English Summary

4 Interplay of noise and delay

4.1 Fundamentals

Langevin equation: $m\ddot{x} = -\underbrace{\eta\dot{x}}_{\text{friction}} + \underbrace{\xi(t)}_{\text{noise}}$ (e.g. Brownian motion)

stochastic differential eq.: $\underbrace{dx_t = \theta(\mu - x_t)\,dt + \sigma\, dW_t}_{\text{Ornstein-Uhlenbeck process}}$ ← noise

θ: return rate, μ: long-term mean, x_t: stochastic process, σ: volatility

W_t: random variable of a Wiener process

($W_0 = 0$, increment: $W_t - W_s \sim N(0, t-s)$ normal distribution with mean 0, variance $t-s$)

Gaussian white noise: $\langle \xi(t) \rangle = 0$ (no net force) ← temporal average

$\langle \xi(t)\,\xi(t') \rangle = \delta(t-t')$ (uncorrelated, no memory)

autocorrelation function: $\Psi(s) = \langle (x(t) - \langle x \rangle)\,(\underbrace{x(t-s)}_{\text{time series shifted by } s} - \langle x \rangle) \rangle$

power spectral density: $S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle x(t)\,x(t-s) \rangle\, e^{i\omega s}\, ds$ (for $\langle x \rangle = 0$)

$\Rightarrow$ Wiener-Khinchin theorem ($S(\omega)$: Fourier transform of $\Psi(s)$)

4.2 Coherence resonance

- constructive influence of noise
- noise-induced oscillations most regular for finite/optimal

noise strength

quantified by:

(i) correlationtime $t_{cor} = \frac{1}{\Psi(0)} \int_0^\infty |\Psi(s)|\, ds$ becomes maximal

(ii) normalized fluctuations of interspik intervals $R = \frac{\sqrt{\langle T_{ISI}^2 \rangle - \langle T_{ISI} \rangle^2}}{\langle T_{ISI} \rangle}$ minimal

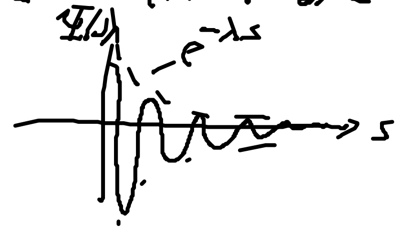
mean

(iii) signal-to-noise ratio $\beta = h \left(\frac{\omega_p}{\Delta\omega} \right)$ maximal

height width of $S(\omega)$ at $h_1 = \frac{h}{\sqrt{e}}$

for linear stochastic processes: $\dot{z} = -(\lambda + i\omega_0) z + \xi(t)$

$$\Psi(s) = \Psi(0) \exp\left[-\frac{2s}{\pi t_{cor}} \right] \cos(\omega_0 s)$$



$\lambda \ll \omega_0$

4.3 Zeitverzögerte Rückkopplung

(extended) time-delayed feedback control = "Pyragas control"

