

## English summary

### 2.3 Hindmarsh-Rose model

$$\left. \begin{aligned} \dot{x} &= c(x - \frac{x^3}{3} - y + z) \\ \dot{y} &= \frac{1}{c}(x^2 + dx - by + a) \end{aligned} \right\} \text{time series, Hopf and SNIPER bifurcation}$$

### 3. Physiological models

#### 3.1 Hodgkin-Huxley model

based on physical, electrical, and chemical fundamentals  
(Ohm's law, Kirchhoff's junction rule, Nernst potential)

$$C_m \dot{V} = -\bar{g}_K n^4 (V - E_K) - \bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_L (V - E_L)$$

$V$ : membrane potential

$E_K, E_{Na}, E_L$ : reversal potential of K, Na, leak

$\bar{g}_K, \bar{g}_{Na}, \bar{g}_L$ : (maximum) conductances of K, Na, leak channels

$$\left. \begin{aligned} n(V): & \text{gating variable (K activation)} \\ m(V): & \text{" " (Na activation)} \\ h(V): & \text{" " (Na inactivation)} \end{aligned} \right\} \begin{aligned} \dot{n} &= \alpha_n(V)(1-n) - \beta_n(V)n \\ \dot{m} &= \alpha_m(V)(1-m) - \beta_m(V)m \\ \dot{h} &= \alpha_h(V)(1-h) - \beta_h(V)h \end{aligned}$$

$$\alpha_n(V) = 0.01 \frac{10 - V}{e^{(10-V)/10} - 1}$$

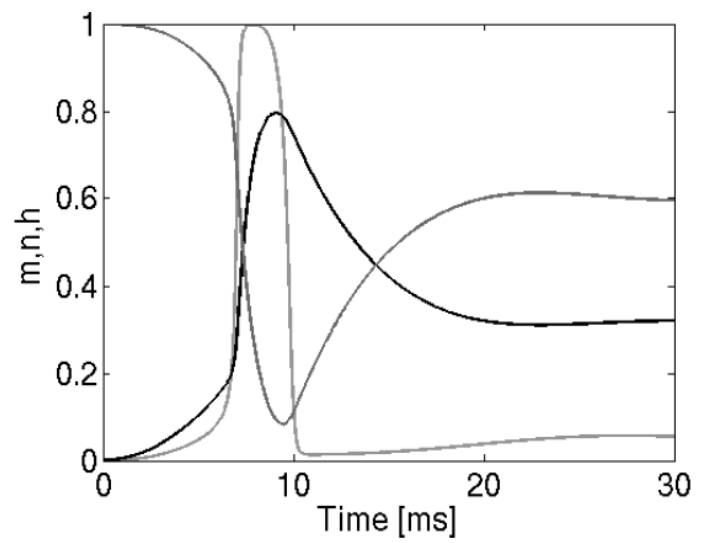
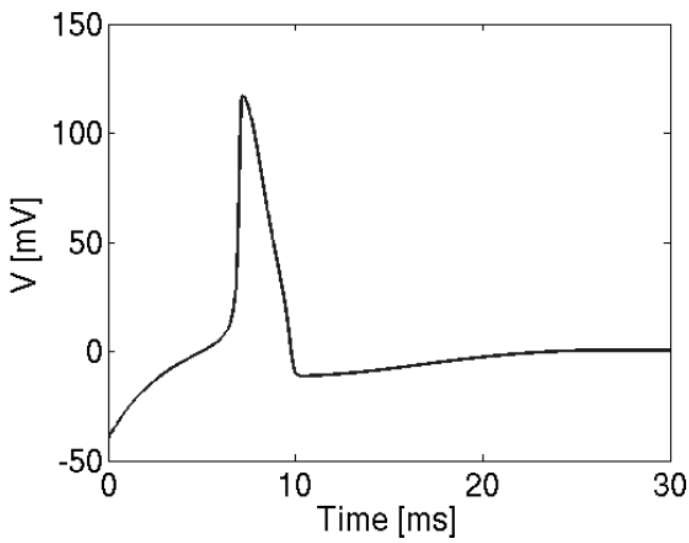
$$\beta_n(V) = 0.125 e^{-V/80}$$

