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Dr. Clive Emary http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/

10. Übungsblatt zur Statistiche Physik I

The Landau theory of phase transitions

Abgabe: Mittwoch, 15th July, bis 16:00 Uhr, Raum E-W 705

Exercise 28 (5 points): *Landau theory*
The Landau theory of second-order phase transitions posits a Gibbs free energy close to the phase transition of the form

$$G(P,T,\eta) = G_0(P,T) + a(T-T_c)\eta^2 + B\eta^4 - \eta h V,$$

where G_0 is the value of the disordered phase, η is the order parameter, h is an external field and $a(P) > 0$ and $B(P) > 0$ are (pressure-dependent) coefficients.

- Consider first the case with no external field, $h = 0$, and find the equilibrium values of the order parameter above and below the critical temperature. Explain how this models a second-order phase transition and calculate the critical exponent β.
- Calculate the susceptibility

$$\chi = \lim_{h\to 0} \left.\frac{\partial \eta}{\partial h}\right|_{T_c},$$

in both phases and calculate the critical exponents γ and γ'.

Exercise 29 (5 points): *Ginzburg-Landau theory*
For an inhomogeneous system near the critical point, the Gibb's free energy can be expanded for long wavelengths as

$$G = \int d^3\mathbf{r} \left\{ a(T-T_c)\eta^2(\mathbf{r}) + B\eta^4(\mathbf{r}) + D|\boldsymbol{\nabla}\eta(\mathbf{r})|^2 - Vh(\mathbf{r})\eta(\mathbf{r}) \right\}.$$

- Show that, in the harmonic approximation (equivalent to setting $B \to 0$), the susceptibility to perturbations with wavevector $\mathbf{k}$ is given by

$$\chi_{\mathbf{k}} = \frac{V}{2\left\{a(T-T_c) + D|k|^2\right\}}; \qquad T > T_c,$$

and thus the size of the fluctuations in the order parameter is

$$\langle |\delta\eta_{\mathbf{k}}|^2 \rangle = \frac{T_c}{2\left\{a(T-T_c) + D|k|^2\right\}}; \qquad T > T_c. \tag{1}$$

[Hint: Use equipartition or the fluctuation-dissipation theorem].

- Show that, in general, the real-space correlation function $g(\mathbf{r}) = \langle \delta\eta(0)\delta\eta(\mathbf{r}) \rangle$ is given by the Fourier transform

$$g(\mathbf{r}) = \sum_{\mathbf{k}} \langle |\delta\eta_{\mathbf{k}}|^2 \rangle e^{i\mathbf{k}\cdot\mathbf{r}}.$$

- Calculate $g(\mathbf{r})$ for the specific fluctuations of Eq. (1). Perform the integration in the complex plane.

Bitte Rückseite beachten!⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/
- **Vorlesung:** Montags & Donnerstags, 14:15 bis 15:45, E-W 202
- **Literatur:**
  - L. D. Landau and E. M. Lifshitz, Statistical Physics, Part 1
  - H. E. Stanley, Introduction to Phase transitions and Critical Phenonena
  - L. E. Reichl, A Modern Course in Statistical Mechanics
  - D. A. McQuarrie, Statistical Mechanics
  - F. Schwabl, Statistische Mechanik
  - M. Kardar, Statistical Physics of Particles & Statistical Physics of Fields
  - H. B. Callen, Thermodynamics and an Introduction to Thermostatistics
- **Übung:** Donnerstags, 10:15 bis 11:45, E-W 733
- **Scheinkriterien:** 50% der Punkte aus den Übungszetteln (Zweierabgabe), aktive Teilnahme an den Tutorien
- **Sprechstunden:**

  Prof. Dr. H. Stark: Fr. 11:30 - 12:30,E-W 709
  Dr. C. Emary: Di, 16:00 - 17:00 Uhr, E-W 705