English summary:

2.2 SNIPER model

Saddle-node infinite period bifurcation

$$\left.\begin{aligned}\dot{x} &= x(1-x^2-y^2)+y(x+b)\\ \dot{y} &= y(1-x^2-y^2)-x(x+b)\end{aligned}\right\} \iff \left\{\begin{aligned}\dot{r} &= r(1-r^2)\\ \dot{\varphi} &= b - r\cos\varphi\end{aligned}\right.$$

| LC for $|b|>1$ with period $T=\frac{2\pi}{\sqrt{b^2-1}}$

· Fixed points and their stability ($\dot{x}=0$, $\dot{y}=0$, linear stability analysis)

$(x_A^*, y_A^*) = (0,0) \Rightarrow$ Eigenvalues $\Lambda_{1,2} = 1 \pm ib \Rightarrow$ unstable focus

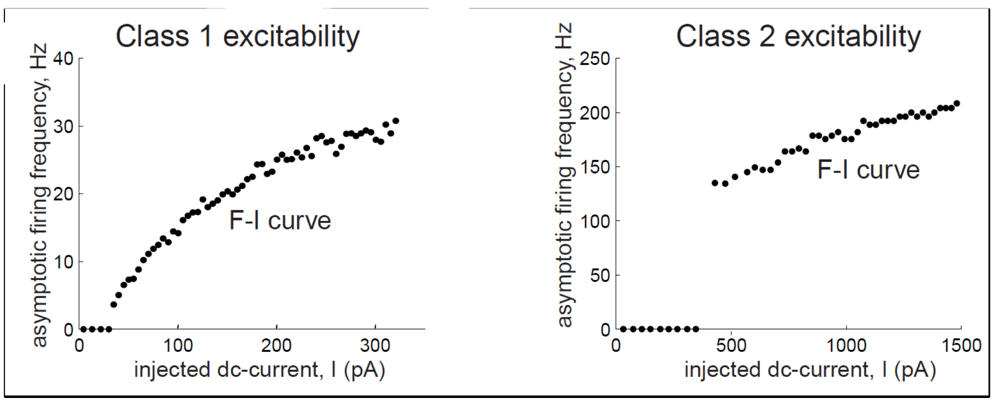
$(x_B^*, y_B^*) = (b, +\sqrt{1-b^2}) \Rightarrow \Lambda_1 = -2,\ \Lambda_2 = +\sqrt{1-b^2} \Rightarrow$ saddle node

$(x_C^*, y_C^*) = (b, -\sqrt{1-b^2}) \Rightarrow \Lambda_1 = -2,\ \Lambda_2 = -\sqrt{1-b^2} \Rightarrow$ stable node

Limit cycle for $|b|>1$: period diverges for $|b| \to 1$

$\Rightarrow$ excitability type I

(frequency = 0 at bif.)



