Prof. Dr. Tobias Brandes
Dr. Clive Emary

7. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht

Abgabe: Fr. 06.07.2012 10:00-12:00, Uhr in der Vorlesung
Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 17 (20 Punkte): *Stochastic Schrödinger equation*

A generic master equation of Lindblad form

$$\frac{d}{dt}\rho(t) \;=\; -i[H,\rho] + \textstyle\sum_\alpha \gamma_\alpha \left[L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\left\{ L_\alpha^\dagger L_\alpha, \rho \right\} \right]$$

can be modelled with the stochastic Schrödinger equation (SSE)

$$\begin{aligned} |d\Psi\rangle \;=\; & \left[-iH - \frac{1}{2}\sum_\alpha \gamma_\alpha L_\alpha^\dagger L_\alpha + \frac{1}{2}\sum_\alpha \gamma_\alpha \langle\Psi| L_\alpha^\dagger L_\alpha |\Psi\rangle \right] |\Psi\rangle dt \\ & + \sum_\alpha \left(\frac{L_\alpha|\Psi\rangle}{\sqrt{\langle\Psi| L_\alpha^\dagger L_\alpha |\Psi\rangle}} - |\Psi\rangle \right) dN_\alpha \end{aligned}$$

where the Poisson increments dN_α satisfy $dN_\alpha dN_\beta = \delta_{\alpha\beta} dN_\alpha$ and $\mathcal{E}(dN_\alpha) = \gamma_\alpha \langle\Psi| L_\alpha^\dagger L_\alpha |\Psi\rangle$, with $\mathcal{E}(x)$ denoting the classical ensemble average.

(a) Write down the SSE for the master equation of a two-level atom coupled to a bath from Aufgabe 12:

$$\begin{aligned} \frac{d}{dt}\rho(t) \;=\; & -i\tfrac{1}{2}\tilde{\omega}_0[\sigma_z,\rho] - \tfrac{1}{2}\gamma_+\Big\{ \sigma_+\sigma_-\rho + \rho\sigma_+\sigma_- - 2\sigma_-\rho\sigma_+ \Big\} \\ & - \tfrac{1}{2}\gamma\Big\{ \sigma_-\sigma_+\rho + \rho\sigma_-\sigma_+ - 2\sigma_+\rho\sigma_- \Big\}. \end{aligned}$$

(b) Write a computer program to numerically solve this SSE.

(c) Plot the Bloch-vector components, $\langle\sigma_i\rangle; i = x, y, z$, and the norm, $\mathrm{Tr}\rho$, as functions of time for a few trajectories starting from the initial state $|\Psi(0)\rangle = 2^{-1/2}(|\uparrow\rangle + |\downarrow\rangle)$. As example parameters choose units such that $\tilde{\omega}_0 = 1$, and take $\gamma_+ = 2\gamma = 0.1\tilde{\omega}_0$. The step size dt should be chosen to ensure good numerical convergence.

(d) Solve the optical Bloch equations for this problem (see Auf. 13) and demonstrate that, once averaged over many trajectories, the results from the SSE approach those from the direct solution of the master equation. How many trajectories do you need to approach the master equation result to within 1%, say?

Bitte Rückseite beachten!⟶

Vorlesung:	• Do. 10:00 Uhr – 12:00 Uhr im EW 203. • Fr. 10:00 Uhr – 12:00 Uhr im EW 203.
Übung:	• Mi. 14–16 Uhr im EW 016 (Clive Emary).
Scheinkriterien:	• Mindestens 50% der Übungspunkte. • Regelmäßige und aktive Teilnahme am Tutorium. • Schriftliche Arbeit.
Literatur:	• G. Schaller, *Non-equillibrium Master Equations*, VL Skript, TU Berlin WS2012. • H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control*, (Cambridge University Press, 2009). • H. Carmichael, *An Open System Approach to Quantum Optics*, Springer Lecture Notes in Physics m 18, (Berlin, Heidelberg, 1993). • H. Carmichael, *Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations* (Springer 2003).