English summary

2 Phenomenological models

few equations, simple nonlinearities, feasible for bifurcation analysis

qualitative agreement of time series

2.1 FitzHugh-Nagumo model

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y \qquad x\text{: activator}, \quad a\text{: bifurcation parameter}$$

$$\dot{y} = x + a \qquad y\text{: inhibitor}, \quad \varepsilon\text{: timescale separation}$$

$\varepsilon \ll 1 \Rightarrow x$ fast, y slow

$$\left.\begin{array}{l} |a|<1\text{: oscillatory (limit cycle)} \\ |a|>1\text{: excitable (fixed point)} \end{array}\right\} \xrightarrow[\text{analysis}]{\text{linear stability}} \text{fixed point } (\dot{x}=0, \dot{y}=0)$$

$x^* = -a$

$y^* = -a + \frac{a^3}{3}$

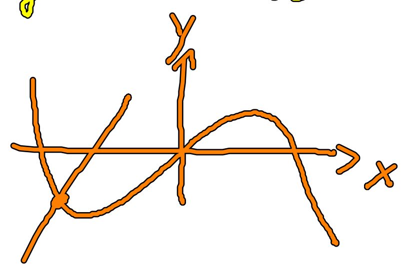
→ unstable for $|a|<1$

→ stable for $|a|>1$

(extended) FitzHugh-Nagumo model ≙ Bonhoeffer-van der Pol model

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y$$

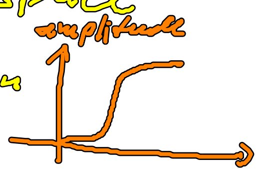
$$\dot{y} = x + a - \gamma y$$

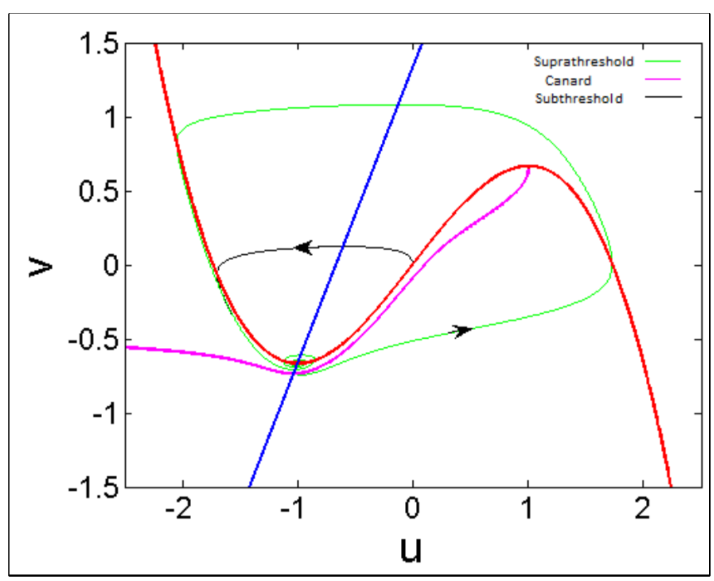


Canard trajectory:

separates subthreshold oscillations from large excursions in phase space

⇒ explosion





excitability type II: at bifurcation point: limit cycle with amplitude 0 and finite frequency $\mathrm{Im}\,\lambda\big|_{|a|=1} = \frac{1}{\sqrt{\varepsilon}}$

2.2 SNIPER model

Saddle-**N**ode **I**nfinite **PER**iod bifurcation

auch als SNIC: **S**addle-**N**ode on an **I**nvariant **C**ycle bekannt.

$$\left.\begin{aligned} \dot{x} &= x(1-x^2-y^2)+y(x+b) \\ \dot{y} &= y(1-x^2-y^2)-x(x+b) \end{aligned}\right\} \quad \begin{aligned} x &= r\cos\varphi \\ &\Longleftrightarrow \\ y &= r\sin\varphi \end{aligned} \qquad \begin{aligned} \dot{r} &= r(1-r^2) \\ \dot{\varphi} &= b - r\cos\varphi \end{aligned}$$

