

# English summary

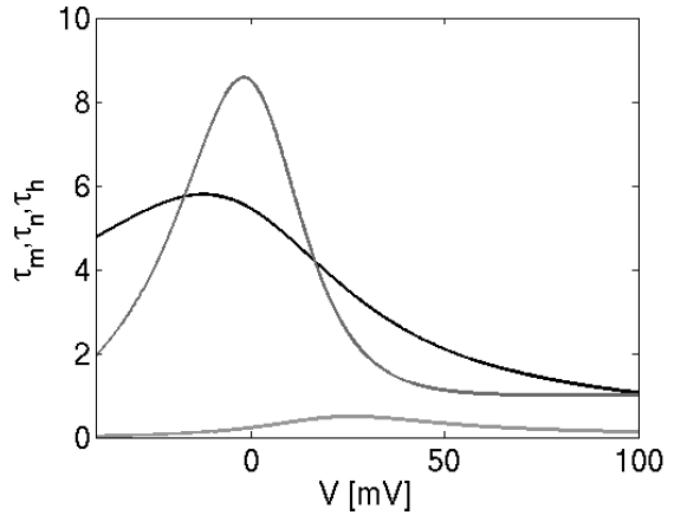
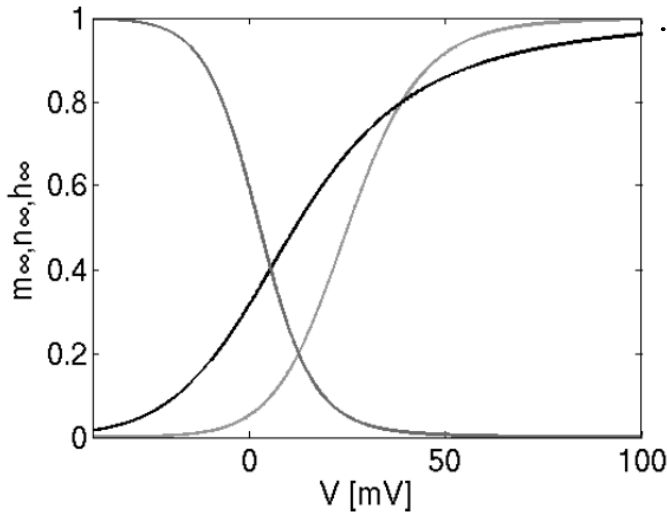
## 3.1 Hodgkin-Huxley model

membrane potential

$$C_m \dot{V} = I - g_K n^4 (V - E_K) - g_{Na} m^3 h (V - E_{Na}) - g_L (V - E_L)$$

gating variables

$$\begin{cases} \tau_n(V) \dot{n} = -n + n_\infty(V) & K \text{ activation} \\ \tau_m(V) \dot{m} = -m + m_\infty(V) & Na \text{ activation} \\ \tau_h(V) \dot{h} = -h + h_\infty(V) & Na \text{ inactivation} \end{cases}$$



Left panel: Steady state activation and inactivation functions. Right panel: Voltage dependent time constants, for sodium activation  $m$  (green) and inactivation  $h$  (pink) and potassium activation  $n$  (black).

## 3.2 Morris-Lecar model

$$C_m \dot{V} = I - g_K w(V) (V - E_K) - \underbrace{g_{Ca} m_\infty(V) (V - E_{Ca})}_{\text{fast Ca dynamics}} - g_L (V - E_L)$$

$$\tau_w(V) \dot{w} = -w + w_\infty(V)$$

and the following functions:

$$m_\infty(V) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{V - V_1}{V_2} \right) \right]$$

$$w_\infty(V) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{V - V_3}{V_4} \right) \right]$$

$$\tau_w(V) = \frac{1}{\cosh \left( \frac{V - V_3}{2V_4} \right)}$$

$\Rightarrow$  homoclinic bifurcation (among others: Hopf, SNIPER)

# 4. Wechselspiel von Rauschen und Zeitverzögerung

bisher: deterministische dynamische Systeme

jetzt: stochastische dynamische Systeme

4.1 Grundbegriffe

4.2 Kohärenzresonanz

4.3 zeitverzögerte Rückkopplung

(gekoppelte Systeme  $\Rightarrow$  Kap 5)