Physics Letters A 170 (1992) 421–428
North-Holland

PHYSICS LETTERS A

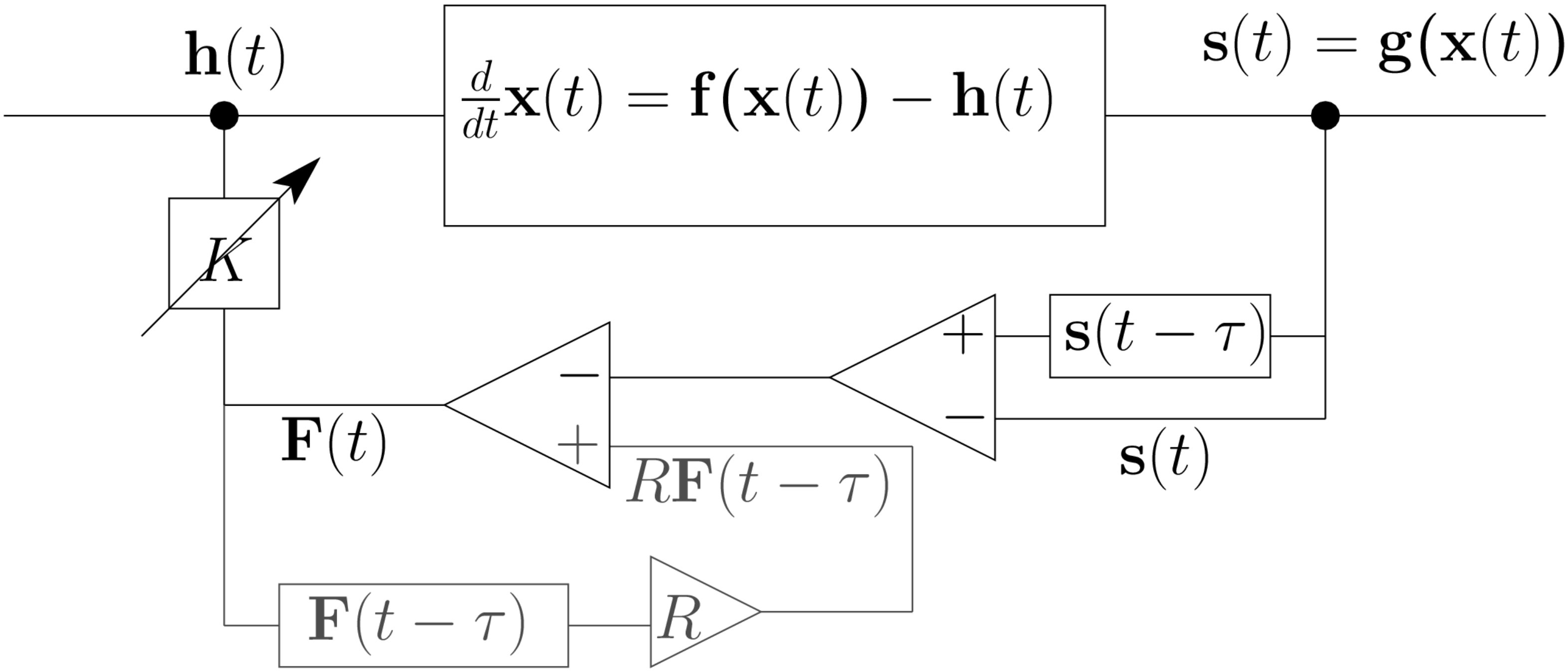
Continuous control of chaos by self-controlling feedback

K. Pyragas [1,2]
Institute for Theoretical Physics, Technical University of Berlin, W-1000 Berlin, Germany

Received 17 August 1992; accepted for publication 16 September 1992
Communicated by J. Flouquet

ursprünglich: Kontrolle instabiler periodischer Orbits / Grenzzyklen in seltsamen Attraktoren chaotischer Systeme

(SS14/15: nichtlineare Dynamik & Kontrolle)



Bsp.: Anwendung von Pyragas-Kontrolle auf die subkritische Hopf-Bifurkation

PRL **98,** 114101 (2007) PHYSICAL REVIEW LETTERS week ending 16 MARCH 2007

Refuting the Odd-Number Limitation of Time-Delayed Feedback Control

B. Fiedler,[1] V. Flunkert,[2] M. Georgi,[1] P. Hövel,[2] and E. Schöll[2]

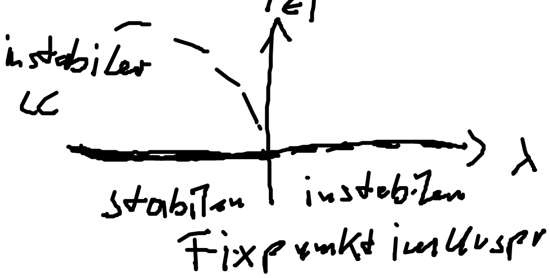
[1]*Institut für Mathematik I, FU Berlin, Arnimallee 2-6, D-14195 Berlin, Germany*