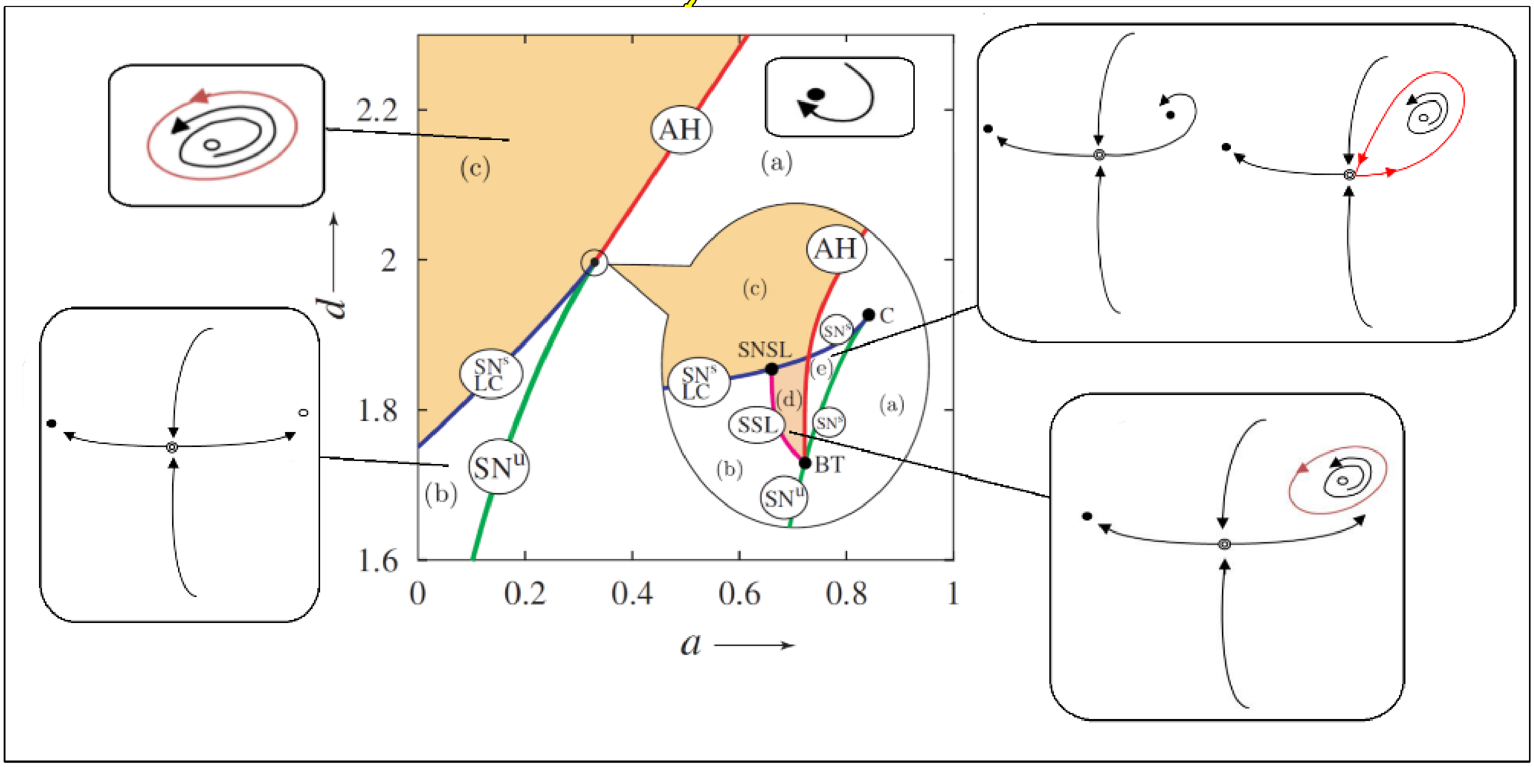
2.3 Hindmarsh-Rose model (2D)

$$\left.\begin{aligned}\dot{x} &= c\left(x - \frac{x^3}{3} - y + z\right)\\ \dot{y} &= \frac{1}{c}\left(x^2 + dx - by + a\right)\end{aligned}\right\}$$

FPs $\iff$ intersection of nullclines

$\Rightarrow$ 1, 2, or 3 FPs (roots of cubic polynomial)

LSA: $\underline{\underline{J}} = \begin{pmatrix} c - c\,(x^*)^2 & -c \\ \frac{1}{c}(2x^* + d) & -\frac{b}{c} \end{pmatrix}$ depends only on x^* coordinate