## 4.1 Grundbegriffe

stochastischer Prozess: zeitentwicklung einer Zufallsvariable

Langevin-Gleichung: fluktuierende stochastische Kraft  $f(t)$

(Rauschen, noise)

z.B. Brownsche Bewegung (1827)

$$m\ddot{x} = -\eta x + f(t) \quad (\text{Bewegung eines Teilchens})$$

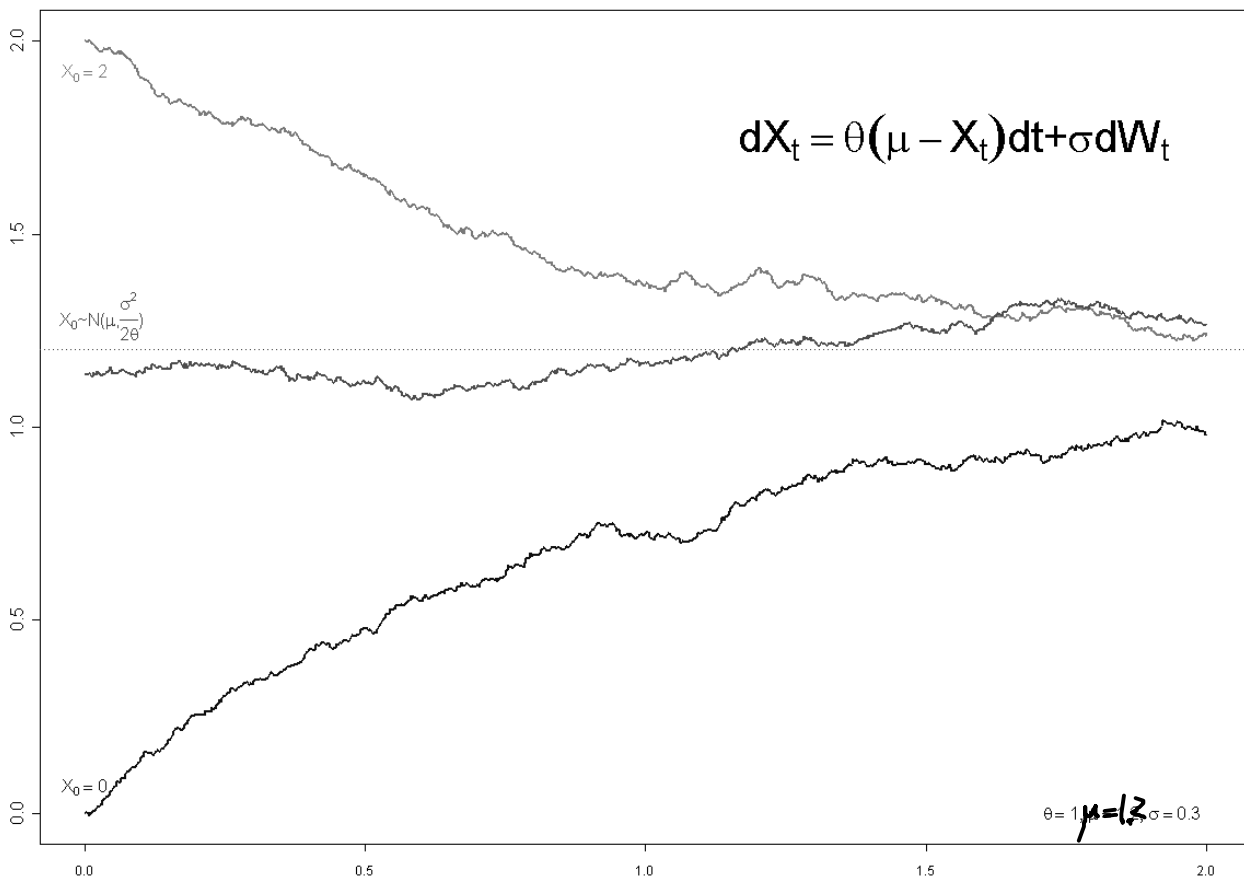
$\underbrace{\hspace{1.5cm}}_{\text{Reibung}} \quad \underbrace{\hspace{1.5cm}}_{\text{Rauschen (Kraft durch zufällige Stöße)}}$

stochastische Dgl:  $dx_t = \theta(\mu - x_t) dt + \sigma dW_t$  Rauschen

$\uparrow$   $\uparrow$   $\uparrow$   $\leftarrow$   $\leftarrow$   
Geschwindigkeit  $\mu$  langfristige Mittelwert  $\sigma^2$  Volatilität  
Zeit  $\uparrow$  Rückstellrate

