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UE: Dr. Clive Emary

Projekte zur TP VI Vertiefung: Nichtgleichgewichtsstatistik

Durchführung

Die Projekte beinhalten Aufgaben aus verschiedenen Bereichen der Nichtgleichgewichtsstatistik und können nach eigenen Vorstellungen bearbeitet werden (Numerik, Analytik, Zusammenfassung der Literatur, Experimente . . .). Die in jeder Projektbeschreibung aufgeführten Punkte können als Leitfaden dienen, Sie können aber auch in Absprache mit den BetreuerInnen eigene Ideen verfolgen.
Die Projekte sind so konzipiert, dass die Bearbeitung mit der angegebenen Literatur und dem Wissen aus der Vorlesung möglich ist. Bei einigen Projekten werden allerdings besondere Vorkenntnisse benötigt (z.B. MATLAB).
Zur vollständigen Bearbeitung gehören folgende Punkte:

1. Bearbeitung des Projekts in Zweier- oder Dreiergruppen
2. Präsentation der Ergebnisse in einem 15 minütigen Kurzvortrag (+5 Minuten Diskussion). Wichtig ist hierbei in erster Linie die verständliche Darstellung. Beschränken Sie sich deshalb auf die zum Verständnis wesentlichen Punkte.
3. Abgabe einer schriftlichen Ausarbeitung mit vollständiger Dokumentation der Lösungswege und vollständigen Quellenangaben bis zum Vorlesungsende. Auch hier steht die Verständlichkeit und übersichtliche Darstellung im Vordergrund. Der Umfang der Ausarbeitung soll fünf bis zehn Seiten umfassen.

Während der gesamten Bearbeitungszeit stehen Ihnen die BetreuerInnen des jeweiligen Projektes für Fragen zur Verfügung. Bitte machen Sie individuell Termine mit den Betreuern aus.

Projekt 1: *Master equation for a bistable chemical reaction system*

Betreuer: Eckehard Schöll, Thomas Isele

The Schlögl model is a simple paradigmatic model for a bistable nonlinear dynamical system. It describes first order nonequilibrium phase transitions between two stable steady states. It arises from the the chemical reaction system

$$A + 2X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} 3X$$
$$A \underset{k_3}{\overset{k_3}{\rightleftharpoons}} X$$

where X and A are chemical species, and k_1, k_2, k_3, k_4 are rate constants. The concentration of A is held constant, while the concentration of X is a variable.

- Perform a literature search on this topic.
- Set up the one-step transition rates $W_{nn'}$ for these reactions, and construct the master equation.
- Derive the hierarchy of moment equations for the factorial moments.
- Derive the mean-field rate equations for the mean concentration x of X and discuss its steady state solutions. In the case of bistability, the two stable steady states may coexist in space, if a diffusion term is added to the nonlinear rate equation. Derive the equal areas rule (Maxwell construction) for spatial coexistence. If the equal areas rule is not satisfied, one state is metastable, and the other is globally stable. Show that the time-dependent spatial profile describes a transition to the globally stable state by a propagating front, and determine the front velocity.
- Determine the stationary probability distribution as solution of the master equation and discuss the occurrence of bistability. Compare the condition of equal probability of the two steady states with the equal areas condition of deterministic bistability.

Literature

[1] Crispin W. Gardiner, *Handbook of stochastic methods*, (Springer, Berlin, 2004), Sect. 7.1.
[2] F. Schlögl, *Chemical reaction models for non-equilibrium phase transitions*, Z. Phys. **253**, 147 (1972).
[3] H. K. Janssen, *Stochastisches Reaktionsmodell für einen Nichtgleichgewichts-Phasenübergang*, Z. Phys. **260**, 67 (1974)
[4] E. Schöll, *Nonlinear spatio-temporal dynamics and chaos in semiconductors* (Cambridge University Press, 2001), Sect. 3.1.

Projekt 2: *Coherence resonance in a laser*

Betreuer: Eckehard Schöll, Valentin Flunkert

In nonlinear stochastic systems described by a Langevin equation, the regularity (coherence) of noise-induced oscillations as a function of noise intensity often exhibits a maximum at a finite noise intensity. This counter-intuitive effect is called *coherence resonance*. There exist many real-world examples like neurons or lasers in the excitable regime. Here we focus on a semiconductor laser with optical feedback, which exhibits coherence resonance below a subcritical Hopf bifurcation of intensity pulsations. This can be described by a Hopf normal form

$$\dot{z} = (\lambda + i\omega_0 + |z|^2 - |z|^4)z + D\xi(t), \tag{1}$$

with $z = re^{i\varphi} \in C$, bifurcation parameter $\lambda < 0$, and Gaussian white noise $\xi(t)$ with intensity D.

