section the banking-network shall be characterized using measures common in the theory of complex networks. We will obtain information about node-strength s_i^{in}, s_i^{out}, node degree k_i^{in}, k_i^{out}, the correlation among those quantities as well as the distribution of tier 1 capital and liabilities among banks.

In 2003 Boss et al [11] conducted a study of the Austrian interbank market which was based on data similar to the data used here. Therefore the characteristics of the US banking-network shall also briefly be compared with their findings.

3.2.1. Degree- and Strength-Distributions

Degree Distributions For the distribution of in- and out-degrees of the 186 nodes in the network a histogram is obtained, see table 3. Since the degree is an integer quantity, no information about the distribution is lost in the histogram when the bin-size is chosen to be one.

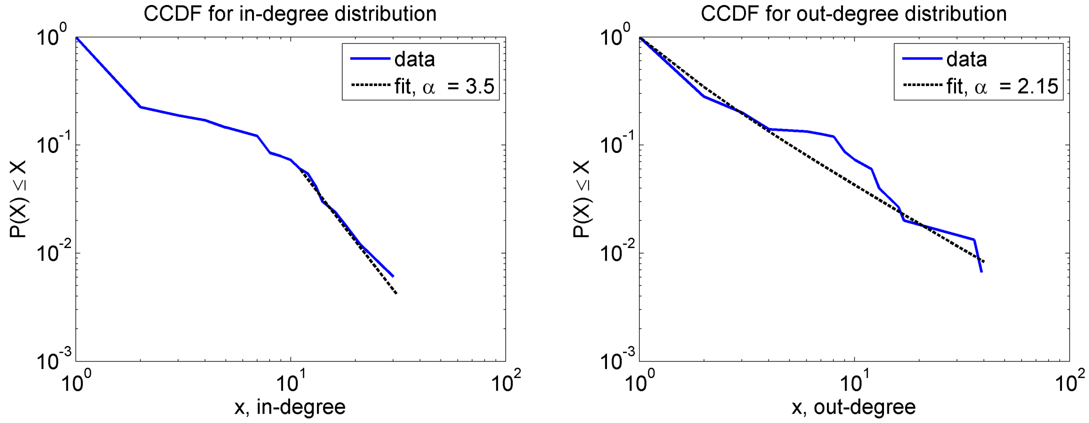


Table 4: CCDF of in- and out-degree distribution. For the in-degree distribution only the tail has been fitted to a power-law exponent $\alpha = 3.5$ while the out-degree distribution exhibits a noisy tail and a fit over the whole range has been attempted.

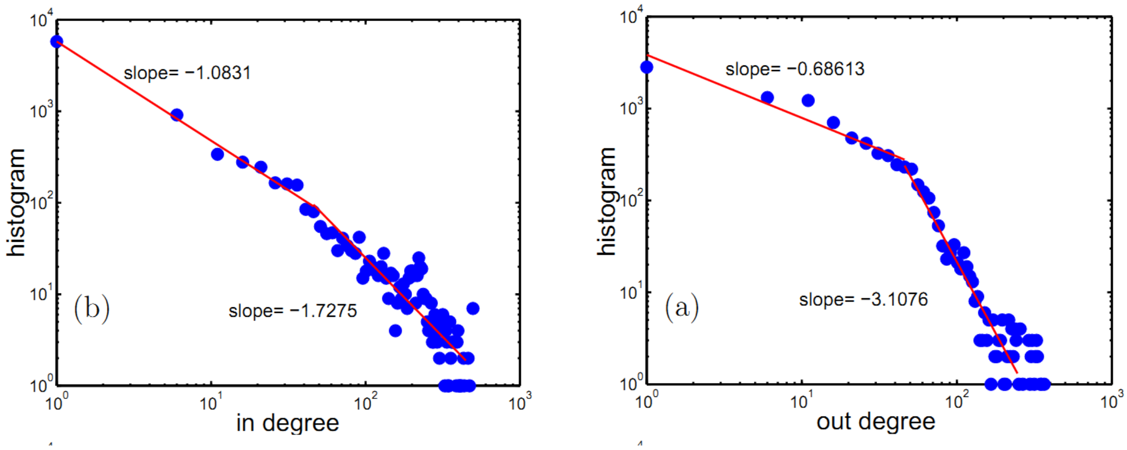


Table 5: Austrian banking-network, in- and out-degree distribution, from [11]. Note that these are log-log plots of histograms, not CCDFs as shown in table 4.

The key features of the degree distributions for the banking-network are:

- most of the banks (80% / 77%) have an in/out-degree of only zero or one
- very few banks have large in/out-degrees: 5.4% with $k_{\text{in}} \geq 10$, 4.8% with $k_{\text{out}} \geq 10$
- bank #1 has both the maximum in-degree (30) and the maximum out-degree (39)
- hardly any banks have degrees between 20 and 30

From the histograms one can obtain the average in- and out-degrees $\langle k_{\text{in}} \rangle = 2.27$ and $\langle k_{\text{out}} \rangle = 2.27$. However, since the degree distributions are skewed the mean degree does not give a clear picture of the network. This is also indicated by the variances of the distributions $\sigma^2_{\text{in}} = 3.9$ and $\sigma^2_{\text{out}} = 4.7$ which are larger than the mean values themselves.

The fact that only a very small number of banks have high in/out-degrees while the bulk of nodes in the network have low degrees hints at the existence of a power-law for the distribution of node-degrees and also indicates a highly connected "core" of the network.

Power-laws have been observed in the degree distribution of the Austrian banking system before. Boss et al [11] report in- and out-degrees for banks of the Austrian banking network following a hockey-stick like shape (see table 5) in double-logarithmic plots with a narrow slope for low degrees and a steeper slope for higher degrees.

To check whether the degree distribution follows a power-law we estimate the exponent using `plfit` and plot the complementary cumulative distribution function (CCDF) of the distributions using the `plplot` function.[21].

The nodes with zero in- or out-degree are excluded from the respective fits and plots, since the logarithm of these values will diverge. In a double-logarithmic plot of the CCDF a power-law for an underlying distribution $p(x) \propto x^{-\alpha}$ will show up as a straight line with slope $\alpha - 1$[22].

Table 4 shows the CCDF as well as the linear fits to estimate the exponent of the distribution. For the in-degree distribution the estimation was done only for the tail (banks with degree $k_{\text{in}} \geq 10$, `plplot` showed a warning for finite-size bias). The out-degree distribution was fitted over the whole range of k_{out} but does not fall nicely on the linear slope.

In both cases the existence of an underlying power-law in the degree distribution is graphically not well confirmed. The `plfit` function does neither estimate the uncertainty of the fitted parameters, nor of the validity of the fit.

Comparing the in- and out-degree distributions in table 4 to the Austian network one notices that the Austrian network covers one more order of magnitude (maximum degree ≈ 150) and contains much more data-points (883 banks). Both the US and the Austrian banking system show a bend in the CCDF/histogram of the degrees, however in the case of the US system the bend is more distinct for the in-degree distribution ($\alpha = 3.4$) while for the Austrian system it is the tail of the out-degree distribution ($\alpha = 3.1$). The skew distribution of node degrees hints at a core-periphery organization that will be discussed in detail later.

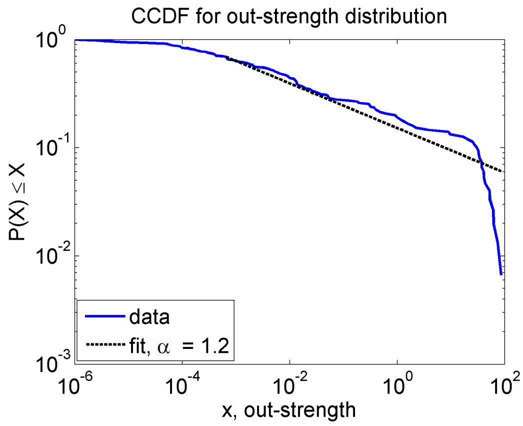
Strength Distributions Each node has weighted incoming and outgoing edges where the weights represent the amount of money lent out or borrowed by a bank from another bank. This information can be used to assign in- and out-strengths (total money lent out/borrowed) s_{in}, s_{out} to the nodes of the network.

Table 6 shows the complementary cumulative distribution function (CCDF) for both the money lent out (in-strength) and borrowed (out-strength) by banks in the dataset. It confirms the picture of a core-network and a periphery that was indicated by the highly heterogenous degree distribution in the previous section. Just as in the case of the degree distribution we saw that only a small fraction ($\approx 10\%$) of the nodes showed a high in/out-degree, for the node strength also a small fraction of the nodes shows an exceptionally high in- and out-strength.

The distribution of node-strengths covers 8 orders of magnitude and shows a sharp bend at $s_{\text{out}} \approx 30$ and $s_{\text{in}} \approx 22$. In both cases this bend separates the banks into two

[21] See 2.2 and 3.1 for more details

[22] See section 2.2 for more details on power-laws



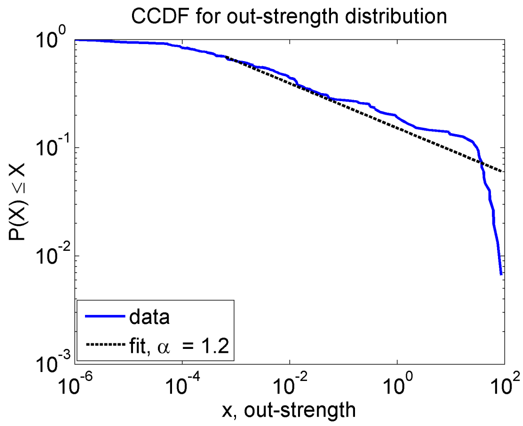


Table 6: CCDF and linear fit for in- and out-strengths of the banking network.

groups. The majority of banks, about 90% account for only 10% of the total amount borrowed/lent. The remaining $\approx 10\%$ account for 90%. Calculating the mean $\langle s_{\text{in}} \rangle = 4.54$ and $\langle s_{\text{out}} \rangle = 4.54$ again does not give a clear picture of the underlying distribution. It is noted, that compared to the degree-distribution the CCDF of the strength-distribution shows a much better agreement with the linear power-law fit.