Ornstein-Uhlenbeck-Prozess

# Ornstein - Uhlenbeck



$W_t$  : Zufallsvariable eines Wiener-Prozesses

$$\Rightarrow W_0 = 0$$

$$W_t - W_s \sim N(0, t-s) \quad \text{Normalverteilung mit Mittelwert 0 und Varianz } t-s$$

(Zentraler Grenzwertsatz: unkorrelierte Zufallsvariablen gehören einer Gauß-Verteilung)

Gaußsches weißes Rauschen  $\langle \xi(t) \rangle = 0$  (im Mittel keine gerichtete Kraft)

$$\langle \xi(t) \xi(t') \rangle = \delta(t-t') \quad (\text{unkorreliert, keine Gedächtniseffekte})$$

Autokorrelationsfunktion (Vergleiche mit sich selbst)

$$\underline{\Psi}(s) = \left\langle \underbrace{(x(t) - \langle x \rangle)}_{\text{um } s \text{ verschoben}} \underbrace{(x(t+s) - \langle x \rangle)}_{\text{Zeitschritt}} \right\rangle_{t \in \mathbb{R}} \quad \text{Mittelung über Zeit}$$

$$\bar{\Psi}(s) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T dt x(t) x(t+s) \quad (\text{wenn } \langle x \rangle = 0)$$

endliches Zeitintervall

Fourier-Transformation:  $\hat{X}(\omega, T) = \frac{1}{2T} \int_{-T}^T dt e^{i\omega t} x(t)$

Spektrale Leistungsdichte (power spectral density):  $S(\omega) = \lim_{T \rightarrow \infty} \frac{1}{T} |\hat{X}(\omega, T)|^2$

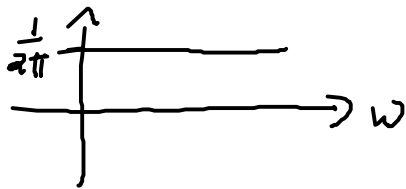
$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle x(t) x(t+s) \rangle e^{i\omega s} ds \quad (\text{wenn } \langle x \rangle = 0)$$

$S$  ist die Fourier-Transformierte von  $\bar{\Psi}(s)$  (Wiener-Khinchin-Theorem)

Bsp.: Gauß'sches weißes Rauschen  $\xi(t)$ :

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle \xi(t) \xi(t+s) \rangle e^{i\omega s} ds$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(s) e^{i\omega s} ds = \frac{1}{2\pi} = \text{const.}$$



alle Frequenzen gleichermaßen vertreten  
 $\Rightarrow$  "weiß"

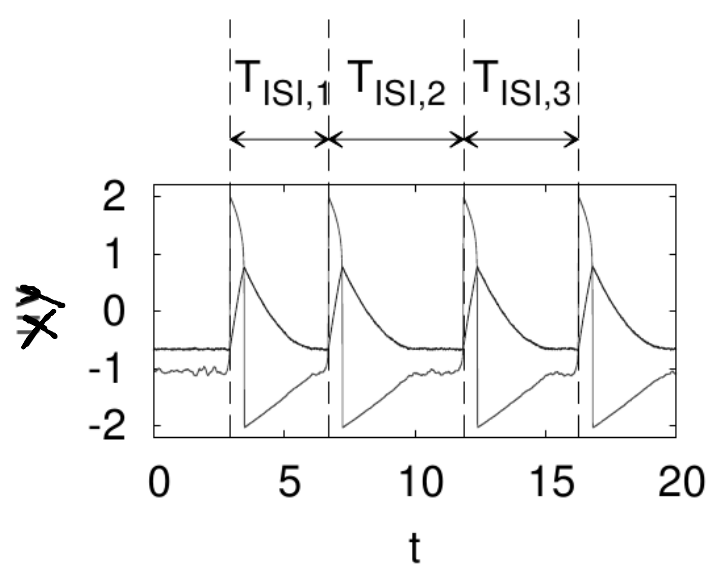
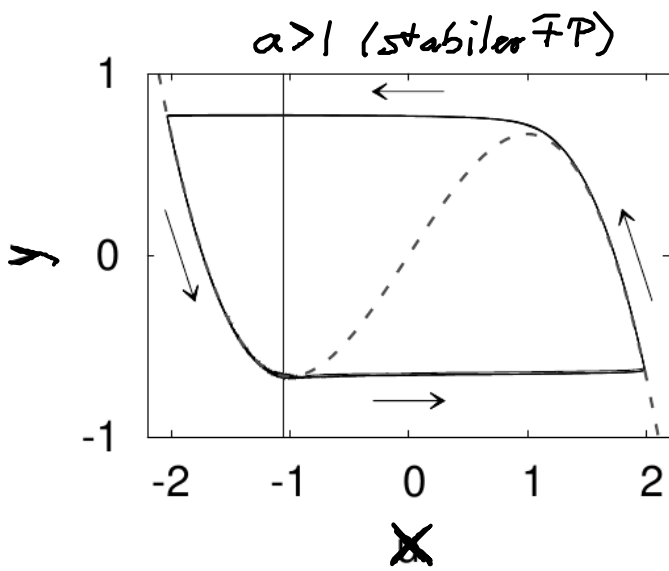
## 4.2 Kohärenzresonanz

