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8. Übungsblatt – Theoretische Festkörperphysik I,II

Abgabe: Fr. 18.06.2010 bis 12:00 Uhr, EW705.
Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 19 (10 Punkte): *Displacement operators*
The single-mode displacement operator is defined as $D(\alpha) \equiv e^{\alpha a^\dagger - \alpha^* a}$ with displacement parameter $\alpha \in \mathbb{C}$.

- Show that the product of two displacement operators is given by

$$D(\alpha)D(\beta) = D(\alpha+\beta)e^{i\Im(\alpha\beta^*)}.$$

- Show that the expectation value of the displacement operator in thermal equilibrium is

$$\langle D(\alpha)\rangle_{\text{eq}} = e^{-\frac{1}{2}|\alpha|^2 \coth(\frac{\beta\Omega}{2})},$$

  with Ω, the mode-frequency. Hint: The Laguerre polynomials and their generating function may be of use.

Aufgabe 20 (10 Punkte): *Jaynes-Cummings model*
A simple model of the interaction between a two-level system (atom) and a single bosonic mode is furnished by the Hamiltonian

$$\mathcal{H}_{\text{JCM}} = \omega_0\sigma_z + \omega b^\dagger b + g(\sigma_+ b + \sigma_- b^\dagger),$$

with $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$. Derive an expression for the atomic inversion $W(t) = \langle\sigma_z(t)\rangle$ as a function of time, given that the atom starts in the excited state and the field starts in a coherent state with photon probability distribution

$$P_n(0) = \rho_{nn}(0) = \langle n\rangle^n e^{-\langle n\rangle}/n!$$

with mean photon number $\langle n\rangle$. For simplicity, consider the system to be on resonance, $\omega = \omega_0$. Plot the results for several values of $\langle n\rangle$ and g ($\omega = \omega_0 = 1$). You should see a distinct "collapse" and "revival" of quantum oscillations. Either numerically or analytically, try to estimate the dependence on g and $\langle n\rangle$ of the time taken to collapse and to revive for $\langle n\rangle \gg 1$.