PHYSICAL REVIEW E VOLUME 50, NUMBER 5 NOVEMBER 1994

Resonancelike responses of autonomous nonlinear systems to white noise

T. Ditzinger, C. Z. Ning,* and G. Hu†
Institut für Theoretische Physik und Synergetik, Universität Stuttgart, Pfaffenwaldring 57/IV, D-70550 Stuttgart, Federal Republic of Germany

PHYSICAL REVIEW LETTERS

VOLUME 71 9 AUGUST 1993 NUMBER 6

Stochastic Resonance without External Periodic Force

Hu Gang
International Centre for Theoretical Physics, Trieste 34100, Italy and Physics Department, Beijing Normal University, Beijing 100875, People's Republic of China

T. Ditzinger,* C. Z. Ning, and H. Haken
Institut für Theoretische Physik und Synergetik, Universität Stuttgart, Pfaffenwaldring 57/IV, D-7000 Stuttgart 80, Federal Republic of Germany
(Received 21 December 1992)

Bestimmung des Fixpunkts.

$$0 = x(1-x^2-y^2)+y(x-b)$$

$$0 = y(1-x^2-y^2)-x(x-b)$$

trivialer Fixpunkt am Ursprung $(x_A^*, y_A^*) = (0,0)$

weitere Fixpunkte für $x-b=0 \Rightarrow x^* = b$

und $1-x^2-y^2=0 \Rightarrow y^2 = 1-b^2 \Rightarrow y^* = \pm\sqrt{1-b^2}$

$$\left.\begin{aligned} \Rightarrow (x_B^*, y_B^*) &= (b, \sqrt{1-b^2}) \\ (x_C^*, y_C^*) &= (b, -\sqrt{1-b^2}) \end{aligned}\right\} \text{ existieren nur für } |b|<1 \quad \left| \begin{aligned} (r_B^*, \varphi_B^*) &= (1, \arccos b) \\ (r_C^*, \varphi_C^*) &= (1, -\arccos b) \end{aligned}\right.$$

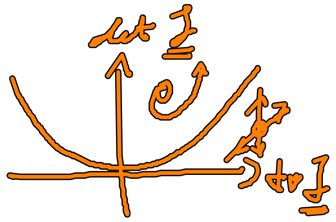
Lineare Stabilitätsanalyse:

Jacobi-Matrix: $\underline{\underline{J}} = \begin{pmatrix} 1-3x^2-y^2+y & -2xy+x-b \\ -2xy-2x+b & 1-x^2-3y^2 \end{pmatrix}$

Eigenwerte $\lambda_{1,2} = \dfrac{\operatorname{tr}\underline{\underline{J}} \pm \sqrt{(\operatorname{tr}\underline{\underline{J}})^2 - 4\det\underline{\underline{J}}}}{2}$

1. Fall $(x_A^*, y_A^*) = (0,0) \Rightarrow \underline{\underline{J}}_A = \begin{pmatrix} 1 & -b \\ b & 1 \end{pmatrix} \Rightarrow \lambda_{1,2} = \dfrac{2 \pm \sqrt{4-4(1+b^2)}}{2}$

$= 1 \pm ib$



(x_A^*, y_A^*) ist ein instabiler Fokus
($\operatorname{Re}\lambda > 0$, $\operatorname{Im}\lambda \neq 0$ für $b \neq 0$)

2. Fall $(x_B^*, y_B^*) = (b, +\sqrt{1-b^2})$

$\underline{\underline{J}}_B = \begin{pmatrix} -2b^2+\sqrt{1-b^2} & -2b\sqrt{1-b^2} \\ -2b\sqrt{1-b^2}-b & -2+2b^2 \end{pmatrix}$

Eigenwerte als Lösung der charakteristischen Gleichung:

$\det(\underline{\underline{J}}_B - \lambda \mathbb{1}) = \ldots = (\lambda+2)(\lambda - \sqrt{1-b^2}) = 0$

