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6. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

Abgabe: Mi. 07.06.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.

Aufgabe 8 (20 Punkte): *Coherent destruction of tunnelling*

The ability of a particle to tunnel between two regions of space is a purely quantum phenomenon that can be suppressed by means of periodic driving. This idea was explored in Grossmann, Dittrich, Jung, Hänggi *Phys. Rev. Lett.* **67**, 516 (1991). By representing the two regions of space by means of a two-level system, tunnelling between both and periodic driving are represented by the Hamiltonian

$$H(t) = g\sigma_x + A\cos(\omega t)\sigma_z. \tag{1}$$

(a) (5) Find the transformation $U(t)$ such that the evolution of the system is governed by the Hamiltonian

$$H'(t) = gU(t)\sigma_x U^\dagger(t). \tag{2}$$

(b) (5) Compute the explicit form of H'

$$H'(t) = gf_x(t)\sigma_x + gf_y(t)\sigma_y, \tag{3}$$

and find the form of the functions $f_x(t)$ and $f_y(t)$.

(c) (5) In the high frequency regime $\omega \gg g$, an appropriate, time-indipendent approximation of the Hamiltonian $H'(t)$ involves averaging over the period,

$$H' \simeq \frac{1}{T}\int_t^{t+T} H'(\tau)d\tau, \tag{4}$$

where T is the period of the Hamiltonian. In this limit, determine the values of A that suppress the Hamiltonian H'. What behaviour is expected in the original picture?
Hint: The definition of the zeroth order Bessel function is

$$J_0(x) = \frac{1}{\pi}\int_0^\pi \cos\left[x\sin(\tau)\right]d\tau. \tag{5}$$

(d) (5) Compare with the usual rotating wave approximation, which is a period-averaging performed in an interaction picture with respect to $H_0 = -\omega\sigma_x$. Find the Hamiltonian $\tilde{H}$ governing the dynamics and the average

$$\tilde{H} \simeq \frac{1}{T}\int_t^{t+T} \tilde{H}(\tau)d\tau, \tag{6}$$

Why is that not an appropriate approximation to study coherent destruction of tunnelling?