- Perform a literature search on coherence resonance.
- Construct the bifurcation diagram.
- Simulate the noise-induced dynamics for $\lambda = -0.26$, $\omega_0 = 2\pi$.
- Calculate and contrast the various coherence measures for this model as functions of the noise intensity D: the correlation time t_{cor}, and the signal-to-noise ratio (SNR) β.
- Apply a mean-field approximations [1] to this model and approximate it by a linear Langevin equation with rescaled $\tilde{\lambda}$. Calculate the power spectral density $S(\omega)$, the autocorrelation function, the correlation time t_{cor}, and the signal-to-noise ratio (SNR) β analytically as a function of D. Determine the optimum noise intensity analytically.
- Discuss the application of this model to semiconductor lasers with optical feedback [2].

Literature

[1] E. Schöll, Notes (2006); V. Flunkert, P. Hövel, and E. Schöll, *Coherence Resonance - Mean Field Approach*, Poster at the Workshop on Constructive Influence on Noise in Complex Systems, Dresden MPIPKS 2006.

[2] O. V. Ushakov, *et al*, *Coherence Resonance Near a Hopf Bifurcation*, Phys. Rev. Lett. **95**, 123903 (2005).

[3] B. Lindner, J. García-Ojalvo, A. Neiman, and L. Schimansky-Geier, *Effects of noise in excitable systems*, Phys. Rep. **392**, 321 (2004).

Projekt 3: *Coloured noise in the FitzHugh-Nagumo model*

Betreuer: Eckehard Schöll, Christian Otto

The FitzHugh-Nagumo model is a paradigm of an excitable system [1]. It describes, for instance, the emission of spikes by neurons:

$$\begin{aligned} \varepsilon\dot{x} &= x - \frac{x^3}{3} - y \\ \dot{y} &= x + a + \eta(t) \end{aligned} \tag{2}$$

where x is the fast activator, y is the slow inhibitor, $\varepsilon \ll 1$ is the time scale ratio of the two variables, and a is the excitation threshold. Here we consider colored noise $\eta(t)$ generated by an Ornstein-Uhlenbeck process [2]

$$\tau_c\dot{\eta} = -\eta + \sqrt{2\sigma^2\tau_c}\xi(t) \tag{3}$$

where $\xi(t)$ is Gaussian white noise, and τ_c is the noise correlation time.

- Perform a literature search on this topic.
- Show that in the limit $\tau_c \ll 1$ one can eliminate η adiabatically, and the colored noise reduces to white noise. Simulate the FitzHugh Nagumo model with white noise.
- Calculate and contrast the various coherence measures for this model as functions of the noise intensity σ^2 and τ_c: the correlation time t_{cor}, and the standard deviation of the interspike interval (ISI) R_T.
- Analytically determine the coherence-resonance curve $R_T(D)$ and the minimum D_{opt} in the limit of white noise $\tau_c \ll 1$ and for $\epsilon \ll 1$ [2,3].
- Discuss applications to chemical systems and spatially extended systems [4,5].

Literature

[1] E. Schöll, Vorlesungsskript "Nichtgleichgewichtsstatistik" WS10/11, Chapter 3.2.

[2] S. Brandstetter, M. A. Dahlem, and E. Schöll, *Interplay of time-delayed feedback control and temporally correlated noise in excitable systems*, Phil. Trans. R. Soc. A **368**, 391 (2010)

[3] B. Lindner, J. García-Ojalvo, A. Neiman, and L. Schimansky-Geier, *Effects of noise in excitable systems*, Phys. Rep. **392**, 321 (2004).

[4] A. G. Balanov, V. Beato, N. B. Janson, H. Engel, and E. Schöll, *Delayed feedback control of noise-induced patterns in excitable media*, Phys. Rev. E **74**, 016214 (2006).

[5] V. Beato, I. Sendiña-Nadal, I. Gerdes, and H. Engel, *Coherence resonance in a chemical excitable system driven by coloured noise*, Phil. Trans. R. Soc. A **366**, 381 (2008).