Eigenwerte $\lambda_1 = -2$, $\lambda_2 = \sqrt{1-b^2} > 0$ für $|b| < 1$

$\Rightarrow$ 1 stabile, 1 instabile Richtung $\Rightarrow$ Sattelpunkt

3. Fall: $(x_C^*, y_C^*) = (b, -\sqrt{1-b^2})$

$\det(\underline{\underline{J}}_C - \lambda \mathbb{1}) = \ldots = (\lambda+2)(\lambda + \sqrt{1-b^2}) = 0$

$\Rightarrow$ Eigenwerte $\lambda_1 = -2$, $\lambda_2 = -\sqrt{1-b^2} < 0$ für $|b| < 1$

$\Rightarrow$ 2 stabile Richtungen $\Rightarrow$ Knoten

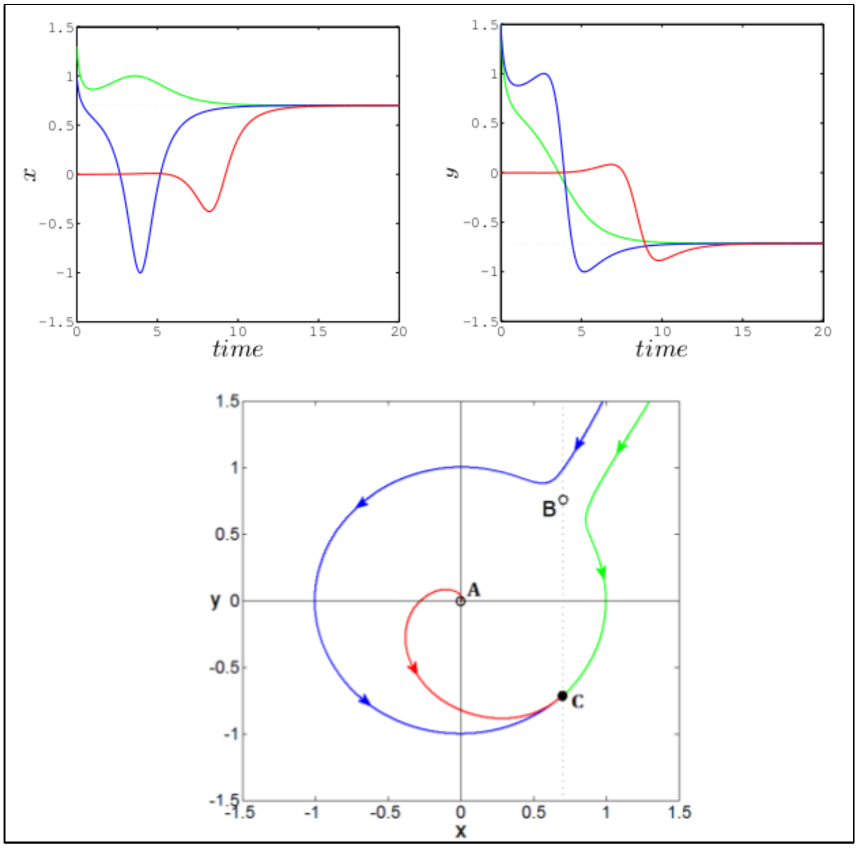
Bei $|b| = 1$: $(x_B^*, y_B^*) = (1, 0) = (x_C^*, y_C^*)$

$\Rightarrow$ Fixpunkte kollidieren mit $\mathrm{Im}\,\Lambda = 0$

$\Rightarrow$ intrinsische Zeit $T \sim \frac{1}{\mathrm{Im}\,\Lambda} \to \infty$

Dynamische Szenarien:

(i) unterhalb der Bifurkation $|b| < 1$, z.B. $b = 0.7$



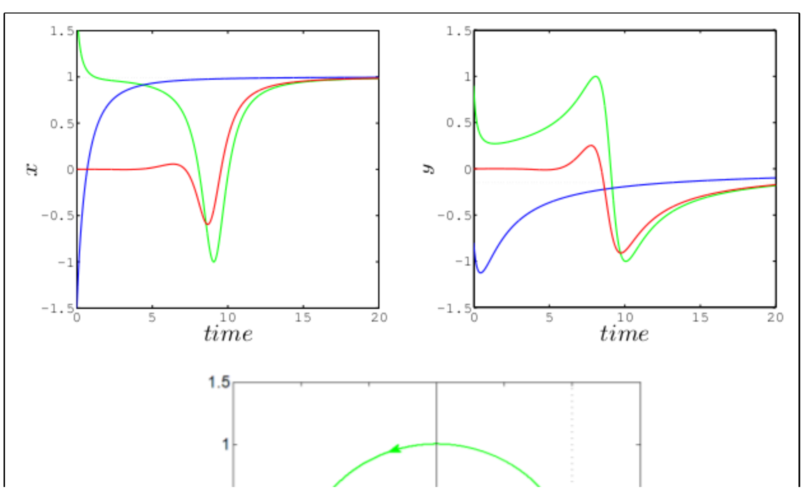
3 Fixpunkte

Fokus A

Sattelpunkt B

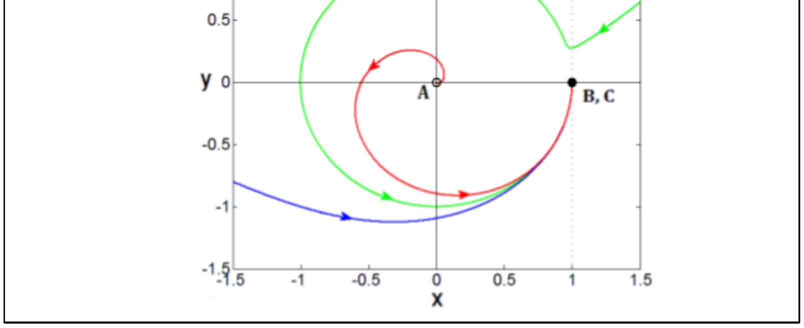
Stabiler Knoten C

(ii) an der Bifurkation $|b| = 1$, z.B. $b = 1$

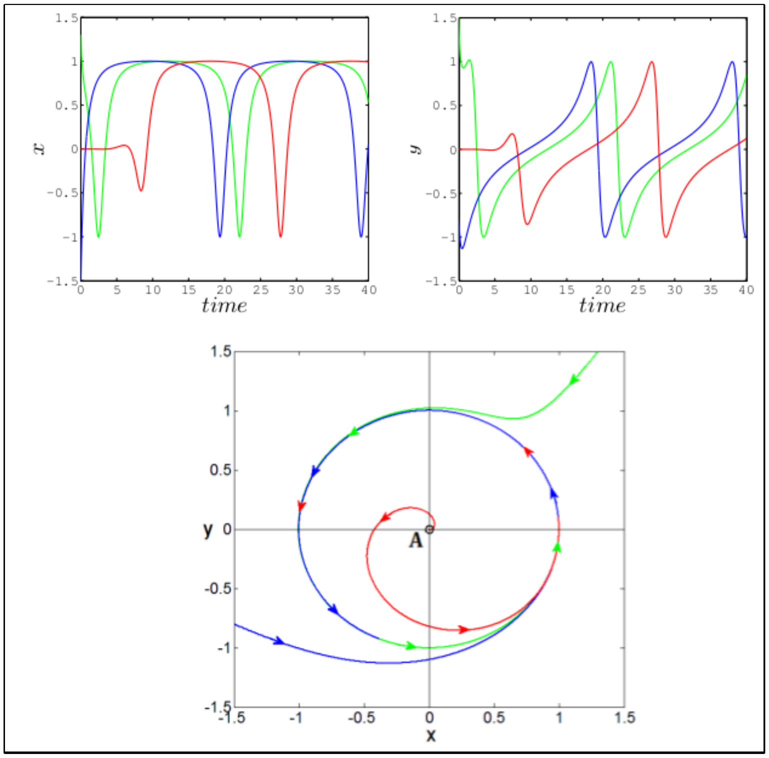


A: Fokus

Fixpunkte B & C kollidieren



(iii) oberhalb der Bifurkation $|b|>1$, z.B. $b = 1.05$



Fokus A

Grenzzyklus mit Radius 1

langsame Dynamik in der Nähe von $(1,0)$ $\Rightarrow$ "Geist"