[2]*Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstraße 36, 10623 Berlin, Germany*

(Received 22 September 2006; published 14 March 2007)

$$\dot{z}(t) = [\lambda + i + (1 + i\gamma)|z(t)|^2]z(t) + b[z(t - \tau) - z(t)]$$

$z \in \mathbb{C}$, Kontrollstärke $b = b_R + i b_I \in \mathbb{C}$



Gedächtnisparameter $R \in [-1, 1]$

extended Pyragas control: $F(t) = b \sum_{n=0}^{\infty} R^n \left[z(t-(n+1)\tau) - z(t-n\tau) \right]$

Bsp: → Applet $\lambda = -0.005 \Rightarrow |z_{LC}| = \sqrt{-\lambda} \approx 0.071$

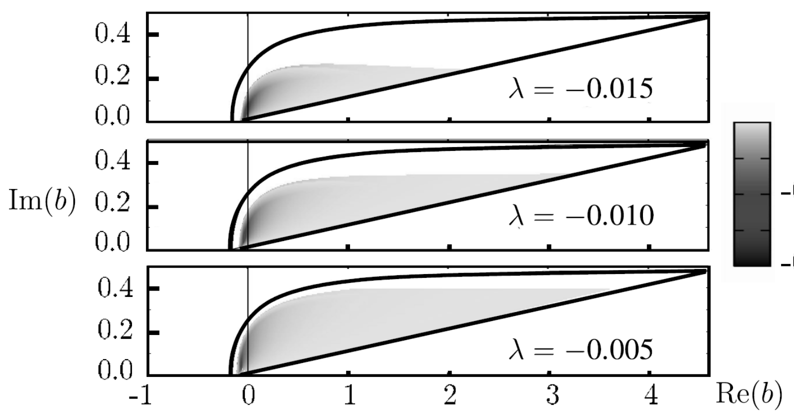


FIG. 3 (color online). Domain of control in the plane of the complex feedback gain $b = b_0 e^{i\beta}$ for three different values of the bifurcation parameter λ. The black solid curves indicate the boundary of stability in the limit $\lambda \nearrow 0$; see (18) and (19). The color-shading shows the magnitude of the largest (negative) real part of the Floquet exponents of the periodic orbit ($\gamma = -10$, $\tau = \frac{2\pi}{1-\gamma\lambda}$).

Anwendung auf neuronale Dynamik:

Eur. Phys. J. Special Topics **187**, 77–85 (2010)

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THE EUROPEAN PHYSICAL JOURNAL
SPECIAL TOPICS

Regular Article

Delay control of coherence resonance in type-I excitable dynamics

R. Aust[1], P. Hövel[1], J. Hizanidis[1,2], and E. Schöll[1]

[1] Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstraße 36, 10623 Berlin, Germany
[2] Optical Communications Laboratory, Department of Informatics and Telecommunications, University of Athens, Athens 15784, Greece

$$\dot{x} = x\left(1 - x^2 - y^2\right) + y\,(x - b) + D\xi + K\,(x_\tau - x) \tag{1a}$$