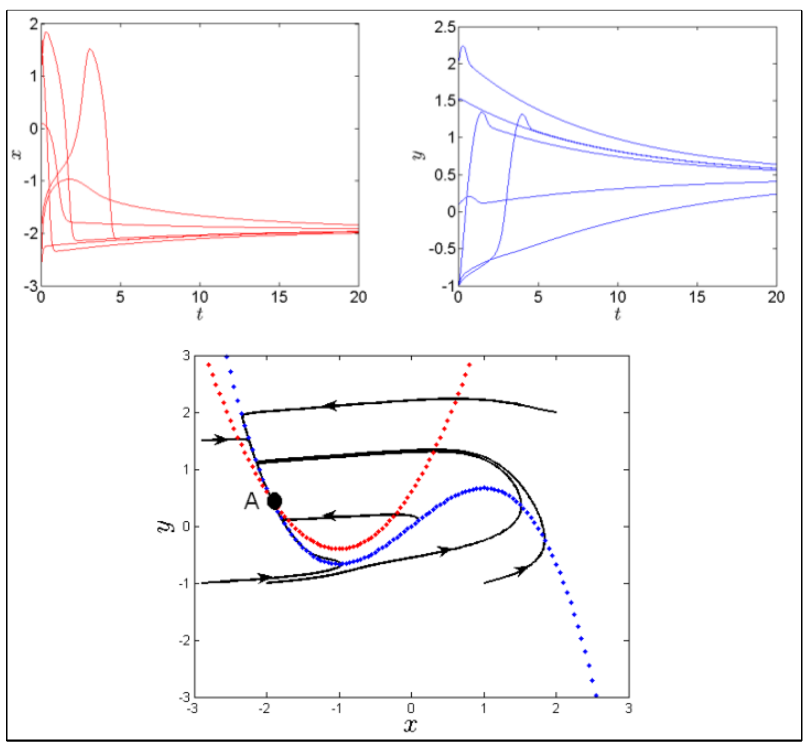
2.3 Hindmarsh-Rose-Modell (Fortsetzung)

Szenario ($b=1$, $c=3$, $z=0$):