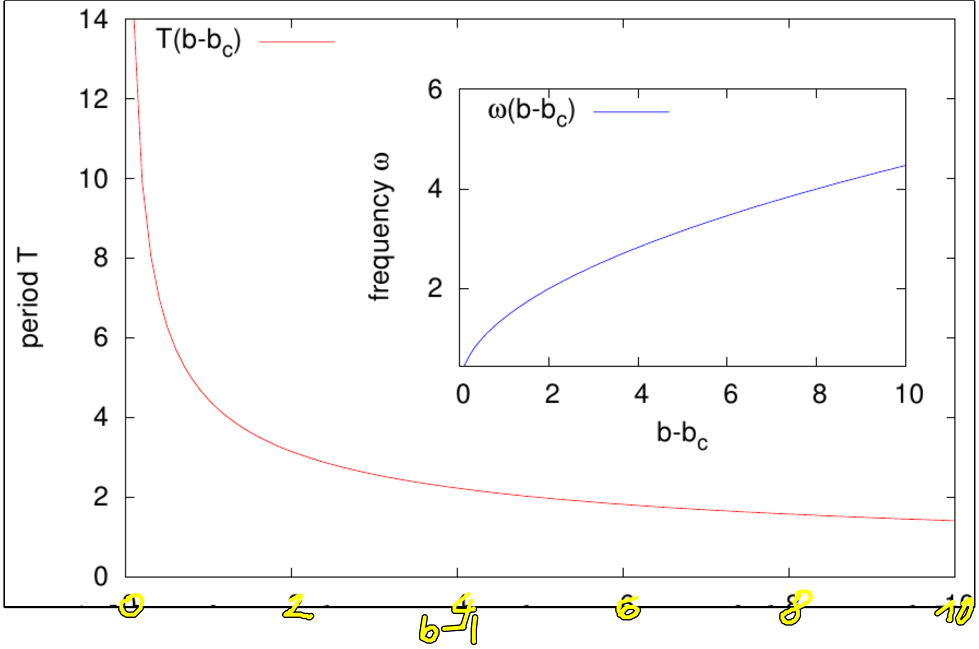
Berechnung der Periode des Grenzzyklus $(b>1)$

$$\dot{\varphi} = b - \cos\varphi \Rightarrow d\varphi = (b - \cos\varphi)\, dt$$

$$\Rightarrow dt = \frac{d\varphi}{b - \cos\varphi}$$

$$\Rightarrow \text{Periode } T = \int_0^{2\pi} \frac{d\varphi}{b-\cos\varphi} = \ldots = \frac{2\pi}{\sqrt{b^2-1}}$$

$\Rightarrow$ Periode T divergiert für $b \to 1$



Am Bifurkationspunkt:
unendliche Periode
endliche Amplitude
$\Rightarrow$ Anregbarkeit Typ I