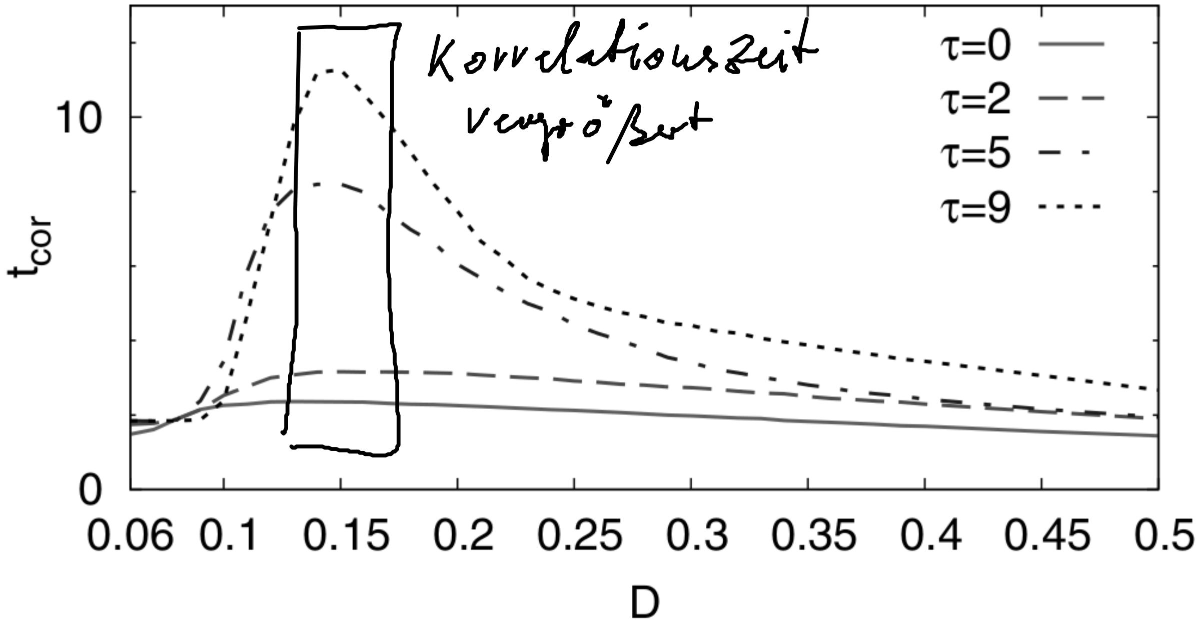
$$\dot{y} = y\left(1 - x^2 - y^2\right) - x\,(x - b) + D\xi + K\,(y_\tau - y)\,. \tag{1b}$$

Here x and y are the variables at time t, while x_τ and y_τ denote the respective variables at a delayed time $t - \tau$. The bifurcation parameter $b \in \mathbb{R}$ is a real constant. K denotes the control strength and τ is the delay time. Random input ξ is realized as Gaussian white noise with mean $\langle \xi(t) \rangle = 0$, variance $\langle \xi(t)\xi(t') \rangle = \delta(t - t')$, and noise intensity D. In polar coordinates the system equations for $D = 0$ and $K = 0$ are given by [22]:

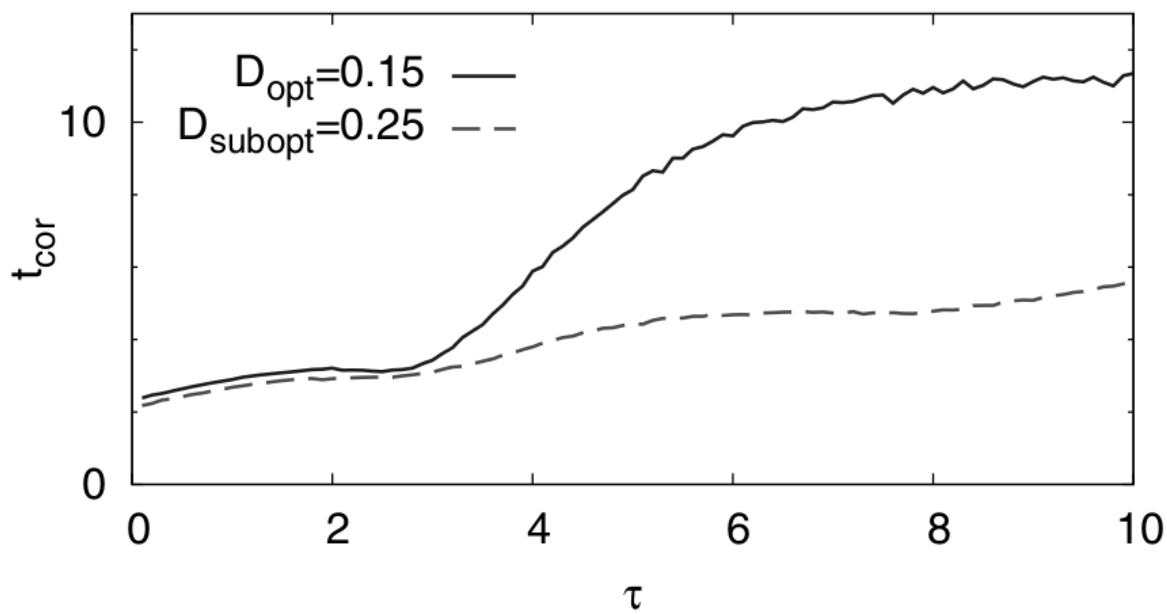
$$\dot{r} = r\left(1 - r^2\right) \tag{2a}$$