- Konstruktiver Einfluss von Rauschen  
 $\Rightarrow$  Regularität verschiedener Oszillationen am größten für eine endliche Rauschstärke

Bsp.:  $\epsilon \dot{x} = x - \frac{x^3}{3} - y$

$$\dot{y} = x + a + D \xi(t)$$

fast periodisches Verhalten



# PHYSICAL REVIEW LETTERS

VOLUME 71

9 AUGUST 1993

NUMBER 6

## Stochastic Resonance without External Periodic Force

Hu Gang

*International Centre for Theoretical Physics, Trieste 34100, Italy  
and Physics Department, Beijing Normal University, Beijing 100875, People's Republic of China*

T. Ditzinger,\* C. Z. Ning, and H. Haken

*Institut für Theoretische Physik und Synergetik, Universität Stuttgart, Pfaffenwaldring 57/IV,  
D-7000 Stuttgart 80, Federal Republic of Germany*

(Received 21 December 1992)

VOLUME 78, NUMBER 5

PHYSICAL REVIEW LETTERS

3 FEBRUARY 1997

## Coherence Resonance in a Noise-Driven Excitable System

Arkady S. Pikovsky\* and Jürgen Kurths\*

*Max-Planck-Arbeitsgruppe "Nichtlineare Dynamik" an der Universität Potsdam Am Neuen Palais 19, PF 601553, D-14415,  
Potsdam, Germany*

(Received 9 August 1996)

We study the dynamics of the excitable Fitz Hugh–Nagumo system under external noisy driving. Noise activates the system producing a sequence of pulses. The coherence of these noise-induced oscillations is shown to be maximal for a certain noise amplitude. This new effect of coherence resonance is explained by different noise dependencies of the activation and the excursion times. A simple one-dimensional model based on the Langevin dynamics is proposed for the quantitative description of this phenomenon. [S0031-9007(97)02349-1]

Maß für Regularität: Korrelationszeit:  $t_{\text{cor}} = \frac{1}{\psi(0)} \int_0^{\infty} |\psi(s)| ds$

# normierte Fluktuationen der interspikeintervalle

$$R = \frac{\sqrt{\langle T_{ISI}^2 \rangle - \langle T_{ISI} \rangle^2}}{\langle T_{ISI} \rangle}$$