2.3 Hindmarsh-Rose-Modell (2D)

$$\dot{x} = c\left(x - \frac{x^3}{3} - y + z\right)$$

$$\dot{y} = \frac{1}{c}\left(x^2 + dx - by + a\right)$$

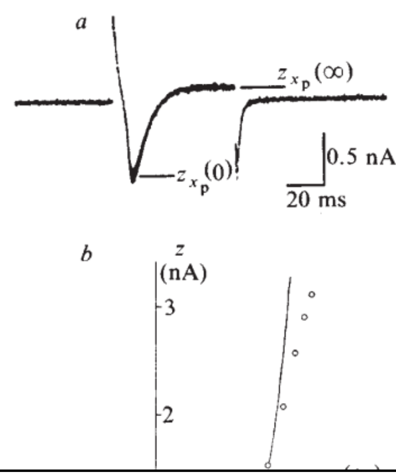
kubische und quadratische Nullkline

7. Egan, R. W., Galen, P. H., Beveridge, G. C., Phillips, G. B. & Marnett, L. J. *Prostaglandins* **16**, 861–869 (1978).
8. Mason, R. T. & Staszewska-Barczak, J. *J. clin. exp. Pharmac. Physiol.* **6**, 678–685 (1979).
9. Takeguchi, C., Kohno, E. & Sih, C. J. *Biochemistry* **10**, 2372–2376 (1971).
10. Winter, C. A., Risley, E. A. & Nuss, G. W. *Proc. Soc. exp. Biol. Med.* **111**, 554–557 (1962).
11. Branceni, D., Azadian-Boulanger, G. & Jequier, R. *Archs int. Pharmacodyn.* **152**, 15–24 (1964).
12. Kemper, F. & Ameln, G. *Z. ges. exp. Med.* **131**, 407–415 (1959).
13. Flückiger, E., Schlach, W. & Taeschler, M. *Schweiz. med. Wschr.* **93**, 1232–1237 (1963).

A model of the nerve impulse using two first-order differential equations

J. L. Hindmarsh & R. M. Rose

Department of Applied Mathematics and Astronomy and Department of Physiology, University College, Cardiff, Cardiff CF1 1XL, UK



Thus the assumed form for our equations is

$$\dot{x} = -a(f(x) - y - z) \tag{7}$$

$$\dot{y} = b(f(x) - q\,e^{rx} + s - y) \tag{8}$$

where $f(x) = cx^3 + dx^2 + ex + h$, and a–h, q, r and s are constants.

After measuring a and b, and fitting cubic and exponential functions to the $z_{x_p}(0)$ and $z_{x_p}(\infty)$ data of Fig. 1b, the solutions of equations (7) and (8) were obtained by numerical integration.

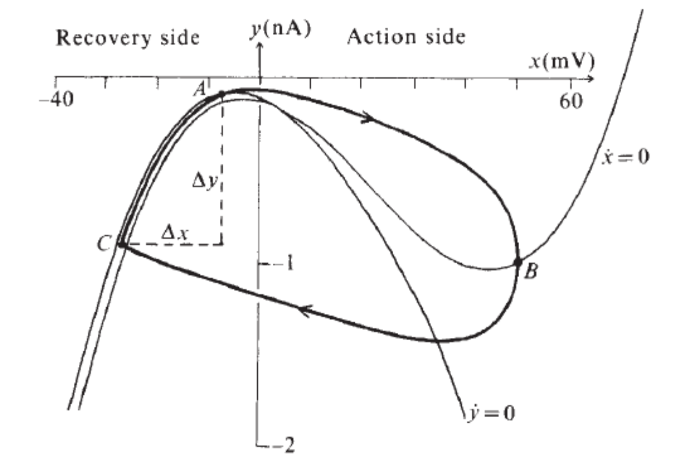


Fig. 3 Phase plane representation of the limit cycle solution to equations (7) and (8). The values of the constants are the same

International Journal of Bifurcation and Chaos, Vol. 17, No. 3 (2007) 985–998

BIFURCATIONS IN TWO-DIMENSIONAL HINDMARSH–ROSE TYPE MODEL

SHIGEKI TSUJI*

†*Aihara Complexity Modelling Project, ERATO, JST, 3-23-5-201 Uehara, Shibuya-ku, Tokyo 151-0064, Japan*

TETSUSHI UETA

Center for Advanced Information Technology, The University of Tokushima, 2-1 Minami-Josanjima, Tokushima 770-8506, Japan