$$\dot{\varphi} = b - r\cos\varphi. \tag{2b}$$

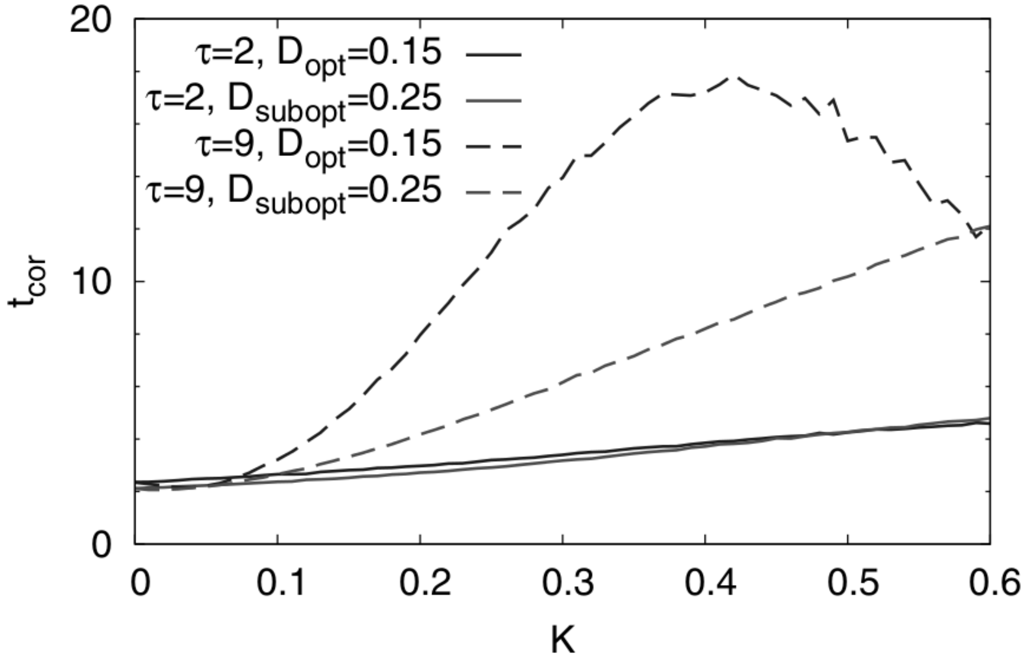
Ziel: Kontrolle (Verstärkung der Regularität) rausch induzierten
Oszillationen



Correlation time in dependence on the noise intensity D for different time delays τ. The solid (green) curve corresponds to the uncontrolled system ($\tau = 0$). The dashed (red), dash-dotted (blue), and dotted (black) curves refer to values of $\tau = 2, 5$, and 9, respectively. Other parameters: $b = 0.95$ and $K = 0.25$.