In a study of the network of telephone calls Onnela and colleagues [41] also observe a non-power-law degree distribution but a power-law strength distribution.

The observed 90-10 relationship might be an instance of a Pareto principle which is well known in economics. The mathematical representation of the Pareto principle is the Pareto distribution which follows a power-law.

Summing up, our dataset is qualitatively similar to the Austrian banking network studied in [11]. Degree- and strength-distributions exhibit a power-law with an exponential cutoff but quantitative differences exist and power-law relationships are not as evident in our data.

Since for now we are only interested in getting a first impression of the network, the question of whether node-degrees and -strengths are described by a Pareto distribution will not be pursued in more detail.

Another possibly interesting quantity that is also not investigated further here is $\Delta s_i = s_i^{\text{in}} - s_i^{\text{out}}$, the net amount borrowed or lent by a bank. This quantity can be used to divide the banks in two groups of net lenders $\Delta s_i > 0$ and net borrowers $\Delta s_i < 0$. It might be used to find out, whether net borrowers are *more vulnerable* and net lenders are more *contagious*.

Why do the top 10% of the banks have such large in- and out-strengths? Is it because they have many connections with similar weight? Or are there maybe only a few extremely strong connections?

A first step to answer this question is to look at the correlation between node-degree and node-strength. Here we examine a linear relationship between in-degree/in-strength and out-degree/out-strength. Table 8 gives the slope of the linear relationship and the R^2 value as a measure of goodness of fit, $R^2 = 0$ indicates no linear $R^2 = 1$ a perfect linear relationship.

The linear regression reveals, that banks with a large number of incoming links also

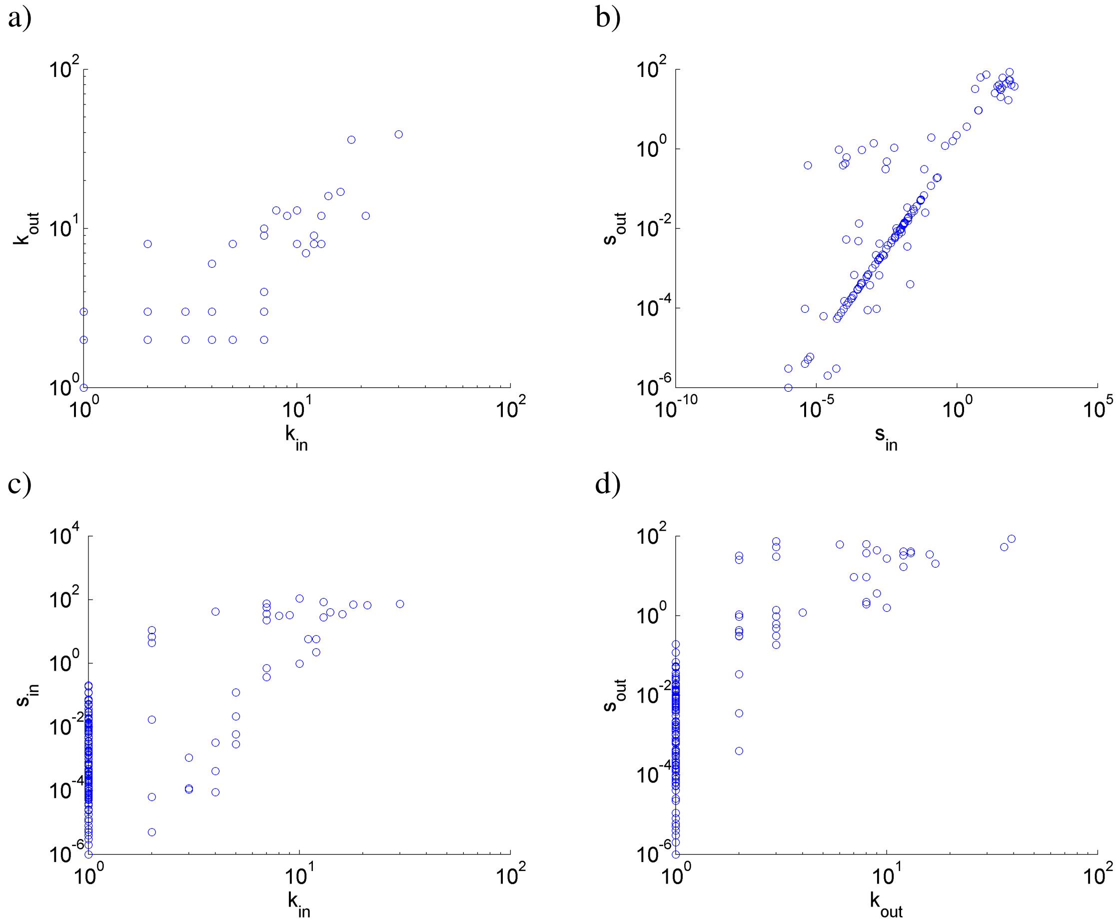


Table 7: Degree- and Strength-Correlations Correlations between a) in- and out-degree, b) in- and out-strength, c) in-degree and in-strength, d) out-degree and out-strength

y	x	a	R^2
k_{out}	k_{in}	1.1	0.77
s_{out}	s_{in}	1.5	0.59
s_{out}	k_{out}	3	0.57
s_{in}	k_{in}	2.1	0.52

Table 8: Correlation between degrees and strengths in the network using a linear regression $y = a \cdot x + b$.

tend to have many outgoing links. A statistically significant correlation ($R^2 > 0.5$) between the number of outgoing links and out-strength (incoming links and in-strength) of the nodes is detected. This is a first indication that high strengths stem from large number of degrees, but further analysis is necessary.

3.2.2. Clustering, Average Shortest Path and Assortativity

Apart from degree- and strength distributions complex networks are characterized by their clustering, average path length and assortativity. In this section these quantities will be obtained for the network. Before one can use the different functions provided by the network-toolboxes it should be decided, whether the bi-directional network of mutual liabilities B or the directed network of netted liabilities J should be analyzed.

In their 2003 study of the Austrian banking system Boss et al work with the bi-directional network, but since we are interested in the propagation of defaults which only occur along paths of the directed network J, analysis will be restricted to this network.

To calculate the clustering coefficient of the network the function `transitivity_BU` (for binary, directed networks) of the Brain Connectivity Toolbox is used.[23] A clustering coefficient of

$$C = 0.241$$

is obtained for the directed network, averaging over individual clustering coefficients gives $\bar{C} = 0.3$. This value is much larger than that of an equivalent random network for with $C = \langle k \rangle / N$ which has $C_{\text{rand}} = 0.01$ The clustering coefficient is also larger than $C = 0.12$ measured for the Austrian banking network. Power-grids in which cascading failures have been studied have clustering coefficients of $C = 0.05$ [12] and $C = 0.10$ [39].

Now the shortest distances between nodes and the average path length is obtained for the directed but unweighted[24] network using `Matlab_BGL`'s `all_shortest_paths` function. Figure 3 gives a graphical representation of the shortest distances in the banking-network.

From the matrix of distances the average path-length $\langle l \rangle$ is calculated. Infinite distances that indicate that some nodes have no path connecting them (46% of all possible connections had infinite distances) were handled by using the formula $\langle l \rangle = \left(\frac{\sum_i \sum_j 1/D_{i,j}}{N(N-1)} \right)^{-1}$ where $D_{i,j}$ is the matrix of distances. We obtain an average path-length of

$$\langle l \rangle = 4.73$$

which is larger than $\langle l \rangle = 2.26$ found for the Austrian banking network. A comparison with a random network is not possible since we are dealing with a directed network.

What do the distance-matrix and the average shortest path tell us about the network?

[23] Averaging over the clustering coefficients that can be obtained for individual nodes of the network through the `clustering_coefficients` function from the `Matlab_BGL` package does not give the same results. This is explained in more detail in [19] p. 3.

[24] obtaied by setting all nonzero weights in the J matrix to one.

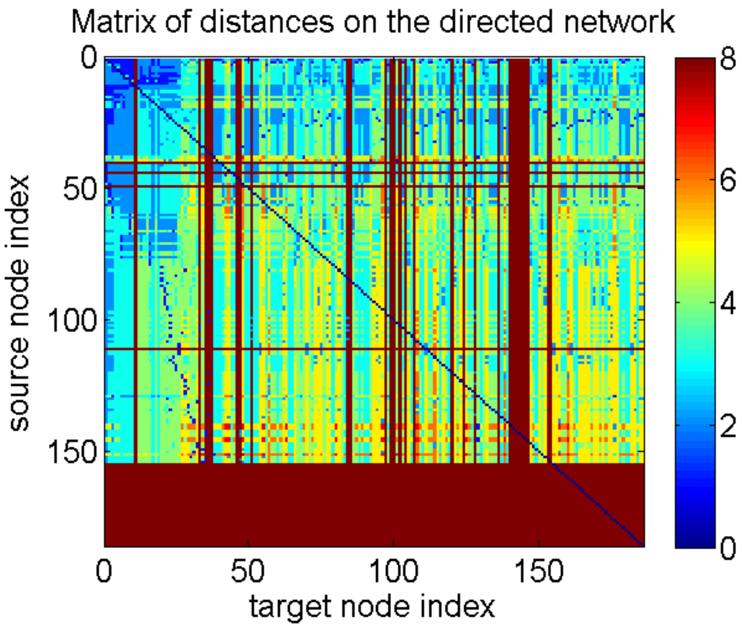


Figure 3: Average pathlength on the directed network of interbank liabilities J. Cool colors indicate small distances, hot colors long distances. Dark red marks infinite distance: there exists no connecting path between source and target node.