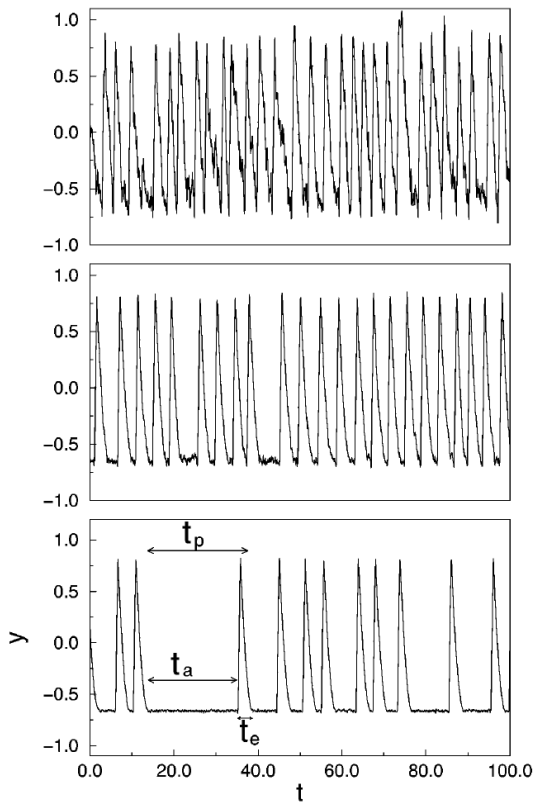


FIG. 1. The dynamics of the Fitz Hugh–Nagumo system [Eqs. (1), (2)] for  $a = 1.05$ ,  $\varepsilon = 0.01$ , and different noise amplitudes: From bottom to top  $D = 0.02$ ,  $D = 0.07$ , and  $D = 0.25$ . The mean durations of pulses are 7, 4, and 3.5, respectively. The activation and the excursion times for one pulse are depicted.

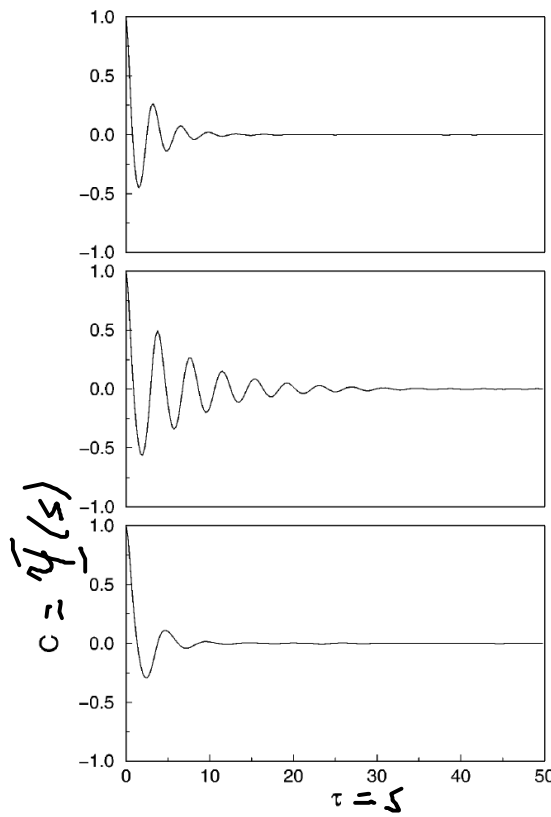
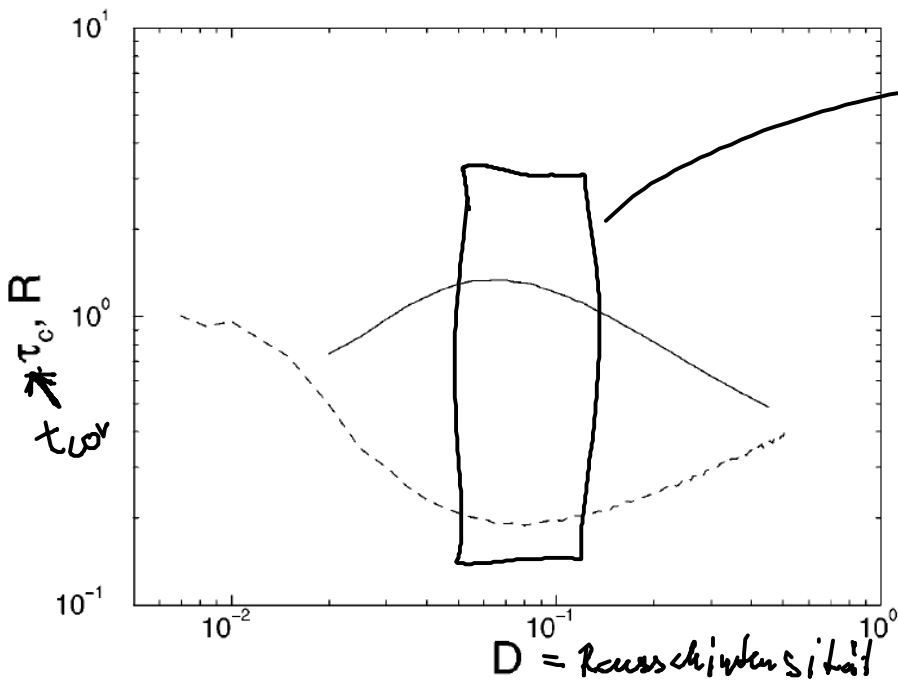


FIG. 2. The autocorrelation function of the regimes presented in Fig. 1.