Projekt 4: *Stochastic resonance*

Betreuer: Clive Emary

The concept of stochastic resonance describes a curious phenomenon in bistable systems subject to both periodic and random forcing: an increase in the input noise can result in an improvement in the output signal-to-noise ratio.

- Perform a literature search on this topic.
- Write a computer simulation of a particle in a double-well potential subject to periodic driving and a stochastic force.
- Use you simulation to calculate the spectral power-density $S(\omega)$ and signal-to-noise ratio
- Compare with known analytic results from the literature.
- What happens for strong driving?

Literature

[1] E. Schöll, Vorlesungsskript "Nichtgleichgewichtsstatistik" WS10/11, Chapter 3.1.
[2] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, *Stochastic resonance*, Rev. Mod. Phys. **70**, 223 (1998).

Projekt 5: *Full counting statistics (FCS) in quantum transport*

Betreuer: Clive Emary

Electronic transport though a nanostructure (e.g. a quantum dot) can, under certain circumstances, be described by the n-resolved quantum master equation

$$\dot{\rho}^{(n)}(t) = \sum_{n'} \mathcal{W}^{(n-n')} \rho^{(n')}(t) \tag{4}$$

where n counts the number of electrons transferred to the collector, $\rho^{(n)}(t)$ is the partial system density matrix associated with each n, and $\mathcal{W}^{(n-n')}$ is a super-operator (Liouvillian) describing the evolution of the system associated with a transfer of $n - n'$ electrons. In this project we want to investigate how well such transport processes can be approximated by Fokker-Planck equations.

- Perform a literature search on this topic.
- Approximate Eq. (4) with a set of coupled Fokker-Planck equations by employing a Kramers-Moyal expansion to second order and derive a formal solution.
- Compare this solution with the standard result (saddle-point approximation) and exact numerical results for the single-resonant level model.
- Calculate the full counting statistics for a double quantum dot in high-bias limit.

Literature

[1] C. Emary, Vorlesungsskript “Theory of nanostructures” WS08/09, available at
`http://wwwitp.physik.tu-berlin.de/cemary/downloads/QME_FCS.pdf`

[2] D.A. Bagrets and Yu. V. Nazarov, *Full Counting Statistics of Charge Transfer in Coulomb Blockade Systems*, Phys. Rev. B, **67**, 085316 (2003).

[3] G. Kiesslich, PhD Thesis TU Berlin (2005): *Nonlinear transport properties of quantum dot systems*, available at
`http://opus.kobv.de/tuberlin/volltexte/2005/1130/`

Projekt 6: *Quantum theory of field damping*

Betreuer: Kathy Lüdge

Consider a single mode of the electromagnetic field inside a cavity. Due to imperfections in the cavity, this mode will be coupled to the field outside the cavity, and this will lead to a damping of the cavity mode.

- Perform a literature search on this topic.
- Derive a quantum master equation to describe the damping of the cavity mode.
- Show that this master equation can be written as a Fokker-Planck equation for the P-representation
- Consider the cavity mode to be prepared in a coherent state. Calculate the time-evolution of the P-representation and illustrate its behaviour through computer visualisation.

Literature

[1] M. O. Scully, M. S. Zubairy, *Quantum Optics*, Cambridge University Press (1997).

Projekt 7: *Resonance fluorescence*

Betreuer: Kathy Lüdge

The phenomenon of resonance fluorescence provides an interesting manifestation of the quantum theory of light. In this process, a two-level atom is typically driven by a resonant continuous-wave laser field and the spectral and quantum statistical properties of the fluorescent light emitted by the atom are measured.

- Perform a literature search on this topic.
- Construct a quantum master equation for a two-level atom in free space driven by a classical field
- Solve the master equation and thus calculate the stationary power spectrum $S(\mathbf{r}, \omega)$ at a point $\mathbf{r}$ away from the atom. Discuss weak and strong field limits.
- Calculate the stationary second-order correlation function $g^{(2)}(\tau)$ and discuss non-classical aspects of the fluorescent radiation.

Literature

[1] M. O. Scully, M. S. Zubairy, *Quantum Optics*, Cambridge University Press (1997).
[2] H. J. Kimble and L. Mandel, *Theory of resonance fluorescence* Phys. Rev. A **13** 2123 (1976).