- If one bank fails then on average 3.59 intermediate bank-failures separate it from any other bank in the network. However not every bank can be reached by defaults as indicated by the infinite distances (dark red) in figure 3.
- Banks #1-30 form a densely connected cluster with shortest distances of 1 or 2 and are well connected (with shortest distances ranging from 1 to 4) by outgoing links to the rest of the network.
- Banks #150-186 have only incoming, no outgoing links.
- Banks #140-146 have only outgoing, no incoming links.

To calculate the assortativity of the network, first the network is changed into an undirected, unweighted network. This lumps together in- and out-degrees[25], e.g. a node with in-degree 4 and out-degree 2 is assigned a total degree 6. Using the Brain Connectivity Toolbox the function `assortativity` gives an assortativity-value

$$r = -0.2844.$$

which corresponds to a disassortative NW. This means that highly connected nodes tend to connect only to weakly connected nodes. These findings are in good agreement with observations of tho Fedwire network of payment flows studied by Soramäki and colleagues [54]: they found a "tightly connected core of banks to which most other banks connect" and a disassortative connectivity.

[25] Alternatively one can use assortativity measures proposed by Foster and colleagues [20] for directed networks that measure in-in, in-out, out-in and out-out degree correlations separately.

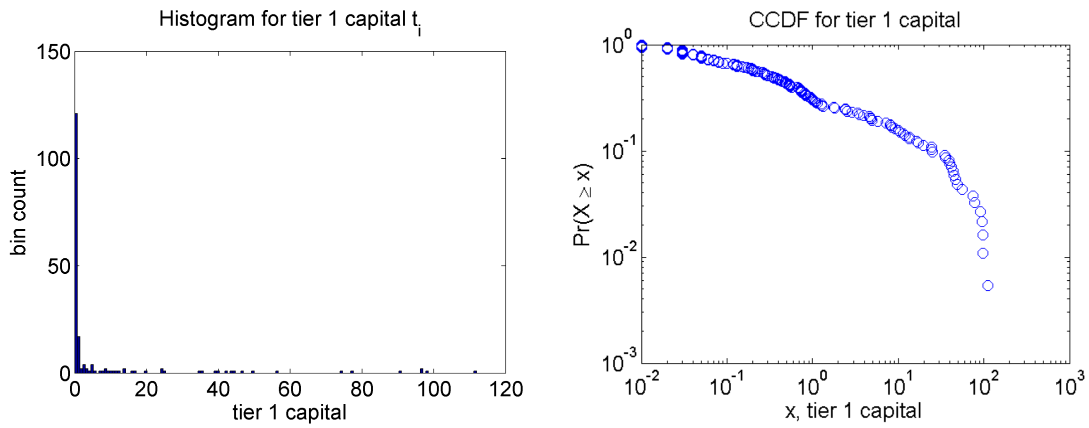


Table 9: Histogram (150 bins) and CCDF for the tier 1 capital distribution.

Given the degree and strength-distributions, the average path lengths, the clustering and assortativity one could now try to find out whether the banking network exhibits properties of well-known network models (e.g. Erdos-Renyi random graph, Watts-Strogatz small world network, scale-free networks, etc) or other empirical networks.

Also an interpretation of the obtained quantities in the specific context of the banking system can now be pursued. So far it has not been of much relevance that the nodes in the network actually represented banks and the edges their interbank borrowing/lending. What implications do high clustering coefficients, disassortativity, hockey-stick like distributions for node-degree and strength mean in the context of the banking network?

The graphical representation of the directed network through the distance-matrix can potentially give further insight into the "core-periphery" structure that was indicated before.

3.2.3. Distribution of Tier 1 Capital and Normalized Liabilities

The update rule that governs the dynamical behavior of the system is

$$s_i^{t+1} = 1 - \Theta\left(\sum_j (1 - s_j^t) J_{j,i} + q_{\text{th}} \cdot t_i\right). \tag{10}$$

This equation contains the interbank-liabilities encoded in the entries of the J-matrix, the tier 1 capital t_i of each bank and the default threshold q_{th}. In this section the distribution of tier 1 capital and the entries of the J-matrix shall be examined more closely, as they form the building blocks of the dynamical model. Their distributions can give helpful insights into the stability of the system.

The distribution of tier 1 capital as depicted in table 9 spans 4 orders of magnitude. The CCDF again exhibits a hockey-stick like form. A comparison between histogram and CCDF reveals a shortcoming of histogram-visualizations of continuously-valued quantities: most of the nodes have a tier 1 capital $t_i < 10^1$ but are lumped into only a few bins so that one cannot tell whether they have different values. The CCDF gives a clearer picture: 90% of the nodes have a tier 1 capital ranging from 10^{-2} to 10^1 and account for 25% of the total tier 1 capital. A mere 10% of the banks with $t_i > 35$ contribute 75% to total tier 1 capital.

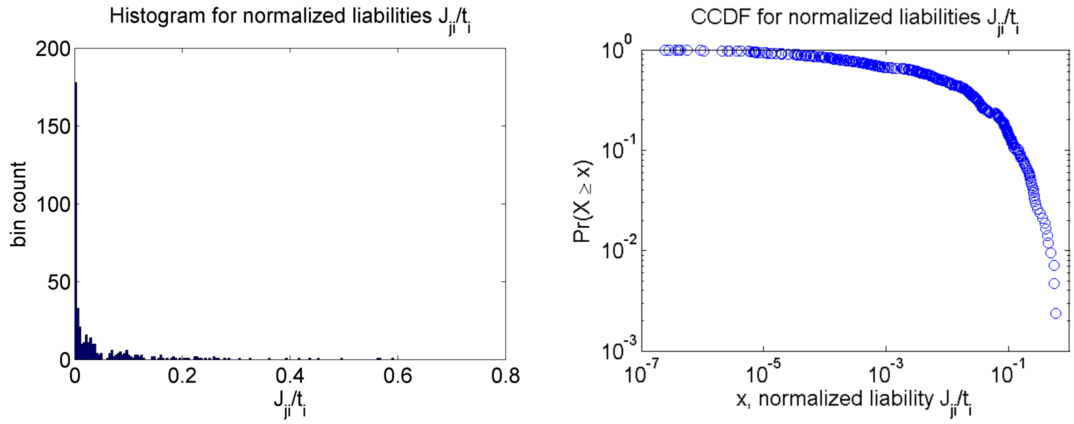


Table 10: Histogram (left) and complementary cumulative distribution function (right) of the normalized liabilities in the network.

Next, the distribution of liabilities in the banking system is scrutinized. This distribution is relevant to the stability of the system: a bank i will default, when one of its neighbors j defaults and the amount $J_{j,i}$ that i had lent to j exceeds q_{th} % of i's tier 1 capital t_i. Considering only *single-neighbor-defaults* stability of the ith bank requires

$$J_{j,i}/t_i < q.$$

If this is the case for all normalized incoming links to a bank i, bank i will not default if any (single) neighbors defaults. Therefore the distribution of normalized liabilities $J_{j,i}/t_i$ determines the stability of the system. The left and right figure in table 10 show this distribution. From the CCDF one can now easily determine what fraction of liabilities for a given threshold q_{th} is "dangerous" to the banks connected by it. As the histogram indicates the highest normalized liability lies around 0.6. Beyond this threshold no bank in the network can be defaulted.

3.2.4. Visualization of Network

In addition to the statistical analysis carried out, a graphical representation of the network can give a more easily accessible impression. Before displaying the network, a few modifications of the data are necessary: we choose for the node-size to scale linearly with tier 1 capital and set its minimum size to 1, so that all nodes remain visible. The size of arcs scales logarithmically with the liability $J_{i,j}$ and is also thresholded to a minimum of 1. Arrows on arcs indicate the direction of payment flows, nodes that are part of the backbone for $\alpha = 0.1$ are marked as red.

Figure 4 shows the visualization of the full network. The division into two classes of banks that was noted earlier for different properties becomes apparent in this figure: the nodes that form the backbone have for the most part a much higher tier 1 capital (node size) and are better connected than the rest of the network. Figure 5 shows only the backbone of the network which consists of a few highly connected nodes owning the bulk of tier 1 capital.