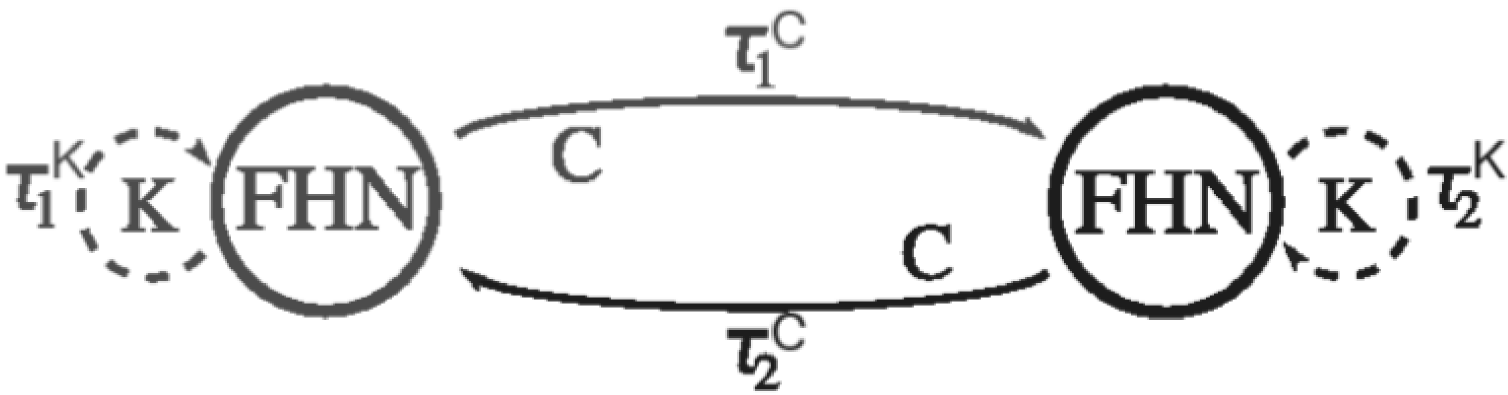


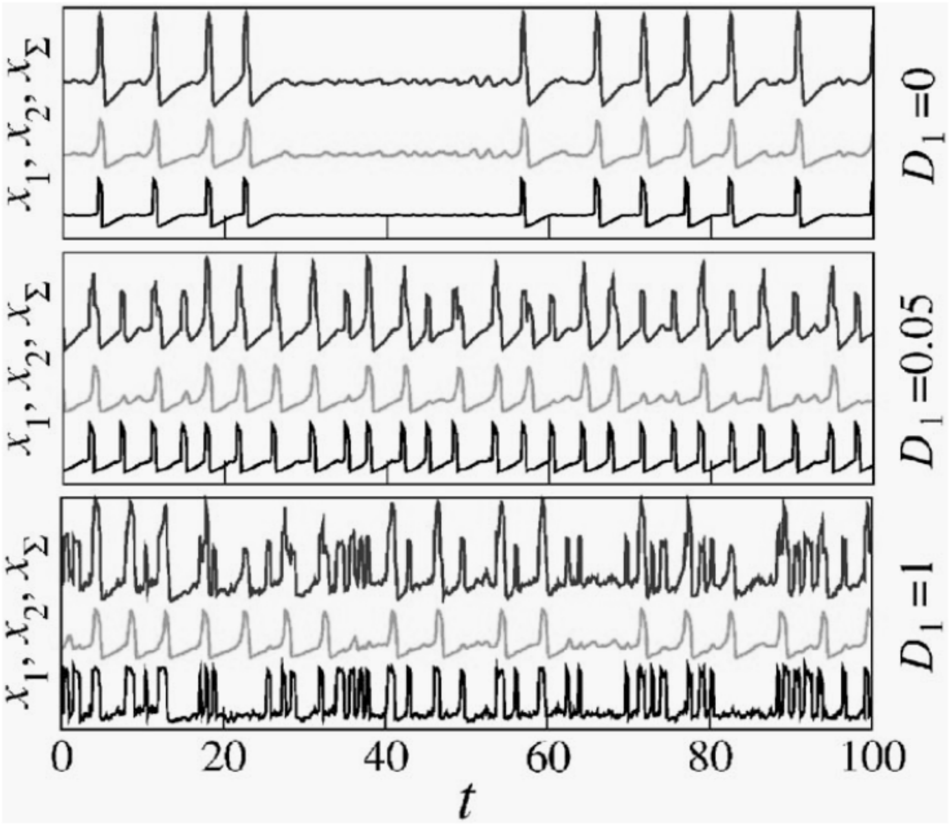
Correlation time t_{cor} in dependence on the time delay τ for two values of the noise intensity D. The dashed (red) curve corresponds to $D_{subopt} = 0.25$ and the solid (blue) curve refers to $D_{opt} = 0.15$. Other parameters: $b = 0.95$ and $K = 0.25$.



