Prof. Dr. Tobias Brandes
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4. Übungsblatt – Theoretische Festkörperphysik I,II

Abgabe: Fr. 14.05.2010 bis 12:00 Uhr, EW705.
Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

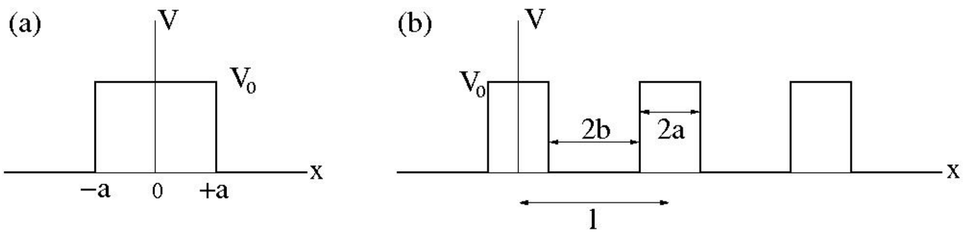
Aufgabe 9 (12 Punkte): *Orthonormalität der Blochfunktionen*

Zeigen Sie die Orthonormalität der Blochfunktionen, $\phi_{\lambda\mathbf{k}}(\mathbf{r}) = V^{-1/2} u_{\lambda\mathbf{k}}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$:

$$\int d^3r \phi^*_{\lambda\mathbf{k}}(\mathbf{r})\phi_{\lambda'\mathbf{k}'}(\mathbf{r}) = \delta_{\lambda,\lambda'}\delta_{\mathbf{k},\mathbf{k}'},$$

mit Hilfe aus der Vorlesung bekannter Eigenschaften.

Aufgabe 10 (15 Punkte): *The Kronig-Penney model*



The solution of the 1D Schrödinger equation for a rectangular barrier of height V_0 and width $2a$ (Fig. (a)) for energy $E < V_0$ can be written

$$\psi(x) = \begin{cases} Ae^{iqx} + Be^{-iqx} & x < -a \\ Ce^{-Qx} + De^{Qx} & -a < x < a \\ Fe^{iqx} + Ge^{-iqx} & x > a \end{cases},$$

with $\hbar q = \sqrt{2mE}$ and $\hbar Q = \sqrt{2m(V_0 - E)}$ for a particle of mass m.

- By considering the boundary conditions at $x = \pm a$, show that the coefficients outside the barrier are related as

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \alpha_1 + i\beta_1 & i\beta_2 \\ -i\beta_2 & \alpha_1 - i\beta_1 \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix},$$

  with $\alpha_1 + i\beta_1 = \left(\cosh 2Qa + \frac{i\epsilon}{2}\sinh 2Qa\right) e^{2iqa}$, $\beta_2 = \frac{\eta}{2}\sinh 2Qa$, $\epsilon = (Q^2 - q^2)/qQ$, and $\eta = (Q^2 + q^2)/qQ$.

Now consider the periodic version of the same potential (Fig. (b)) with valley width $2b$ and unit-cell length $l = 2(a + b)$. In the nth valley, the wavefunction can be written

$$\psi(x) = A_n e^{iq(x-nl)} + B_n e^{-iq(x-nl)},$$

for $a - l < x - nl < -a$.

- Shows that coefficients of successive valleys are related as

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = P \begin{pmatrix} A_n \\ B_n \end{pmatrix}; \quad \text{with} \quad P = \begin{pmatrix} (\alpha_1 - i\beta_1)e^{iql} & -i\beta_2 e^{iql} \\ i\beta_2 e^{-iql} & (\alpha_1 + i\beta_1)e^{-iql} \end{pmatrix}.$$

Bitte Rückseite beachten!⟶

4. Übung TFP SS10

- Show that the normalisability of the wavefunction implies that eigenvalues of matrix P can be written $p_\pm = e^{\pm ikl}$ with real parameter k fulfilling

$$\cos kl = \cosh 2Qa \cos 2qb + \tfrac{\epsilon}{2} \sinh 2Qa \sin 2qb$$

  for $E < V_0$.

- Show that the analogous condition for $E > V_0$ is

$$\cos kl = \cos 2q'a \cos 2qb - \frac{{q'}^2 + q^2}{2q'q} \sin 2q'a \sin 2qb$$

  with $\hbar q' = \sqrt{2m(E - V_0)}$.

- Plot the resulting dispersion relation, E vs. k, for parameters $a = b = 1$ and $2mV_0/\hbar^2 = \pi^2/4$. Discuss what happens for $2qb = N\pi$, with N =integer, for $E < V_0$, and at $2q'a + 2qb = N\pi$ for $E > V_0$.