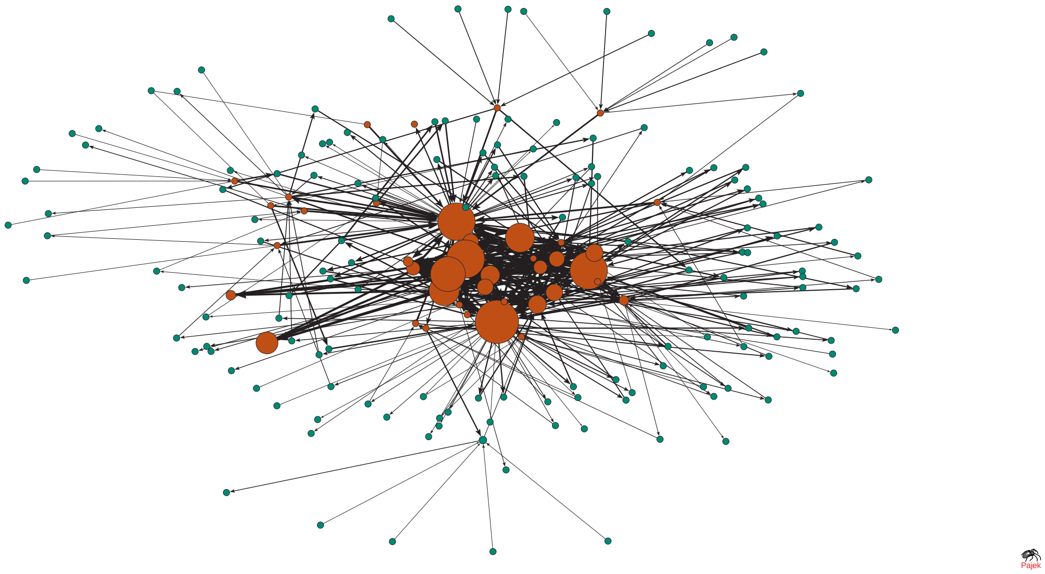


Figure 4: Directed network of interbank liabilities, node size indicates tier 1 capital, arrow size indicates liability, red nodes are part of the backbone for significance-level $\alpha = 0.1$.

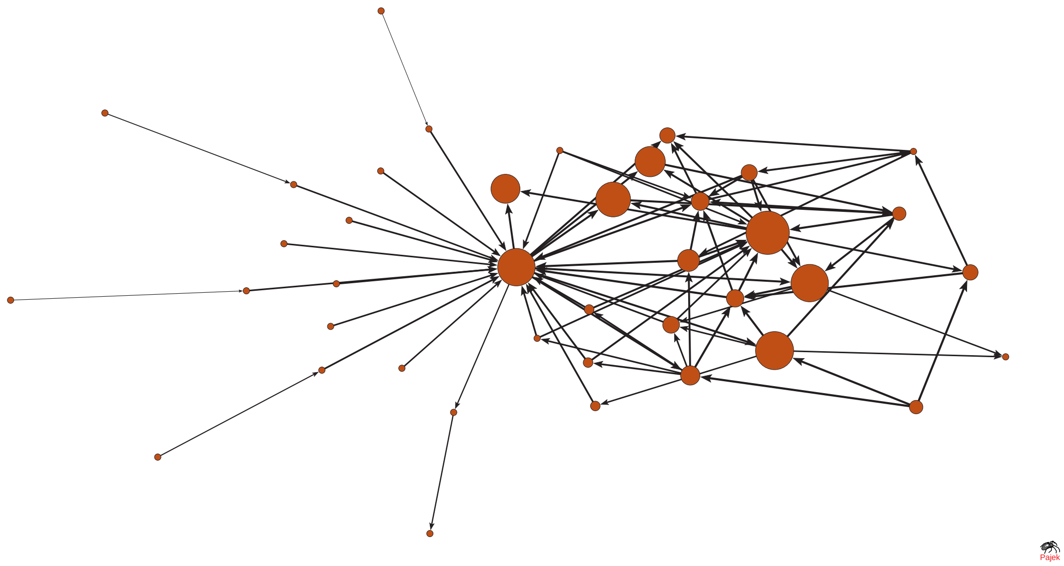


Figure 5: Backbone of the interbank network for significance-level $\alpha = 0.1$.

3.3. Simulation of Cascades

Now that we have a better understanding of the structure of the banking network we will study the effects of single-bank defaults in the system. To do this we will first introduce some terminology that enables us to talk about stability-properties of the network a bit more precisely.

3.3.1. Definitions: Local and Global Stability

When talking about cascades on the network we will need to distinguish between:

Topological and dynamical properties. *Properties that are derived only from the adjacency matrix J are called* topological. *Properties that depend on the choice of parameters in the dynamical model which is used to simulate cascades are called* dynamical properties.

Local stability For dynamical systems it is common to obtain the stability of a fixed point by linearizing the system around that point. This stability is then called *local stability*. In ecological networks the *global stability* describes the ability of a system to withstand greater perturbations [25] and generally it describes a system for which the state-vector remains within a bounded area of phase space (Lyapunov stability). Following these ideas we see local stability as a property of single nodes of the network and global stability as a property of the whole network.

Local stability of nodes leads us to two new concepts:

Vulnerability. *A node is vulnerable if there is at least one neighbor whose default can lead to the default of the node itself.*

Contagiousness. *A node is contagious if there is at least one neighbor that will fail if the node itself fails.*

From these definitions it follows, that contagious nodes are always connected to vulnerable nodes by outgoing links. It is also possible for nodes to be both vulnerable and contagious.

The classification of nodes according to the above definitions depends on the choice of the threshold parameter q_{th} and can be used to visualize how nodes turn from vulnerable to "immune" (from contagious to "healthy") as the system-wide threshold is increased. Figures in table 11 illustrate this transition.

For $q_{\text{th}} = 0$ nearly all nodes are contagious and nearly all nodes are vulnerable. Nodes that are not contagious have an index > 150 and it has been observed in section 3.2.2 that these nodes have only incoming links, therefore cannot infect their neighbors. Nodes #140-146 are not vulnerable, this again can be explained with the distance matrix: these nodes have no incoming connections and cannot be influenced by next-neighbor defaults.

We notice that banks with low indices (those who were identified to have high interconnectivity) are vulnerable and also contagious up to very high default-thresholds compared to the majority of the banks.

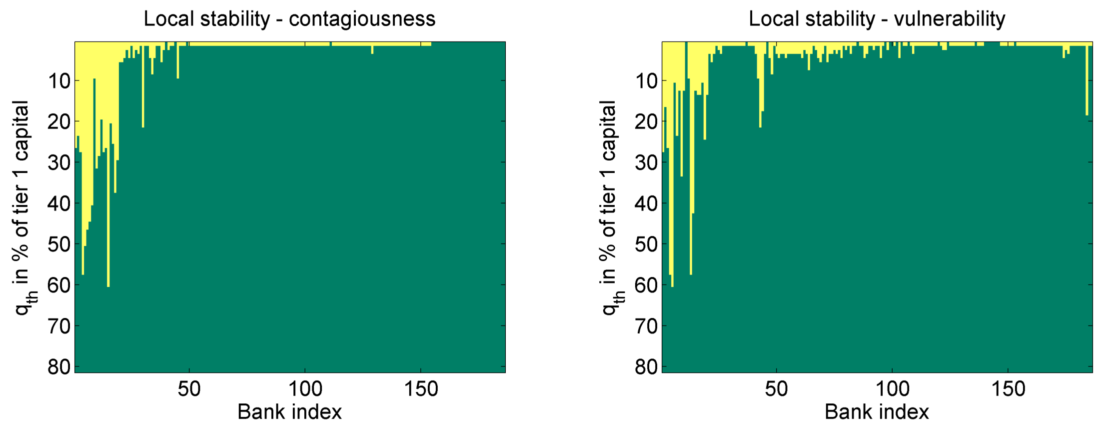


Table 11: Local stability of nodes. Left – nodes turn from contagious (yellow) to "healthy" (green) as the threshold q_{th} is increased. Right – nodes turn from vulnerable (yellow) to "immune" (green) as threshold q_{th} is increased.

Above $q_{\text{th}} = 0.6$ no node is vulnerable or contagious anymore. It is interesting to note in this context that in the Basel I agreement the minimum tier 1 capital ratio was set to $4\ \%$.[26]

How are the node-properties of contagiousness and vulnerability related to the distribution of normalized liabilities, which is a link property, obtained in section 3.2.3? We notice that the threshold $q_{\text{th}} = 0.6$ that was obtained from the normalized liabilities distribution gives indeed the correct value for the transition to complete local stability in the system.[27] The distribution of transition-thresholds for the nodes will be related but is not identical to the distribution of normalized liabilities, because transition thresholds describe node-properties, the normalized liabilities describe link-properties.

It should be kept in mind that the transition vulnerable – "immune" (contagious – "healthy") is only valid when considering defaults caused by a single neighbor. Banks now classified as "healthy" can still default if e.g. two of their debtors default.

Global stability Global stability is a property of the whole network obtained by applying the dynamics given by equation 4. Because of the finite size of the system (at most all N nodes of the network can fail) the trajectory of the state vector in phase space will remain bounded. In that sense the system is globally stable. But intuitively, global stability should decrease as the size of cascades following the perturbation of a single node increases. The following definition of stabiliy takes that into account:

Global stability. *The global stability of a network is inversely proportional to the average number of nodes that fail during a cascade triggered by a single node.*

A quantitative measure of global stability is g:

$$g \equiv \frac{1}{\langle \text{avalanche size} \rangle} \tag{11}$$

[26] This is not exactly the same as the quantity considered here. The tier 1 capital ratio is the ratio of tier 1 capital to risk-weighted assets of the banks. The quantity we consider here is the ratio of liabilities to tier 1 capital.

[27] However this finding is trivial, given that implementation of the vulnerable/contagious-definitions is correct.

