June 4, 2009

Technische Universität Berlin
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Dr. Clive Emary http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/

6. Übungsblatt zur Statistiche Physik I

Hydrodynamic equations, Energy fluctuations

Abgabe: Mittwoch, 10th June, bis 16:00 Uhr, Raum E-W 705

Exercise 17 (7 points): *Linearised hydrodynamic equations*
Consider the hydrodynamic equations as derived in the lectures:

$$\begin{aligned}\frac{\partial}{\partial t}n &= -\boldsymbol{\nabla}\cdot(n\boldsymbol{u})\\ mn\frac{d}{dt}\boldsymbol{u} &= -\boldsymbol{\nabla}(nk_BT)+\eta\nabla^2\boldsymbol{u}+\frac{1}{3}\eta\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{u})\\ \frac{d}{dt}T &= \frac{2\kappa}{3nk_B}\nabla^2T-\frac{2}{3}T\boldsymbol{\nabla}\cdot\boldsymbol{u}+\frac{2}{3nk_B}T'_{ij}\nabla_iu_j.\end{aligned}$$

- Set $n=\bar{n}+\delta n$, $T=\bar{T}+\delta T$ and $\boldsymbol{u}=0+\boldsymbol{u}$, with equilibrium values $\bar{n}$ and $\bar{T}$, and linearise the hydrodynamic equations in terms of δn, δT and $\boldsymbol{u}$.
- For harmonic variations, $e^{i(\boldsymbol{k}\cdot\boldsymbol{q}-\omega t)}$, the eigenmodes of the system are given by the solutions of the matrix equation

$$\omega\begin{pmatrix}\delta n\\ u_\alpha\\ \delta T\end{pmatrix}=\mathbf{M}(\mathbf{k})\begin{pmatrix}\delta n\\ u_\beta\\ \delta T\end{pmatrix}. \tag{1}$$

  Find matrix the $\mathbf{M}(\mathbf{k})$. Note: there are three velocity components, u_α, so $\mathbf{M}(\mathbf{k})$ is actually a 5×5 matrix. However, it can be specified with just three equations.
- In zeroth-order hydrodynamics, we set $\mu=\kappa=0$. In this case, find the five solutions of Eq. (1). Give eigenfrequencies and eigenvectors and discuss the character of the solutions.
- With μ and κ finite (first-order hydrodynamics), find the eigenmode frequencies correct to second order in the wavevector, k^2. Discuss the results.

Exercise 18 (3 points): *Energy fluctuations in the canonical ensemble*
Show that $\ln\mathcal{Z}(\beta)$ can be thought of as a "generating function" for the cumulants of the internal energy of the canonical ensemble, i.e.,

$$\langle U_S^n\rangle_c=(-1)^n\frac{\partial^n}{\partial\beta^n}\ln\mathcal{Z}(\beta).$$

Use this expression to relate the second cumulant of the internal energy and the heat capacity at constant volume.

Bitte Rückseite beachten!⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/
- **Vorlesung:** Montags & Donnerstags, 14:15 bis 15:45, E-W 202
- **Literatur:**
  - D. A. McQuarrie, Statistical Mechanics
  - L. E. Reichl, A Modern Course in Statistical Mechanics
  - F. Schwabl, Statistische Mechanik
  - M. Kardar, Statistical Physics of Particles & Statistical Physics of Fields
  - M. Plischke and B. Bergersen, Equilibrium Statistical Physics
  - H. B. Callen, Thermodynamics and an Introduction to Thermostatistics
- **Übung:** Donnerstags, 10:15 bis 11:45, E-W 733
- **Scheinkriterien:** 50% der Punkte aus den Übungszetteln (Zweierabgabe), aktive Teilnahme an den Tutorien
- **Sprechstunden:**

  Prof. Dr. H. Stark: Fr. 11:30 - 12:30,E-W 709
  Dr. C. Emary: Di, 16:00 - 17:00 Uhr, E-W 705