$$\alpha_m(V) = 0.1 \frac{25 - V}{e^{(25-V)/10} - 1}$$

$$\beta_m(V) = 4 e^{-V/18}$$

$$\alpha_h(V) = 0.07 e^{-V/20}$$

$$\beta_h(V) = \frac{1}{e^{(30-V)/10} + 1}$$

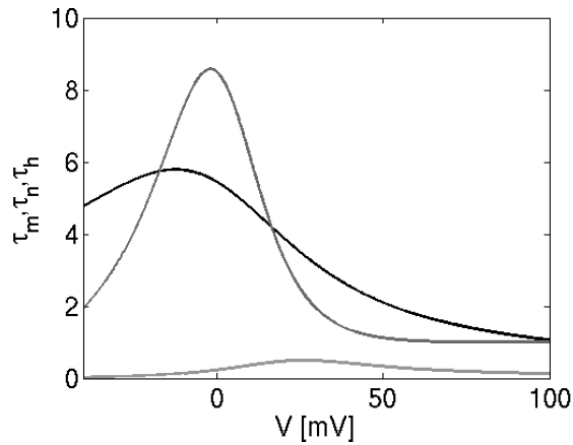
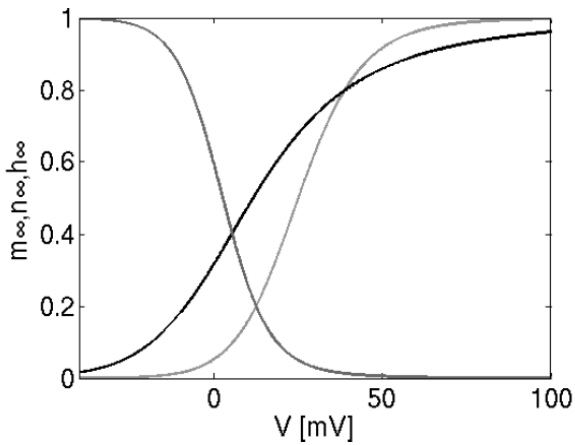


Left panel: Membrane voltage. Right panel: Activation potential, for sodium activation  $m$  (green) and inactivation  $h$  (pink) and potassium activation  $n$  (black).

### 3.1 Hodgkin-Huxley-Modell (Fortsetzung)

alternative Schreibweise der  $u, m, h$ -Gleichungen:

$$\tau_k = \frac{1}{\alpha_k + \beta_k}, \quad k_\infty = \frac{\alpha}{\alpha_k + \beta_k}, \quad k = n, m, h$$



Left panel: Steady state activation and inactivation functions. Right panel: Voltage dependent time constants, for sodium activation  $m$  (green) and inactivation  $h$  (pink) and potassium activation  $n$  (black).

$$\tau_m(V) \frac{dm}{dt} = -m + m_\infty(V)$$

$$\tau_h(V) \frac{dh}{dt} = -h + h_\infty(V)$$

$$\tau_n(V) \frac{dn}{dt} = -n + n_\infty(V)$$

Lösung (für konstantes  $V$ ): ("voltage clamp")

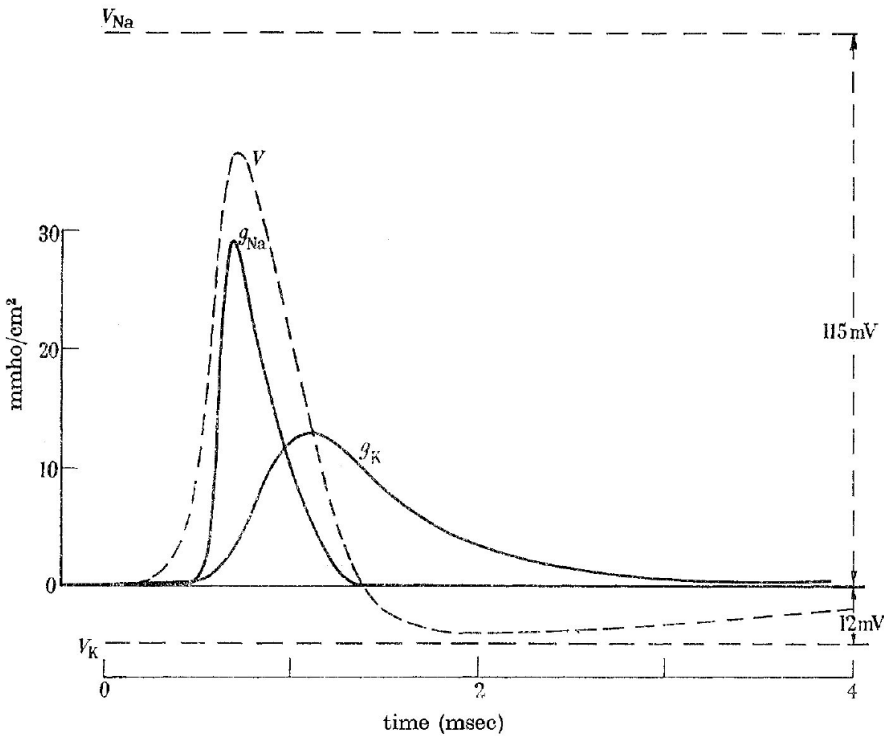
$$n(t) = n_\infty + (n_0 - n_\infty) e^{-t/\tau_n} = n_0 - (n_0 - n_\infty) (1 - e^{-t/\tau_n})$$