HIROSHI KAWAKAMI

The University of Tokushima, 2-24, Shinkura, Tokushima 770-8501, Japan

HIROSHI FUJII

Department of Information and Communication Sciences, Kyoto Sangyo University, Kamigamo-Motoyama, Kita-ku, Kyoto 603-8555, Japan

KAZUYUKI AIHARA†

**Institute of Industrial Science, The University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Japan*

Bestimmung der FP: $\dot{x} = 0$, $\dot{y} = 0$

⇒ Lösen der Gleichung: $\alpha x^3 + \beta x^2 + \gamma x + \delta = 0$

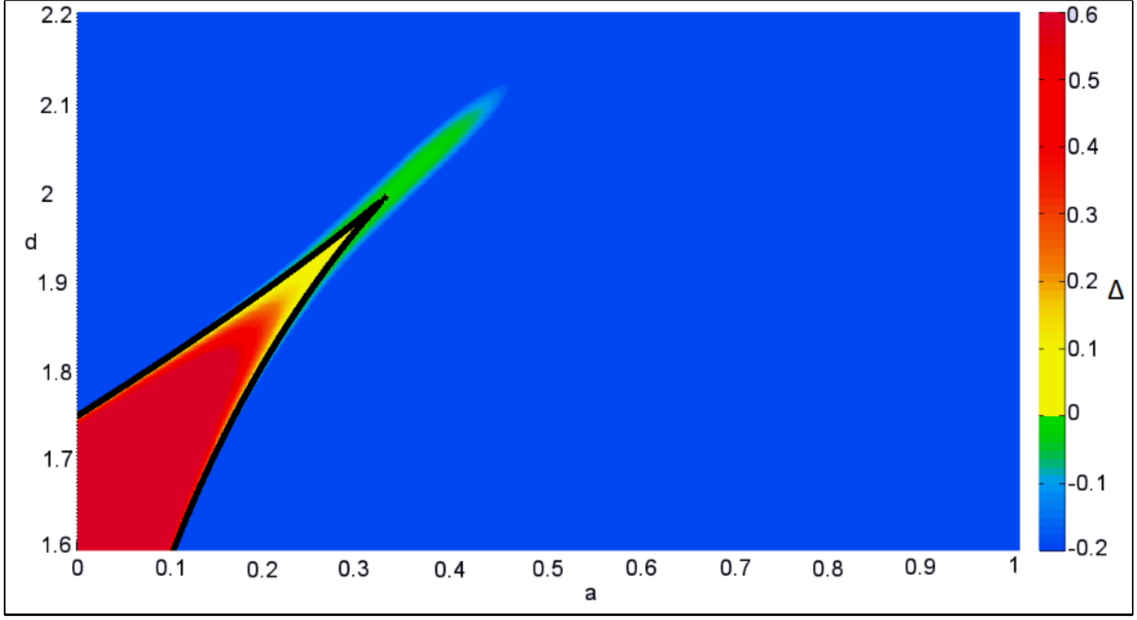
hier (für $b=1$, $c=3$, $z=0$): $\alpha = 1$, $\beta = 3$, $\gamma = 3(d-1) + 3a$

⇒ Berechne Diskriminante: $\Delta = \beta^2\gamma^2 - 4\alpha\gamma^3 - 4\beta^3\delta - 27\alpha^2\delta^2 + 18\alpha\beta\gamma\delta$