(a) $a=0.6$, $d=1.7$

Stabiler Knoten

$\lambda_{1,2} < 0$



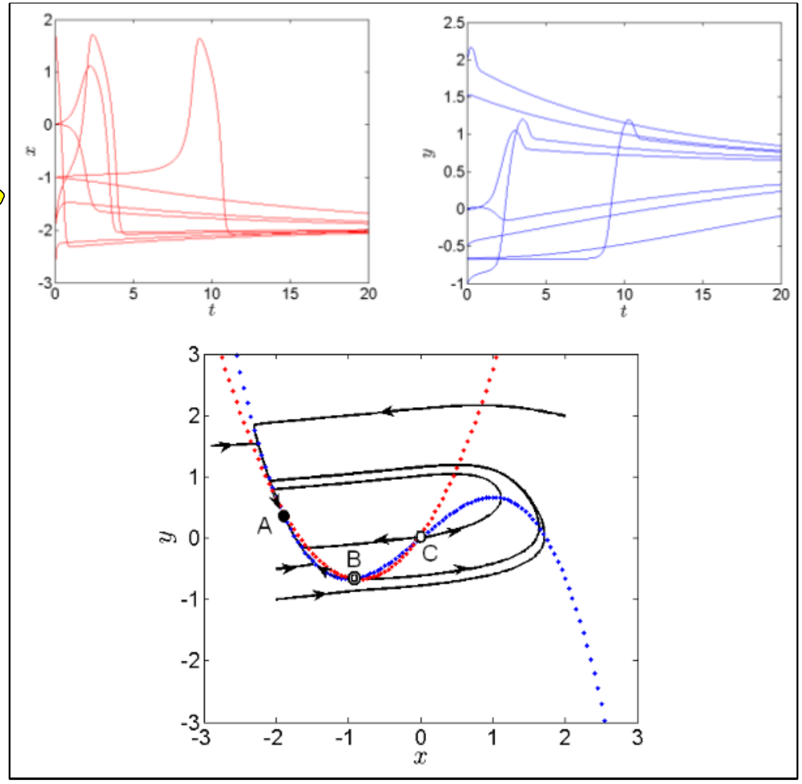
(b) $a = 0.05$, $d = 1.7$

3 Fixpunkte:

A: stabiler Knoten $\lambda_{1,2} < 0$

B: instabiler Knoten $\lambda_{1,2} > 0$

C: Sattelpunkt $\lambda_1 < 0$, $\lambda_2 > 0$

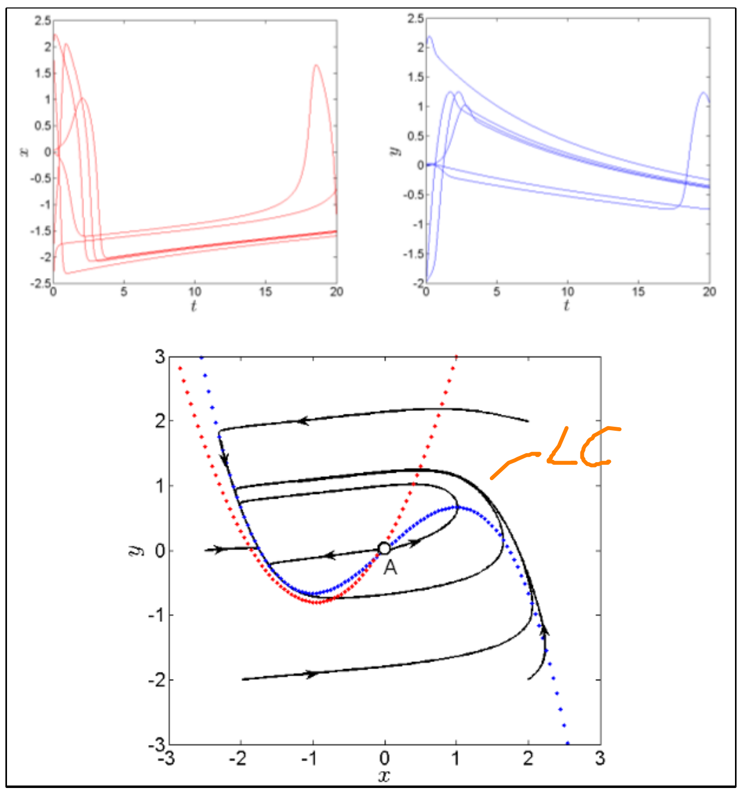


(c) $a = 0.1$, $d = 1.9$

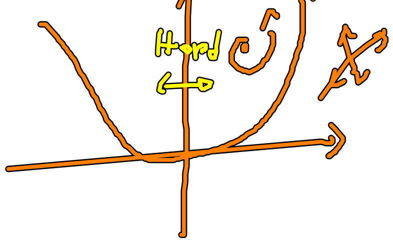
1 Fixpunkt

instabiler Knoten

$\lambda_{1,2} > 0$



übergang in stabil Fokus → instabiler Knoten



Hindmarsh-Rose-Modell mit 3 Variablen

$$\dot{x} = y - a x^3 + b x^2 - z + I$$ kubisch

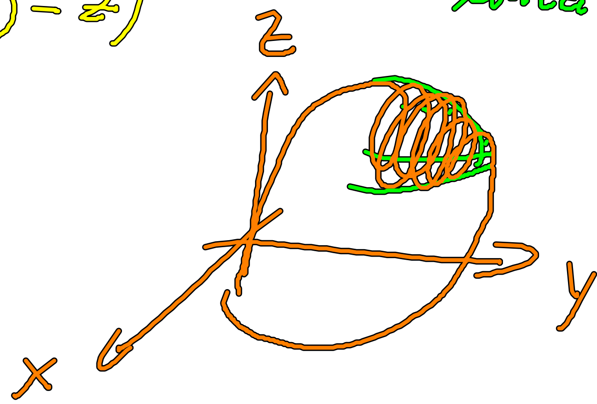
$\dot{y} = c - dx^2 - y$

quadratisch

$\dot{z} = \varepsilon(s(x - x_0) - z)$

linear

$\Rightarrow$ Bursting



3 Physiologische Modelle

- Physiological models:
  - e.g. Hodgkin-Huxley equations
  - Many physiological details and processes
  - Detailed description of single cell
  - Many equations, many parameters
  - Applicable to ensembles of many oscillators?
  - Feasible for bifurcation analysis?