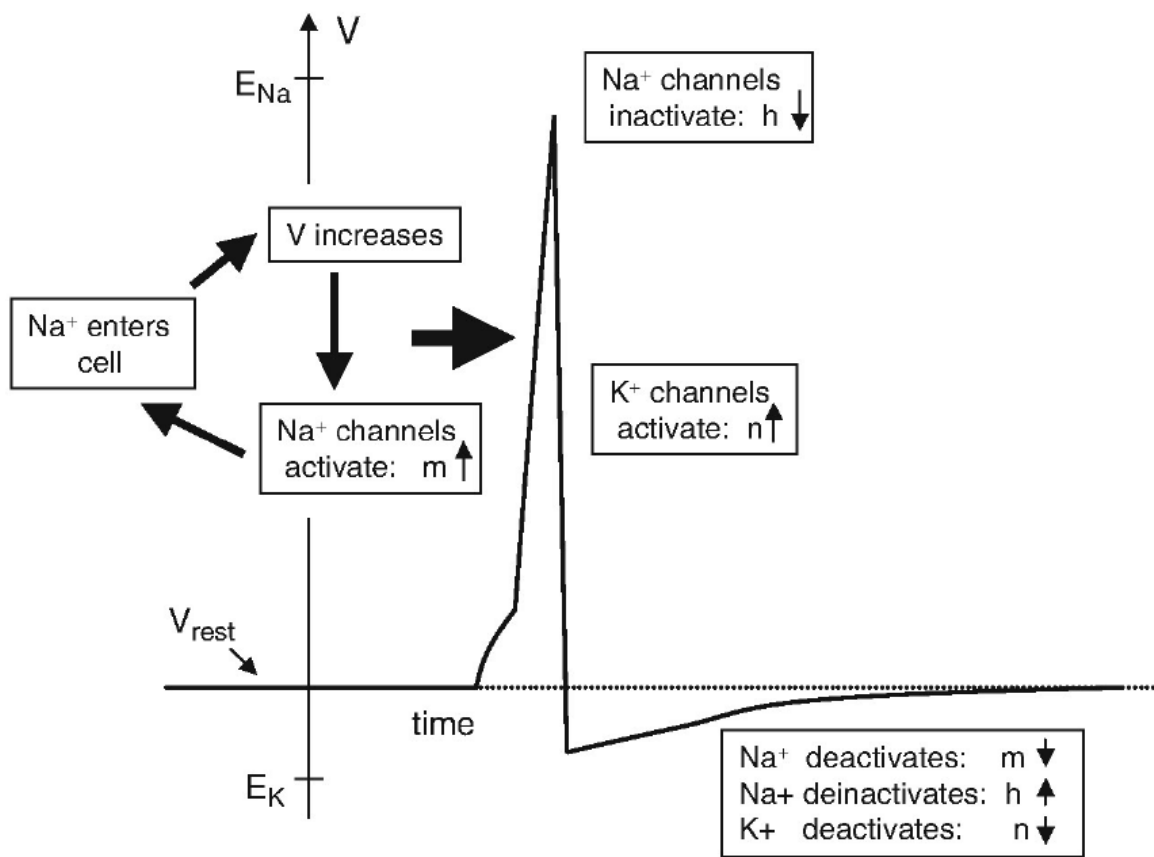
$$m(t) = m_0 - (m_0 - m_\infty) (1 - e^{-t/\tau_m})$$