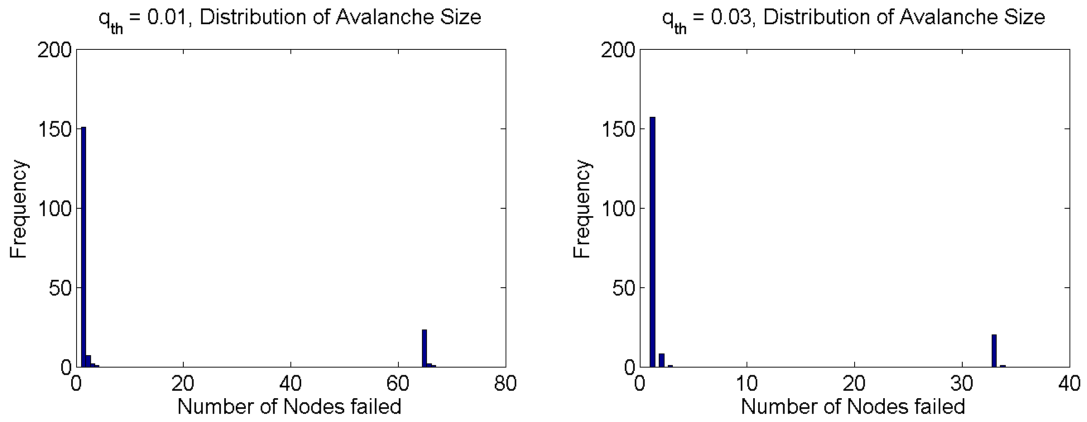


Table 12: Global stability. Distribution of cascade size for $q_{\text{th}} = 0.01$ (left) and $q_{\text{th}} = 0.03$ (right). Note the different scaling of the x-axis in left and right histogram.

where the avalanche size is averaged over all possible avalanches that can be triggered by perturbing each single node of the system. Every perturbation is trivially a cascade of size one, therefore if no propagation of defaults occurs for any perturbation on the system g will take on a maximum value $g_{\text{max}} = 1$. If propagating cascades occur, g decreases. For a network of finite size N it reaches a minimum value if every perturbation affects the whole network $g_{\text{min}} = \frac{1}{N}$[28].

For our banking-network cascade statistics including the stability measure g can be calculated for a given topology J, tier 1 capital and default threshold q_{th} using the `MATLAB` function `mc_stability`[29].

To calculate global stability g the average of the avalanche size distribution has to be used. Therefore we will quickly take a look at this distribution for two different default-thresholds q_{th} – table 12 shows two histograms of avalanche sizes. This also allows us to check for signs of self-oganized criticality (SOC)[30] in the model. There is good reason to expect signs of SOC, because in many other models of cascading failures avalanche-distributions follow power-laws. In the BTW sand-pile model [5], a showcase model for SOC, the size distribution of avalanches (cascades) follows a power law. This result is quite robust even if the topology of the sand-box is changed from a regular grid to undirected, directed [42], random[31] [58] and small-world networks [62].

The cascade size distributions in table 12 however show, that our dynamical system does not exhibit any signs of self-organized criticality. Instead cascades follow a "many or nothing" behavior. Either only the perturbed node and rarely a few of its immediate neighbors default (this happens in most of the cases) or a large number of nodes (for $q_{\text{th}} = 0.01$ more than 60 nodes, for $q_{\text{th}} = 0.03$ more than 30 nodes) default. For other values of q_{th} (not displayed here) the avalanche size ditribution has a similar shape.

[28] In studies of foodwebs several other definitions of (global) stability are used: "The stability of the ecosystem is (...) defined by the rate at which small perturbations to the populations of various species decay with time", [52].
[29] For details see the appendix.
[30] See appendix for a short explaination of self organized criticality.
[31] Watts does not use a sand-pile model.

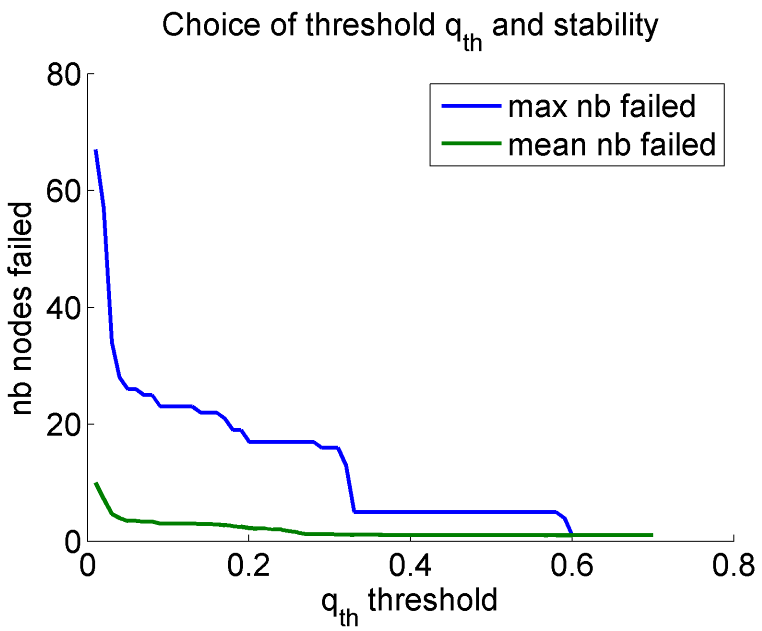


Figure 6: Dynamics and global stability. Influence of dynamical property q_{th} on the global stability of the network: Maximum (blue) and mean (green) number of nodes that failed in one cascade.

3.3.2. Dynamical Properties and Stability

In the previous section local and global stability were introduced. Now the dependence of global stability on the dynamical parameter q_{th} is studied.

First we will re-examine the cascade size distributions in table 12 that we obtained in the previous section. The maximum number of nodes that default in one cascade obviously depends on the threshold q_{th}. As this threshold increases the cascade size decreases and cascades become confined to smaller and smaller parts of the network. To obtain more information about global stability we now vary the dynamic parameter q_{th} from 0.01 to 0.8 and determine average and maximum cascade size. Figure 6 shows the obtained relationship.

We see, that the mean is much smaller than the maximum, which can easily be explained by the bimodal distribution. The mean cascade size smoothly decreases from about 10 (for $q_{\text{th}} \rightarrow$) as $q_{\text{t}} \searrow 0$ to 1 at the predicted value of $q_{\text{th}} = 0.6$. The graph of the maximum cascade size exhibits four distinct sections: (i) for low values of q_{th} there is a steep decline in cascade size followed by (ii) an intermediate regime with a low slope, that drops to (iii) a platea and finally (iv) goes to 1 for $q_{\text{th}} \geq 0.6$.[32]

To illustrate the dynamics of a cascade we plot the pattern of defaults for the largest cascade for $q_{\text{th}} = 0.01$ in figure 7. One of the first banks to default is bank # 1 which was identified in section 3.2.1 to have highest connectivity. From there the tightly connected cluster of banks #1-#30 and a number of banks with higher indices (presumably periphery) are affected. After 6 rounds of defaults the system settles to a new equilibrium.

[32]In the single-neighbor default approximation for fixed q_{th} this relationship can be linked to the number of vulnerable nodes in a cross-section of the vulnerability-plot (right plot in figure 11).

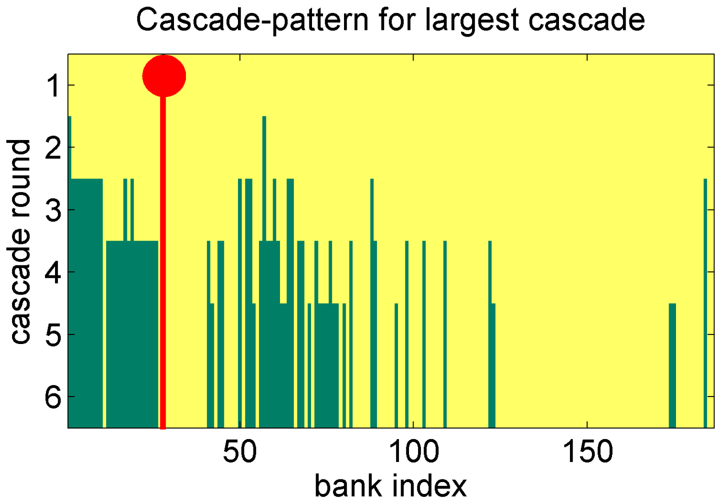


Figure 7: Dynamics and global stability. Pattern of default for the largest cascade in which 67 banks failed. The big red dot indicates the initial perturbation.

3.3.3. Topological Properties and Stability

The topology of a network sets limits to the dynamical processes that can take place on it and thereby also determines stability. This is illustrated by considering the extreme cases of completely disconnected and fully connected (all-to-all) networks: In a disconnected network no propagation of cascades can occur at all, while in an all-to-all topology cascades can travel through the whole network affecting every node.

As seen in the matrix of shortest distances in section 3.2.2 and confirmed by the local stability measures in table 11 not all banks can trigger cascades and not all banks can be reached by cascades, the largest cascade affects 67 out of 186 banks. Therefore current topology puts a lower limit to the global stability measure g that lies above the theoretical limit $g_{\text{min}} = 1/N = 1/186$.

Also it is found that above a threshold $q_{\text{th}} = 0.6$ all nodes are locally stable, which implies that no global cascades can occur. We therefore expect $g(q_{\text{th}}$ to reach 1 for $q_{\text{th}} = 0.6$.

To study how changes in the topology affect the (global) stability we trigger cascades on two additional network topologies that are derived from the original network: In one case the weights of a randomly chosen pair of links are swapped (*random weight exchange*, RWE) in the other case for a pair of two banks with outgoing links their target nodes are exchanged (*random link exchange*, RLE). See figure 8 for an illustration of the concept.

Both random weight exchange and random link exchange preserve the in- and out-degree sequence of the network but change the node in- and out-strengths. The difference is, that the random weight exchange does not change the creditor-debtor relationship: After applying a random weight exchange to the network a debtor will owe a *new amount* to the *old creditor* while the total amount of debt in the system is held constant. After a random link exchange debtors owe a *new amount* to a *new creditor*.