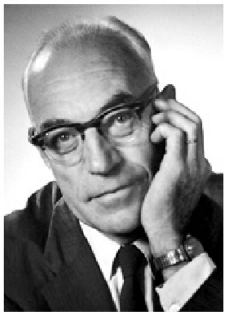
3.1 Hodgkin-Huxley-Modell

3.2 Morris-Lecar-Modell

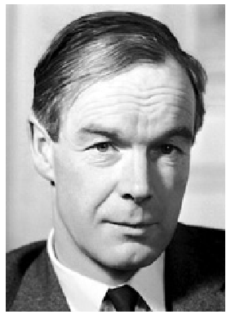
3.1 Hodgkin-Huxley-Modell

The Nobel Prize in Physiology or Medicine 1963

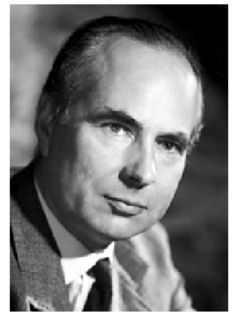
"for their discoveries concerning the ionic mechanisms involved in excitation and inhibition in the [...] nerve cell membrane"



Sir John Carew Eccles



Alan Lloyd Hodgkin



Andrew Fielding Huxley

*5.02.1914
†20.12.1988

*22.11.1917
†30.05.2012

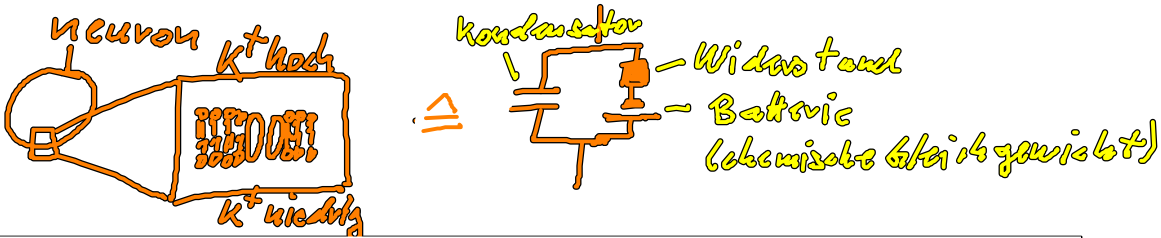
J. Physiol. (1952) 117, 500–544

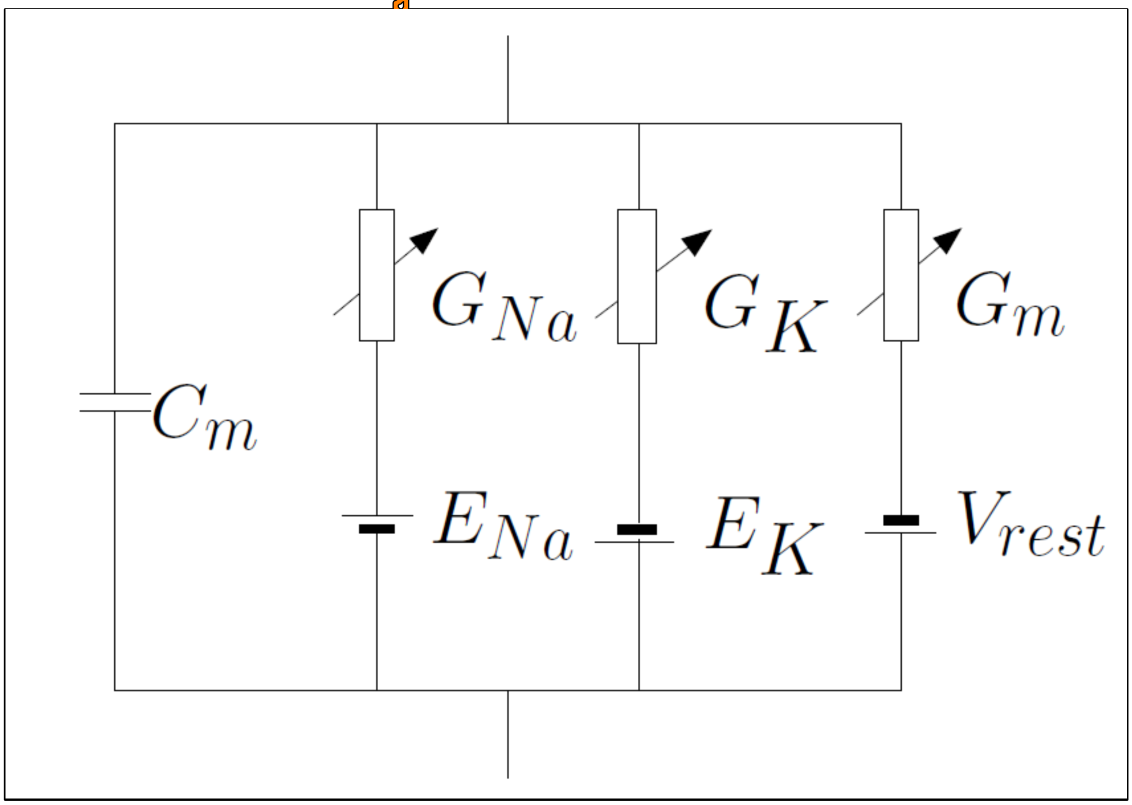
A QUANTITATIVE DESCRIPTION OF MEMBRANE CURRENT AND ITS APPLICATION TO CONDUCTION AND EXCITATION IN NERVE

