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Technische Universität Berlin
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Dr. Clive Emary http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/

9. Übungsblatt zur Statistiche Physik I

Van der Waal's gas

Abgabe: Mittwoch, 8th July, bis 16:00 Uhr, Raum E-W 705

Exercise 25 (2 points): *Equation of state*
Show that the Helmholtz free energy

$$F(T,V,N) = -Nk_BT\left(1+\ln\left(\frac{V-Nb}{N\Lambda^3}\right)\right) - \frac{aN^2}{V},$$

with Λ the thermal de Broglie wavelength, gives rise to the van der Waal's equation of state for a non-deal gas

$$\left(P + a\frac{N^2}{V^2}\right)(V-Nb) = Nk_BT.$$

What is the heat capacity C_V of the gas?

Exercise 26 (3 points): *Critical point*
Determine the temperature T_c, pressure P_c, and volume V_c of a van der Waal's gas at its critical point. Use these results to rewrite the equation of state in terms of reduced variables, e.g. $P_r \equiv P/P_c$, and comment on the universality of the van der Waal's equation.

Exercise 27 (5 points): *Critical exponents*
Systems close to a critical point are often described by a set of *critical exponents*. Calculate the following exponents for the van der Waal's gas

- Critical isotherm (δ)

  $$(P_r - 1) = \mathcal{D}|\rho/\rho_c - 1|^\delta \mathrm{sgn}(\rho-\rho_c); \qquad T = T_c,$$

  with $\rho = N/V$ and $\rho_c = N/V_c$.

- Disordered phase compressibility (γ)

  $$\kappa_T/\kappa_T^0 = \mathcal{C}\epsilon^{-\gamma}(1+\ldots); \qquad T > T_c,\ \rho = \rho_c$$

  with κ_T^0 the compressibility of an ideal gas at $T = T_C$ and $V = V_c$. Here, we have defined $\epsilon = T_r - 1$.

- Disordered phase specific heat (α)

  $$C_V = \mathcal{A}\epsilon^{-\alpha}(1+\ldots); \qquad T > T_c,\ \rho = \rho_c$$

- Order parameter (β)

  $$(\rho_{\mathrm{liquid}}(T) - \rho_{\mathrm{gas}}(T))/2\rho_c = \mathcal{B}(-\epsilon)^\beta$$

  The calculation of this exponent is more complicated because it is defined for the ordered phase and thus the Maxwell construction is required to obtain physically meaningful results. Since the maths is a bit annoying, you can just outline the solution. **A bonus 5 points** for actually performing the calculation.

Bitte Rückseite beachten!⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/
- **Vorlesung:** Montags & Donnerstags, 14:15 bis 15:45, E-W 202
- **Literatur:**
  - H. E. Stanley, Introduction to Phase transitions and Critical Phenonena
  - L. E. Reichl, A Modern Course in Statistical Mechanics
  - F. Schwabl, Statistische Mechanik
  - M. Kardar, Statistical Physics of Particles & Statistical Physics of Fields
  - H. B. Callen, Thermodynamics and an Introduction to Thermostatistics
- **Übung:** Donnerstags, 10:15 bis 11:45, E-W 733
- **Scheinkriterien:** 50% der Punkte aus den Übungszetteln (Zweierabgabe), aktive Teilnahme an den Tutorien
- **Sprechstunden:**

  Prof. Dr. H. Stark: Fr. 11:30 - 12:30,E-W 709
  Dr. C. Emary: Di, 16:00 - 17:00 Uhr, E-W 705