The dependence of the average cascade size and its inverse the global stability measure g on the threshold value q_{th} is shown in two figures in table 13. We observe the

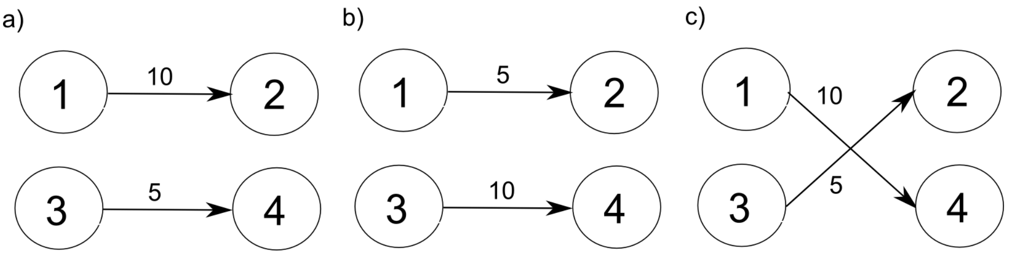


Figure 8: Randomization of topology. a) Original network, b) Random weight exchange, c) Random link exchange.

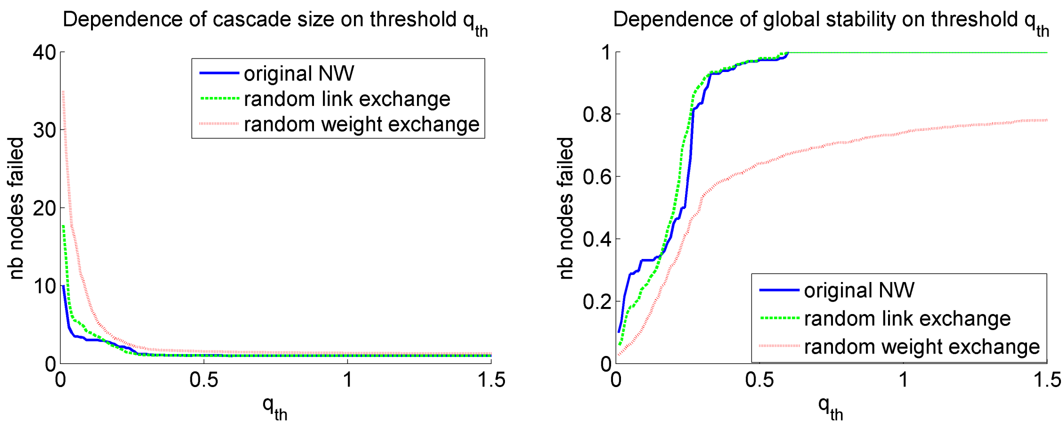


Table 13: Topology and global stability. Left – average size of cascades depending on q_{th}. Right – global stability g depending on q_{th}. Results were obtained by averaging over 10 networks, of which each had undergone 1000 random exchanges of weights or links.

following behavior:

- for low values of q_{th} the original network is the most stable among the three with an average cascade size of 10 followed by the RLE-network and the RWE-network
- as q_{th} increases all three networks become more stable
- global stability reaches maximum (unity) for the original network and the RLE network at $q_{\text{th}} = 0.6$
- the RWE-network does not stabilize at $q_{\text{th}} = 0.6$. Even for values of $q_{\text{th}} \geq 1$, which means that banks are allowed to lose more than their tier 1 capital before they default complete global stability is not reached.

These changes (RLE, RWE) in topology are far from realistic in the real banking network. Why should banks decide to exchange their mutual liabilities at random or pick a new creditor/debtor at random? But even for these rather simple, randomized changes of topology a detailed explaination (e.g. the RWE-network does not stabilize) yet remains to be found. In any case, these results provide first evidence that topology does have a strong influence on stability for the banking network.

3.4. Predicting Stability from Topology

Banks are different: some can default without causing any damage while others ("super-spreaders") can trigger cascades that affect a major part of the financial system. In the framework developed so far, banks which can lead to the default of at least one of their neighbors are called *contagious*, superspreaders are a subset of those. The question now arises of how to identify such banks.

Obviously these banks can be identified through the iterated application of the update rule after an initial perturbation. This lead us to the concept of *global stability* in section 3.3.1.

But is it possible to identify those banks on the basis of their topological properties in the network without introducing a dynamical update rule? More precisely:

> Can the *local stability* (contagiousness and vulnerability) of banks and their impact on *global stability* (magnitude of cascade they can cause) be predicted solely from the topology of the network?

We will use three different concepts from the theory of complex networks that measure the topological "importance" of nodes:

- Using the *backbone-reduction* banks are more important if they remain part of the backbone for lower significance levels α_{th}.
- Using *eigenvector centrality* they are more important if they have a higher centrality value.
- Using *k-cores* banks are more important if they have a higher core number.

These topological concepts are not identical, yet one would suspect that banks that are "important" in one measure will also be classified as important in another.

While these measure only take into account the liability matrix J, local and global stability are determined by additional information: the banks' tier 1 capital t_i and the global default-threshold q_{th}.

In an attempt to give a quantitative answer to the above question an approach using set-overlaps is chosen: Let $N = \{n_i\}$ be the set of nodes that make up the banking-network. Then the topologically most important nodes are a subset of N:

- $B = \{\alpha_i\} \subseteq N$ contains the nodes that are part of the backbone for a given value of α_{th} with $\alpha_i \leq \alpha_{\text{th}}$
- $K = \{k_i\} \subseteq N$ contains the nodes that are part of the k-core for a given threshold k_{th} with $k_i \geq k_{\text{th}}$
- $E = \{e_i\} \subseteq N$ contains the node that have an eigenvector centrality above a threshold e_{th}: $e_i \geq e_{\text{th}}$

In the same way, in order to evaluate *local stability*, the vulnerable and contagious banks[33] are grouped into sets:

[33]Definitions according to section 3.3.1.

- $V = \{v_i\} \subseteq N$ contains the nodes that are classified as vulnerable for a given threshold q_{th}
- $C = \{c_i\} \subseteq N$ contains the nodes that are classified as contagious for a given threshold q_{th}

And finally, to evaluate *global stability* the set $G = \{g_i\} \subseteq N$ contains all banks that can trigger cascades which affect a fraction $g_i \geq g_{\text{th}}$ of the network. For this a threshold $q_{\text{th}} = 0.01$ is chosen

Now the *Jaccard-index* can be used to calculate the overlap of the above sets. For two sets A and B this index is defined as

$$J(A,B) = \frac{A \cap B}{A \cup B} \tag{12}$$

and can take values between 0 (no overlap) and 1 (identical sets).

Before the overlap between those sets can be calculated we have to choose the different threshold values that determine which banks are elements of the respective sets. Instead of arbitrarily choosing five different thresholds ($\alpha_{\text{th}}, k_{\text{th}}, e_{\text{th}}, q_{\text{th}}, g_{\text{th}}$) a different approach is used: For each measure all banks are ranked according to each of the three measures of importance. For example using the backbone-reduction as a measure, banks with low α_{th} are given a high importance ranking.[34] Then the top 10% of nodes according to each measure are chosen as elements of the sets B, K, E, V, C and G.

Figure 9 shows the result of the rankings of all three importance measures, the top 10 % are the first 19 banks. Again we notice an approximate 90-10 relationship. Most of the banks are relatively "unimportant" while a few are highly important.

	E	K	B	C	V	G
E, EV centrality	1	0.63	0.39	0.9	0.65	0.73
K, Core number	0.63	1	0.38	0.56	0.62	0.70
B, Backbone-reduction	0.39	0.38	1	0.39	0.34	0.43
C, Contagious	0.9	0.56	0.38	1	0.58	0.67
V, Vulnerable	0.65	0.63	0.34	0.58	1	0.55
G, Global stability	0.73	0.70	0.43	0.67	0.55	1

Table 14: Jaccard index measuring set-overlap between the top 10% of banks for each measure.

Table 14 contains the computed overlap among the top 10% of banks for each measure, we observe the following:

[34] The numerical measures of importance necessary for the ranking are obtained in the following way: For the backbone-reduction α_{th} is decreased from 1 to 0.01 in steps of 0.01. The importance is measured by the number of steps it takes until the bank is not part of the backbone for α_{th} anymore. Numerical values for vulnerability and contagiousness are obtained similarly by decreasing q_{th} from 0.8 in steps of 0.01 and counting the number of steps it takes for a bank to change from vulnerable to non-vulnerable or from contagious to non-contagious. For the core-number and the eigenvector centrality the direct numerical values are used.

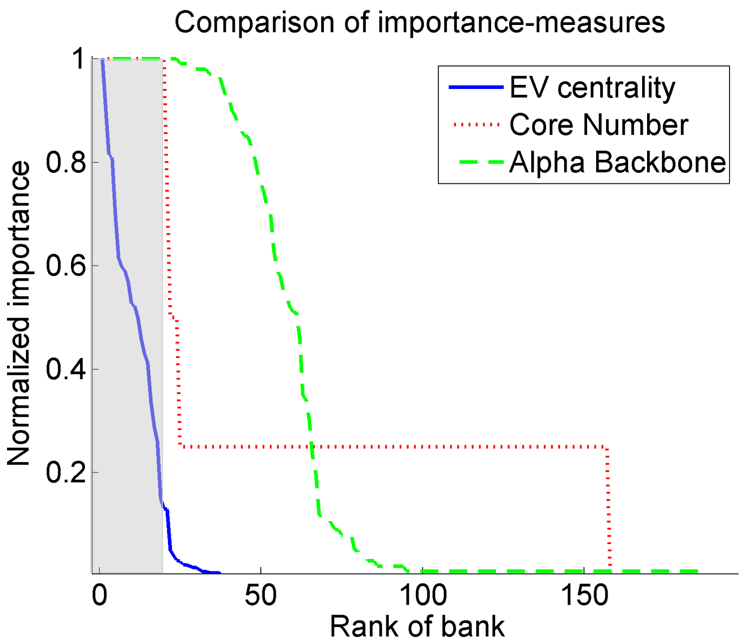


Figure 9: Comparison of the three different measures of importance: eigenvector centrality, backbone-reduction and core number. The shaded area marks the top 10 % for each measure.