Physically, the appearance of coherence resonance is

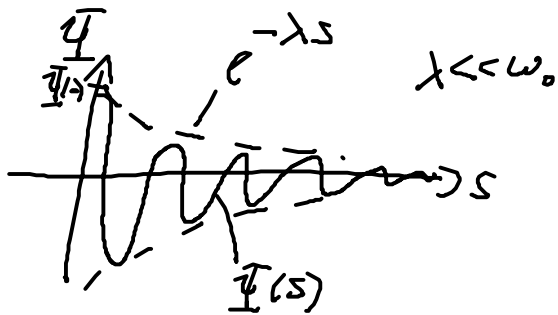


• Korrelationszeit maximal  
 • Fluktuation von  $T_{ISI}$  minimal  
 $\Rightarrow$  optimale Rauschstärke

FIG. 3. Correlation time  $\tau_c$  (solid line) and the noise-to-signal ratio  $R$  [Eq. (5), dashed line] vs noise amplitude for the Fitz Hugh–Nagumo system with  $a = 1.05$ ,  $\varepsilon = 0.01$ .

Ein linearer stochastischer Prozess:  $\dot{x} = -(\lambda + i\omega_0)x + \xi(t)$

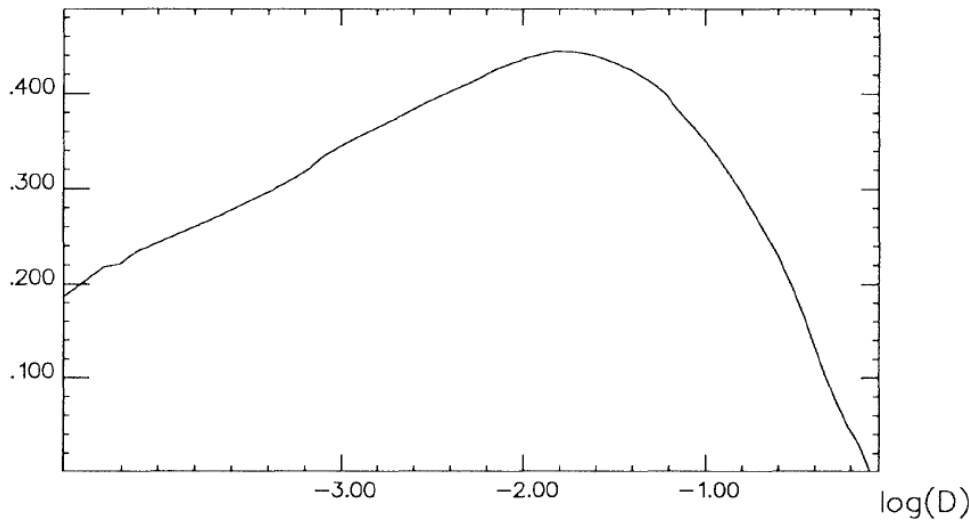
$$\Rightarrow \bar{\psi}(s) = \bar{\psi}(0) e^{-\lambda s} \cos(\omega_0 s)$$



$$t_{cor} = \int_0^{\infty} e^{-\lambda s} |\cos \omega_0 s| ds \approx \frac{2}{\pi \lambda}$$

$$\Rightarrow \bar{\psi}(s) = \bar{\psi}(0) e^{-\frac{2}{\pi} t_{cor} s} \cos(\omega_0 s)$$

$\beta$



Signal-Rauschen  
Verhältnis

(SNR) (SNR)

SNR maximal  
für optimale  
Rauschintensität

FIG. 5. The SNR  $\beta = h(\Delta\omega/\omega_p)^{-1}$  vs  $\log(D)$ . A stochastic resonance maximum can be seen.