(a) $\Delta > 0$: 3 reelle Lösungen

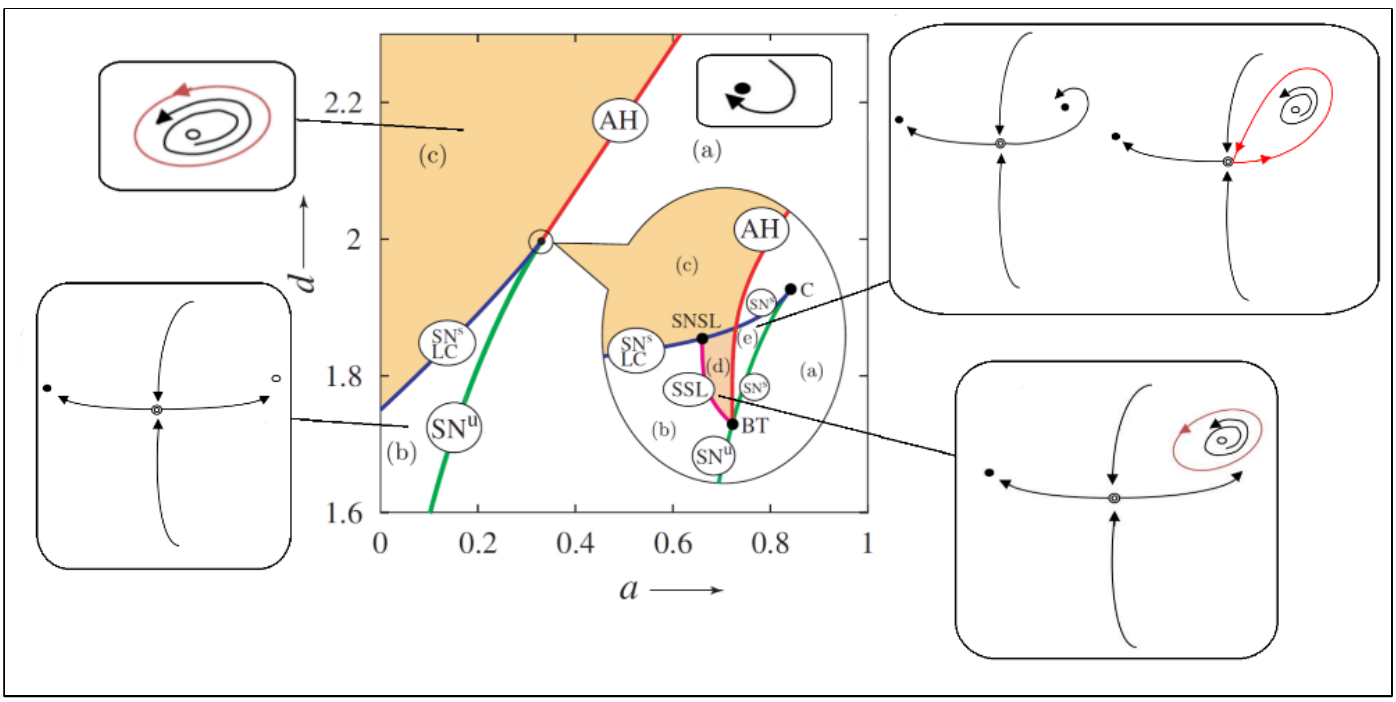
(b) $\Delta = 0$: 1 Lösung mit Vielfachheit 2, 1 weitere Lösung

(c) $\Delta < 0$: 1 reelle Lösung, 2 komplex konjugierte Lösungen



Eigenwerte per linearer Stabilitätsanalyse

$$\underline{\underline{J}} = \begin{pmatrix} c - c(x^*)^2 & -c \\ \frac{1}{c}(2x^*+d) & -\frac{b}{c} \end{pmatrix}$$



Label	Description
(a)	1 stable fixed point
(b)	1 saddle, 1 stable, 1 unstable fixed point
(c)	1 unstable fixed point, 1 stable limit cycle
(d)	1 saddle, 1 stable fixed points, 1 unstable fixed point, 1 stable limit cycle
(e)	1 saddle, 2 stable fixed points
AH	(Andronov-)Hopf bifurcation
SNLC	Saddle-node bifurcation on a limit cycle
SN	Saddle-node bifurcation (of equilibria)
BT	Bogdanov-Takens bifurcation
C	Cusp bifurcation
SSN	Saddle-separatrix loop bifurcation
SNSL	Saddle-node on separatrix loop bifurcation

Fortsetzung folgt ...