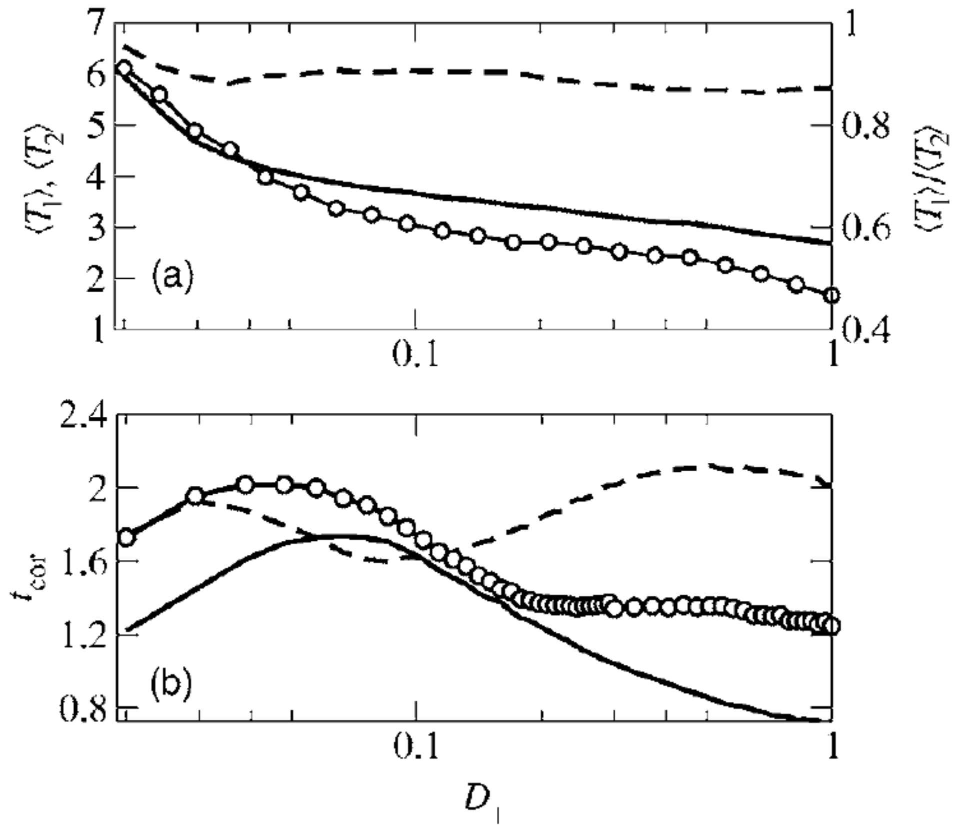
Correlation time t_{cor} in dependence on the control strength K for two values of the noise intensity D and two values of the delay time τ. The gray (red) and black (blue) curves depict the cases of D_{subopt} and D_{opt}, respectively. The solid and dashed lines correspond to $\tau = 2$ and $\tau = 9$, respectively. Other parameter: $b = 0.95$.