- the top 10% of the banks according to the 3 different measures eigenvector centrality, core number and backbone reduction are not identical
- EV centrality is the best indicator for local and global stability with 90% overlap with contagious banks, 65 % with vulnerable and 73 % with super-spreaders
- backbone-reduction is the least accurate indicator of local and global stability

However, these results are still very preliminary. The sensitivity of the overlap on the margin (here: top 10%) has to be checked, also the global stability measure is dependent on q_{th} which was arbitrarily set to 0.01. If the results turn out to hold for other parameter values as well the relationship obtained can be used to single out banks that pose an increased danger to the financial system.

Why does the eigenvector-centrality give best identification of superspreaders? A first guess could be, that while the backbone-reduction and k-core operate on a local, next-neighbor level, the eigenvector is determined by the full matrix and therefore the global topology. So it seems plausible that global importance measures give better estimates of global stability properties. However this only provides a first hint for further, more rigorous analysis.

Some further remarks on the methods used are necessary: calling the exploitation of the set-overlap a “prediction” might be viewed as an exaggeration. The set-overlap is a mere correlation, a causal relationship must be established through more detailed analysis.

4. Discussion and Conclusion

4.1. Summary of Results

What have we accomplished in the understanding of cascades on the empirical banking network? Better answers to the three questions put forward in the introduction can now be given:

1. **What is the network like?**

   After calculating the matrix of directed, netted liabilities J the analysis of the degree-distributions, node-strengths, tier 1 capital and normalized liabilities showed a 90-10 principle: a small number of banks ($\approx 10\%$) are highly connected, own most of tier 1 capital and has an overwhelming contribution the majority to the overall amount of debt in the system.[35] This was indicated by the skewed nature of the distributions and the sharp cutoffs in the complementary cumulative distribution functions.

   The matrix of shortest distances in the network revealed a small, densely connected cluster of banks which was well conntected to the rest of the network. Also it was found, that there exist some banks in the network which have no incoming or no outgoing links.

   The interpretation of these findings in the context of the banking network remains to be done.

2. **How do dynamics and topology influence stability?**

   After defining local stability (by a classification of vulnerable and contagious nodes) and global stability (as the inverse average size of cascades triggered) the influence of the threshold value q_{th} and the topology (random link exchange and random weight exchange) has been evaluated. Avalanches follow a bimodal size-distribution with an "many-or-none" behavior, their size and the number of vulnerable/contagious banks decreases with increasing q_{th}. Global stability g is lowest for random link exchange, and highest for the original network. At $q_{\text{th}} = 0.6$ the original network and the RLE-network become locally and globally stable. For the random weight exchange, global stability g does not reach its maximum within this range of q_{th}.

3. **Can one identify "super spreaders" based on their topological features?**

   It turns out that the three different methods for defining topologically "important" nodes – backbone-reduction, eigenvector centrality and core number – do not identify the same banks. Nevertheless the picture of a few very important and many less important nodes is confirmed. Using set-overlaps as a measure shows that dangerous banks which can trigger large global cascades are best identified by the eigenvector-centrality measure. This is not a trivial result because in addition to topological features, stability also depends on the choice of the dynamical model and its underlying additional parameters.

[35] Note, however that the banks that are in the top 10 % for one property need not to be the same that are in the top 10 % for another property.

Connection to existing cascading failure literature In a number of studies of cascading failures on complex networks where power-laws in the cascade-distributions are found, these systems are said to exhibit signs of self organized criticality. Why does our model not show such behaviour?

The reason might be, that in models of cascading failures that exhibit SOC a conserved quantity is redistributed after a node-failure. In our model there is no (obvious) conserved quantity.

4.2. Open Questions and Possible Extensions

A number of questions remain open:

- Does a power-law describe the in- and out-degree distribution well? Does the network-structure follow a well known model or exhibit the same features as other empirical networks? – This is an important question, if there is a model describing the network one might for example be able to continuously vary model-parameters to study "what-if questions".
- The 90-10 principle seems to be ubiquitous (degrees, strengths, tier1, backbone, k-core, eigenvector-centrality). But are the top 10 % banks always the same?
- Why does the random weight exchange-NW not reach $g = 1$ within range of q_{th} suggested by local stability?

Possible extensions are:

- The interpretation of all the quantities obtained for the network in the context of economics. How can for example q_{th} be set by regulators?
- The 90-10 structure and the matrix of distances suggests a highly connected core and a sparsely connected periphery. Using appropriate measures one could try to further quantify the core-periphery structure.
- To simulate effects of a liquidity crunch on the banking network[36] one could make q_{th} dependent on the number of banks that have failed.
- As Haldane and May [26] point out *homogeneity* among banks might increase system fragility. They suggest that more *modularity* and *diversity* in the financial system can increase its stability. A recent article [13] also provides a number of suggestions. Translating these suggestions into concrete changes of the topology of the current model and testing their influence on the global stability will be very interesting.

[36] as for example done in [21]

4.3. Conclusion

This thesis has contributed to the understanding of complex economic networks in two ways:

- The banking network we studied has been characterized as a a network with highly heterogenous node properties and strong core-periphery structure,
- It has been demonstrated that on this network global stability can be estimated through topological measures of importance without considering detailed dynamics of cascading failures.

It is difficult to ascertain whether the real banking-network is globally stable. However with some confidence we can assume that in the network we studied, there are institutions whose default can lead to a collapse of a major part of the financial system. Saving these banks in the event of financial turmoil is therefore necessary as long as the financial system maintains its current topology. But changes to the network through policy measures which limit for example the size and activity of banks will hopefully make banks that are “too big to fail” a thing of the past.

A. Appendix

A.1. (Self-Organized) Criticality and Complexity

Physicists are sometimes said to be obsessed with power laws and indeed in many recent publications scientists try to identify power law-relationships in data. Why is that? As Cosma Shalizi, a physicist/computer scientist/mathematician remarks, physicists care about power laws because “they’re a sign of something interesting and complicated happening”.[37]

Before physicists extended their research to living and manmade systems the occurence of power laws was studied in the context of phase-transitions and critical phenomena of inanimate systems like fluids or magnets. One candidate that explains the occurence of power law relationships is self-organized criticality which tries to explain why systems arrive at their critical point even in the absence of external control. In this section a brief overview of these two important concepts *criticality* and *self-organized criticality* will be given.

Complex networks played a central role in this thesis. In section 2 we introduced networks. But what is meant by “complex”? *Complexity* is now a widely used buzzword in many areas of natural sciences as well as economics and we will attempt to give a short explaination of what complexity is.

Criticality “Criticality refers to the behavior of extended systems at a phase transition where observables are scale free, that is, no characteristic scales exist for the observables. At a phase transition, the many constituent ’parts’ give rise to macroscopic phenomena that can not be understood by considering the laws obeyed by a single part alone”[38] Well known examples of such phase transitions in physics are[39]

- the liquid-gas transition of a fluid
- the paramagnetic-ferromagnetic transition of certain materials at the Curie point
- the occurence of superfluidity below a critical temperature
- the occurence of superconductivity below a critical temperature
- the occurence of a fully connected cluster in a percolation model on a lattice

The *scaling hypothesis* states, that near the critical point of a system the correlation length η (which takes on different meanings depending on the system that is being referred to) is the only characteristic length of the system [29]. For example in the paramagnetic-ferromagnetic transition the correlation length can be intuitively understood as the average size of a cluster of spins with same orientation. Experimental observation suggests that the correlation length η diverges as a system approaches the critical point.[40] Therefore, following the scaling hypothesis, the system will no longer

[37] `http://www.cs.umich.edu/~crshalizi/notebooks/power-laws.html`
[38] [15], preface.
[39] [29], chapter 16.
[40] This can also be shown more rigorously using the renormalization group.

have a characteristic length-scale. In mathematical terms this means, that the correlation function Γ will follow a power law:

$$\Gamma \propto x^{-p}$$

One of the most striking features of the theory of phase transitions is the fact that the exponent p is only determined by the dimension of the system. Also other observable thermodynamic properties like the heat capacity or susceptibility do not depend on microscopic details as the system approaches the critical point. Therefore even grossly simplifying descriptions like the Ising model often give good estimations of the system properties near its critical point.

Self-Organized Criticality However in experiments it turns out, that the tuning of the system-parameters (e.g. temperature, pressure) towards the critical point is rather difficult which makes it very unlikely for power laws observed in nature to be caused by critical phenomena. But in the late 80ies the Danish theoretical physicist Per Bak came up with the concept of **self-organized criticality**. Which states that through self-organization nonequilibrium systems tend towards their critical point.

The most popular illustration of this concept is that of a sandpile on which sand is trickling. This sandpile is a driven nonequilibrium, open system because the trickling sand is constantly changing its state. From time to time the angle of repose will be too steep and an avalanche of sand will bring the system back into a new equilibrium.