$$h(t) = h_0 - (h_0 - h_\infty) (1 - e^{-t/\tau_h})$$

⇒ zeitabhängige / spannungsabhängige Leitfähigkeiten

$g_K, g_{Na}, g_L$

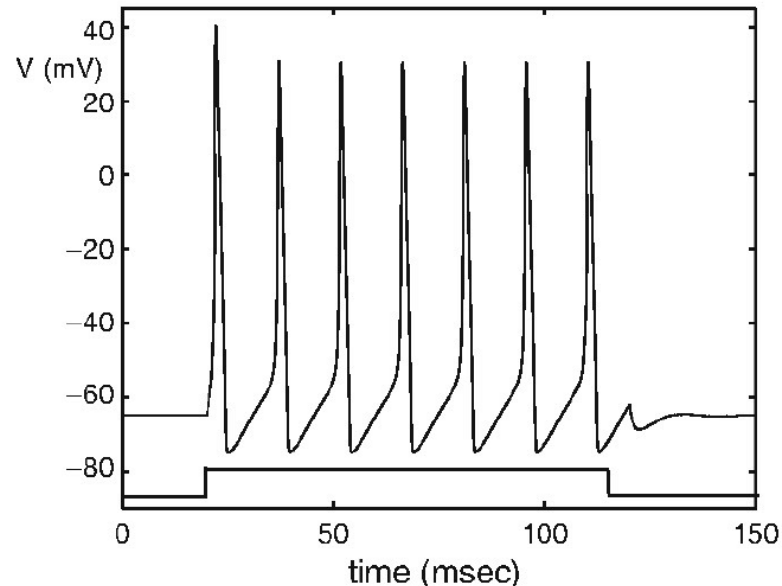
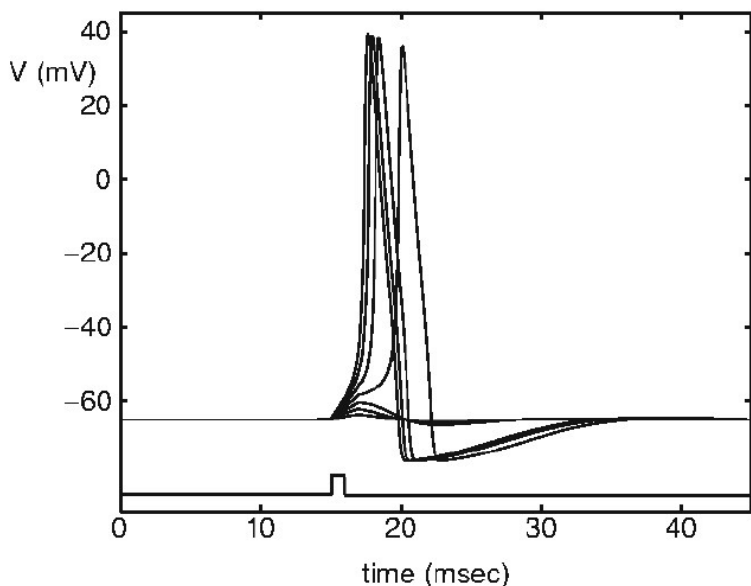




dynamische Verhalten: zusätzlicher Strom als Bifurkationsparameter  $I$

$$C_m \dot{V} = I - I_{ion}$$

$I$  erhöht  $\Rightarrow$  Fixpunkt (Ruhezustand) wird instabil in einer Hopf-Bifurkation.



### 3.2 Morris-Lecar-Modell

# VOLTAGE OSCILLATIONS IN THE BARNACLE GIANT MUSCLE FIBER

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2-Variablen Modell:

$$C_m \dot{V} = I - g_L (V - E_L) - g_{Ca} \underbrace{m_\infty(V)}_{\text{Ca immer im Fixpunkt}} (V - E_{Ca}) - g_K w(V) (V - E_K)$$

Ca immer im Fixpunkt  
(Dynamik sehr schnell)

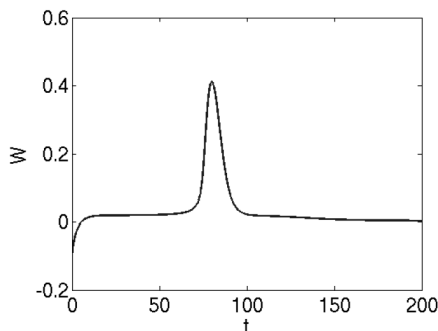
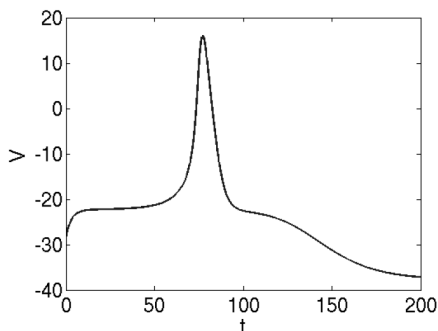
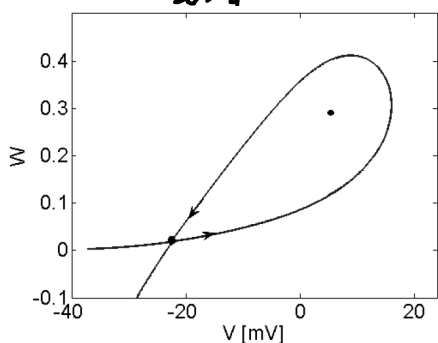
$$\tau_w(V) \dot{w} = w_\infty(V) - w$$