5. Dynamik gekoppelter Elemente

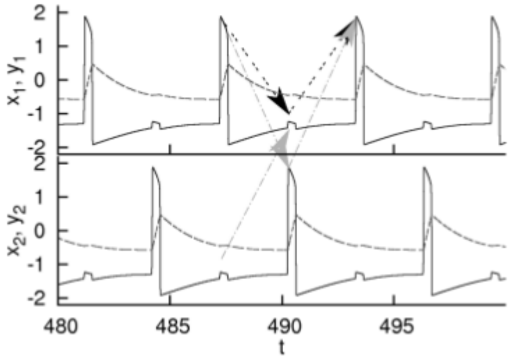




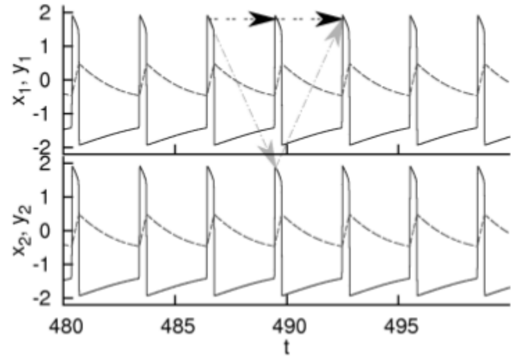
x1, x2, xΣ
D1 =0
D1 =0.05
D1 =1
0
20
40
60
80
100
t



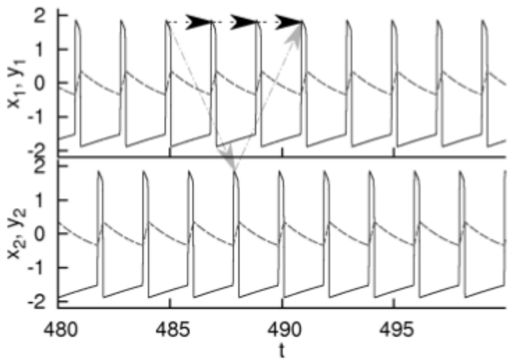
⟨T1⟩, ⟨T2⟩
⟨T1⟩/⟨T2⟩
(a)
7
6
5
4
3
2
1
1
0.8
0.6
0.4
0.1
1
tcor
(b)
2.4
2
1.6
1.2
0.8
0.1
1
D1


x1, y1
x2, y2
480
485
490
495
t

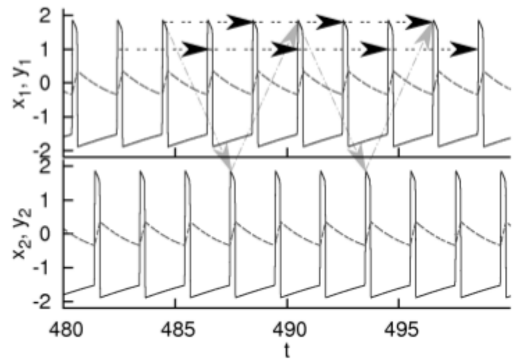
(a)


x1, y1
x2, y2
480
485
490
495
t

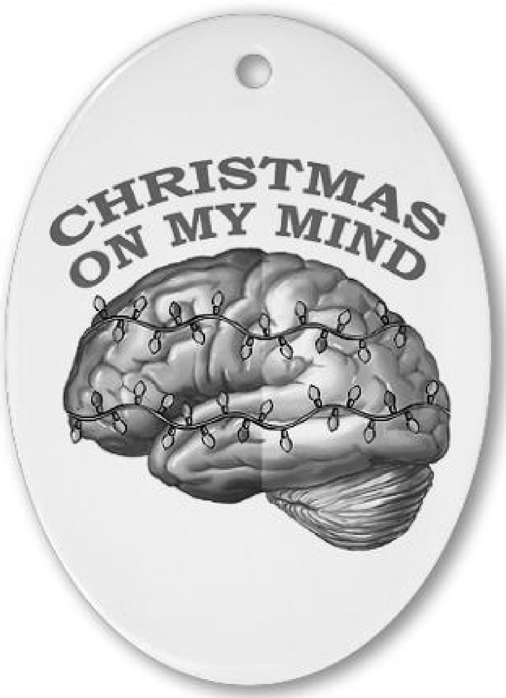
(b)


x1, y1
x2, y2
480
485
490
495
t

(c)


x1, y1
x2, y2
480
485
490
495
t

(d)


CHRISTMAS
ON MY MIND