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Technische Universität Berlin
Institut für Theoretische Physik
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Dr. Clive Emary http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/

7. Übungsblatt zur Statistiche Physik I

Ensembles

Abgabe: Mittwoch, 17th June, bis 16:00 Uhr, Raum E-W 705

Exercise 19 (3 points): *Phase space volume for noninteracting particles*
Compute $\Omega(U)$, the number of states with energy less than or equal to U for a gas of N noninteracting particles in a box of volume V.

Exercise 20 (1 points): *Saddle-point approximation*
Show that for twice-differentiable function $f(x)$ with a unique global maximum at x_0 we may approximate

$$\int_a^b e^{Nf(x)}dx \approx \sqrt{\frac{2\pi}{N|f''(x_0)|}}e^{Nf(x_0)}.$$

as $N \to \infty$ limit with $a < x_0 < b$.

Exercise 21 (2 points): *Microcanonical Entropy*
The total number of configurations with total energy U of an isolated box divided in two compartments can be written as

$$g(U) = \int dU_1 \exp\left[\frac{S_1(U_1) + S_2(U_2)}{k_B}\right],$$

with U_i and S_i the internal energy and entropy of compartment $i = 1, 2$ such that $U = U_1 + U_2$. Use the result of Ex. 20 to show that, in the thermodynamic limit, the total entropy of the system is

$$S(U) = S_1(U_1^*) + S_2(U - U_1^*),$$

where U_1^* is the value of U_1 that maximises the sum of individual entropies $S_1(U_1) + S_2(U - U_1)$.

Exercise 22 (3 points): *Equipartition theorem*
Prove the equipartition theorem

$$\langle x_k \frac{\partial H}{\partial x_k}\rangle = k_B T; \qquad x_k = q_k, p_k,$$

for the canonical ensemble with Hamiltonian H.
Given that the Hamiltonian of a non-ideal gas may be written as

$$H = \sum_i \left(\frac{1}{2m}\boldsymbol{p}_i^2 + V_{\text{walls}}(\boldsymbol{q}_i)\right) + V_{\text{int}}(\{\boldsymbol{q}_i\}),$$

with V_{walls} the confining potential of the walls and V_{int} describing interactions between the particles, use the equipartition theorem to derive the Virial equation of state:

$$PV = Nk_BT - \frac{1}{3}\sum_{i=1}^{N}\langle \boldsymbol{q}_i \cdot \frac{\partial V_{\text{int}}(\{\boldsymbol{q}_i\})}{\partial \boldsymbol{q}_i}\rangle.$$

Bitte Rückseite beachten!⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/
- **Vorlesung:** Montags & Donnerstags, 14:15 bis 15:45, E-W 202
- **Literatur:**
  - D. A. McQuarrie, Statistical Mechanics
  - L. E. Reichl, A Modern Course in Statistical Mechanics
  - F. Schwabl, Statistische Mechanik
  - M. Kardar, Statistical Physics of Particles & Statistical Physics of Fields
  - M. Plischke and B. Bergersen, Equilibrium Statistical Physics
  - H. B. Callen, Thermodynamics and an Introduction to Thermostatistics
- **Übung:** Donnerstags, 10:15 bis 11:45, E-W 733
- **Scheinkriterien:** 50% der Punkte aus den Übungszetteln (Zweierabgabe), aktive Teilnahme an den Tutorien
- **Sprechstunden:**

  Prof. Dr. H. Stark: Fr. 11:30 - 12:30,E-W 709
  Dr. C. Emary: Di, 16:00 - 17:00 Uhr, E-W 705