und folgende Parameter:

$$m_\infty(V) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{V - V_1}{V_2} \right) \right] = \left[ 1 + \exp \left( -2 \frac{V - V_1}{V_2} \right) \right]^{-1}$$

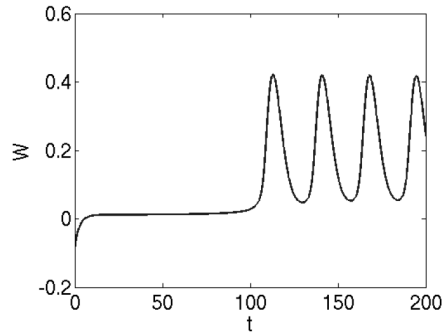
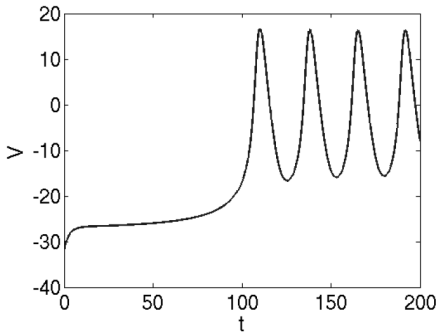
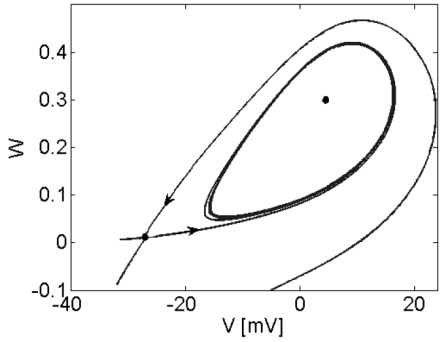
$$w_\infty(V) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{V - V_3}{V_4} \right) \right] = \left[ 1 + \exp \left( -2 \frac{V - V_3}{V_4} \right) \right]^{-1}$$

$$\tau_w(V) = \frac{1}{\cosh \left( \frac{V - V_3}{2V_4} \right)}, \quad V_1, V_2, V_3, V_4: \text{Parameter zum Einstellen des Fixpunkts}$$



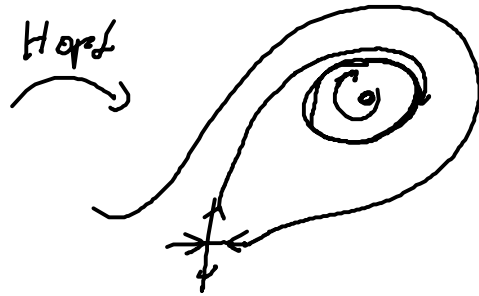
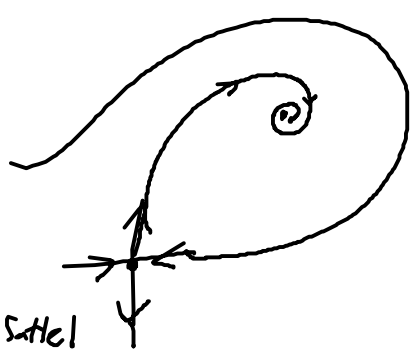
The phase portrait of the homoclinic bifurcation present in the Morris-Lecar model at  $I_0 = 35$  and corresponding time series for  $v$  and  $w$ .

homokline  
Bifurkation



The phase portrait of Morris-Lecar at  $I_0 = 39.5$  (oscillatory regime) and corresponding time series for  $v$  and  $w$ .

## Einschub: homokline Bifurkation (Nachtrag zu Kap. 1)



stabiler Grenzzyklus  
entsteht aus dem Fokus



Sattelpunkt wird Teil  
des Grenzzyklus