In an extremely simple model of a sandpile Bak, Tang and Wiesenfeld [5] were able to show, that the size-distribution of those avalanches follows a power law. In the model the sandpile consists of sites on a regular grid with four nearest neighbors. The trickling sand is modelled by randomly increasing a variable $z(x, y)$ at each of the sites by 1. If z exceeds a critical value $z > K$ it is reset to $z(x, y) - 4$ and z of its four neighbors is increased by one, mimicking the redistribution of sand grains. The size of an avalanche is then measured by the number of sites that topple until the system reaches its equilibrium. The observation of scale free (power power law) distributions of the avalanches was quite robust: even when details of the model such as the place at which the sand was placed or the critical angle of repose were changed, the qualitative overall behaviour remained the same. The occurance of an avalanche is seen as a critical event during which the system rapidly changes into a new state, large avalanches occur for the same reasons small ones do. The system is not tuned towards the critical state from the out side but rather approaches a critical state in which avalanches occur through self-organization.

In his book with the modest title "‘How Nature Works"’ [4] Per Bak suggested that self-organized criticality might be a mechanism which is responsable for many of the power laws observed in nature in areas as different as earthquakes and evolution.

But for seeking signs of self-organized criticalty everywhere he has also received some criticism, Shalizi remarked ”Prof. Per Bak and his disciples sometimes seem to want to claim everything obeying a power law distribution for self-organized criticality.”[41] Nevertheless his contributions to the field of critical phenomena are widely recognized and self-organized criticality remains a field of interest to many physicists.

[41] http://cscs.umich.edu/~crshalizi/reviews/self-organizing-economy/

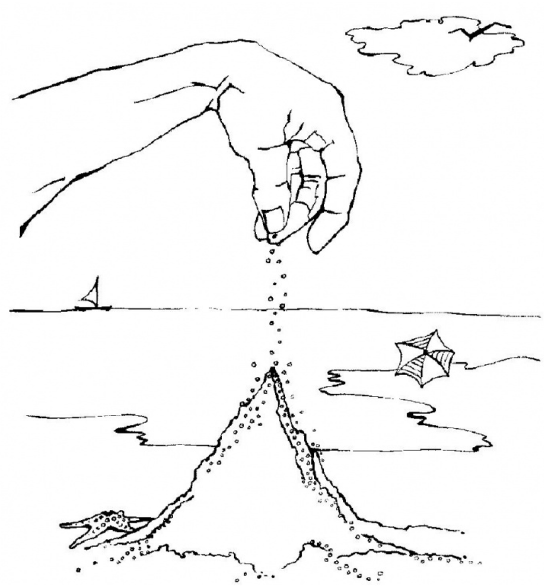


Figure 10: Self organized criticality. Illustration of the sandpile model proposed by Per Bak and colleagues, the sandpile is a nonequilibrium system that exhibits self organized critical events (avalanches of sand). From [4].

New of self-organized criticality with different boundary conditions, different update rules and other topologies, e.g. [62] are now studied.

Complexity

> "I think the next century will be the century of complexity."
>
> — Stephen Hawking, Complexity Digest 2001/10

> "Every decade or so, a grandiose theory comes along, bearing similar aspirations and often brandishing an ominous-sounding C-name. In the 1960s it was cybernetics. In the 1970s it was catastrophe theory. Then came chaos theory in the '80s and complexity theory in the '90s."
>
> — Steven Strogatz, Sync, 2003 [57]

What is complexity, what do people mean when they say that a system is complex? A solid, axiomatic foundation of what constitutes a complex system has not been developed yet, but finding concepts or even laws that might govern systems as different as the human brain, the world economy, mega-cities or novel fluids recently became a fashionable endeavour in various fields of science.

Complexity is studied not only by physicists, but also by computer scientists, economists [8], biologists and urban planners [7].[42]

[42] [35] provides a good introduction.

In their book “Complexity and Criticality” [15] Christensen and Moloney define from the physicist’s perspective[43]:

> “For our purposes, *complexity* refers to the repeated application of simple rules in systems with many degrees of freedom that gives rise to emergent behaviour not encoded in the rules themselves.”

Cellular automata like the update-rule of the banking network are examples of such simple systems exhibiting unexpectedly complex behaviour. More famously Conway’s Game of Life and the one-dimensional elementary cellular automata introduced and studied by Stephen Wolfram are systems that a small set of very simple rules, yet produce a remarkable variety of patterns and “behaviour”. This led scientists to conclude that a whole new way of doing science is necessary.

Exemplary for the “old way” of science was physics, the most precise of the natural sciences. It had brought the *reductionist approach*[44] to a remarkable success and was able to describe and predict the behaviour of systems as small as atoms and as large as the universe.

Proponents of a complexity science argue that it is not feasible anymore to study complicated systems like the ones mentioned using the reductionist approach.

A.2. Economics, Physics and Econophysics

In the most general sense science is problem solving, fields of science differ only in the problems they are trying to solve and the methods they employ.

The goal of this short chapter is to highlight through a few examples *how economics has been influenced by physics and how econophysics fits in between economics and physics*. It is only supposed to serve as an idiosyncratic collection of ideas, facts and references for further exploration of this question.

The first chapters of Eric Beinhocker’s “The Origin of Wealth” [8] and also Robert Heilbroner’s classical work “The Worldly Philosophers” [27] give much more comprehensive answers and provide more references for exploration.

Economics and Physics Economics and physics by themselves are two established scientific disciplines. While physics seeks to explain the world of natutural phenomena, economics tries to explain (some aspects of) human behavior. Both disciplines have started with qualitative investigations of their subject and developed more and more sophisticated quantitative methods. While the mathematization of physics is associated with the publication of Newton’s Principia and the development of calculus in the late 18th century the mathematization of economics did not take place until the 19th century [27]. The mathematization of economics is well illustrated for example in [8].

Most notably economists borrowed quite a number concepts from mechanics and statistical physics: The set of ideas and theories often refered to as *mainstream economics* relies heavily the existence of market-*equilibria*. Market *forces* bring lead to the

[43]Quantum mechanic Seth Lloyd of MIT has compiled a list of over 30 definitions `http://web.mit.edu/esd.83/www/notebook/Complexity.PDF`

[44]Understanding complicated systems by dividing them into smaller parts, studying those parts extensively and then putting the smaller parts back together to understand the larger system.

readjustment of prices and let the market return to *equilibrium* when it is perturbed by external events like tax-cuts, change in interest rates or increased government spending. In mathematical finance a model of derivative pricing developed by Black and Scholes, based on *stochastic equations of motion* which were originally devised for the study of Brownian motion received a Nobel prize in economics in 1997.

Also influential economists have studied under advisors who were physicists themselves: The American physicist Josiah Williard Gibbs who contributed substantially to the development of thermodynamics was thesis advisor to Vilfredo Pareto, one of the first economists to apply mathematical analysis in economics. Edwin Bidwell Wilson another student of Gibbs was mentor to Paul Samuelson who is widely accepted to be one of the most influential American economists of the last century. His textbook "Economics: An Introductory Analysis" widely makes use of the concept of equilibrium, stability and comparative statics (a concept originally derived from the Chatelier-Principle in chemical thermodynamics [61]).

Econophysics What is econophysics? In one of the earliest textbooks by Mantega and Stanley [44] the following description is given:

> Econophysics describes the "activities of physicists who are working on economics problems to test a variety of new conceptual approaches deriving from the physical sciences".

The ideal gas law is often given as an example that illustrates the power of the machinery of statistical physics. This law is derived from considerations quite independent of the microscopic description of the gas. The fact that such simple macroscopic laws emerge from microscopically untracable behaviour motivated physicists to look for the application of physical concepts in economics long before the financial crisis of 2008.

In fact the term econophysics was first used by H. E. Stanley on a conference in 1995 in Kolkata.[45] Since then econophysics has gained lots of momentum in the physics community with incrasing publications in journals[46], conferences devoted entirely to econophysics and also some interest by the media[47].

Some exemplary economic phenomena tackled by econophysics include explanations of [53]:

- the distribution of price-fluctuations on the stock market (of course following a power law)
- the distribution of income and wealth across a society (also following a power law)
- the propagation of bank defaults in the banking network and more generally cascades in complex networks

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[45] [53] p. 5

[46] Recent econophysics papers can be found on `arxiv.org` – sections: Physics and Society, Condensed Matter and Statistical Mechanics `http://www.hep.yorku.ca/menary/econophysics/` contains a huge collection of links related to econophysics

[47] For example `http://www.zeit.de/2011/33/CH-Oekonophysik` and more at the chair of Dirk Helbing at ETHZ `http://www.soms.ethz.ch/`

A.3. MATLAB-Routines

The following routines were written for the simulation:

adj2pajek_old
Creates a file that can be read in pajek from an adjaceny matrix, the size of vertices and two properties that determine their color (e.g. part of the backbone) can be chosen

bb_extraction
Calculates the backbone of a directed, weighted network represented by an adjacency matrix for a given significance level. It also returns degree- and strength-distribution and statistics mentioned in 3.1.3

mc_stability
Calculates the q_{th}-stability relationship for either a given network or a RLE/RWE-network based on the given network. Data to calculate mean and standard deviation of the relationship for RWE/RLE-network is also returned.

node_coloring
Contains a set of scripts that classify nodes into contagious and vulnerable nodes, create the plots from section 3.3.1 and the labeling of backbone-nodes used in plot 4

nw_statistics
Contains a set of scripts that calculate most of the measures used to characterize the network in section 3.2.

randomize_nw
Given the adjacency matrix of a directed network this function returns a new network with randomly exchanged links or weights.

update_nw
Evaluates the update rule 4 on an arbitrary network for a given initial condition, stops if equilibrium is reached and returns equilibrium state.

vital_few_trivial_many
Contains a set of scripts that assign measures of importance to the nodes of the banking network as described in section 3.4.

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