By A. L. HODGKIN and A. F. HUXLEY

From the Physiological Laboratory, University of Cambridge

"conductance-based model"





Kirchhoff-Gesetz ($\sum_{k=1}^{n} I_k = 0$ | innerhalb einer Masche)
| an einem Knoten

$$I_m = -\left(\underbrace{I_{Na} + I_K + I_{leak}}_{I_{ionic}}\right)$$

Membran

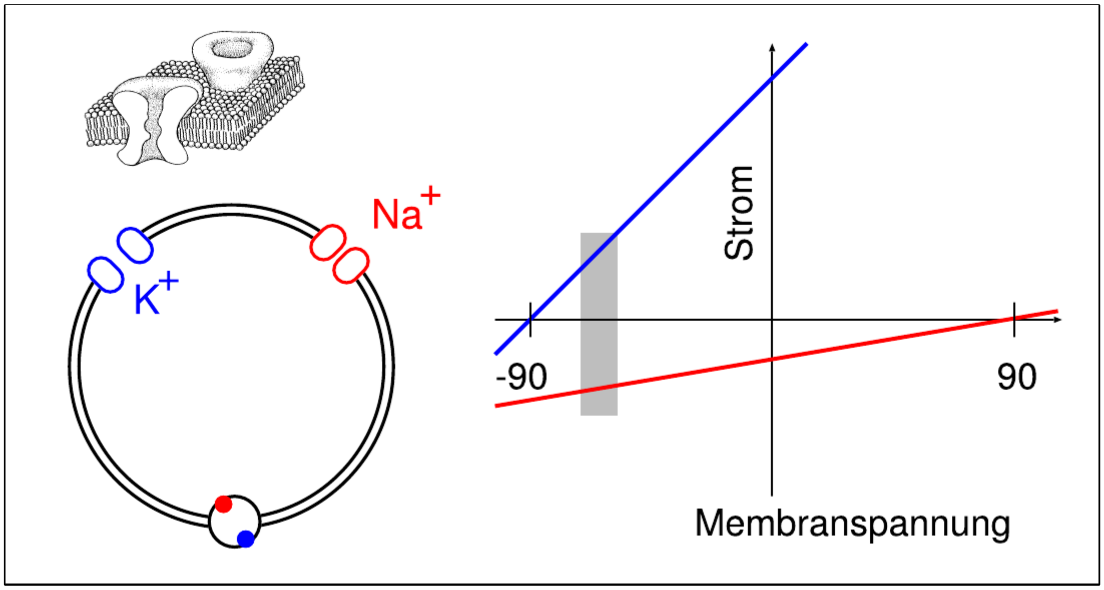
Spannung am Kondensator (Membran): $Q = CU$

$$\dot{Q} = C\dot{U} \Rightarrow C\dot{U} = I_m = -I_{ionic}$$

Ohmsches Gesetz: $U = RI \underset{g=\frac{1}{R}}{\Longrightarrow} g(V - E_{ion}) = I_{ion}$

E_{Ion} (Bakterie) ergibt sich aus Nernst-Potenzial bei unterschiedlichen Ionenkonzentrationen: $E_{Ion} \sim \ln \frac{[Ion]_{in}}{[Ion]_{out}}$

Ursache: Balance zwischen Diffusion und elektrostatischen Drift

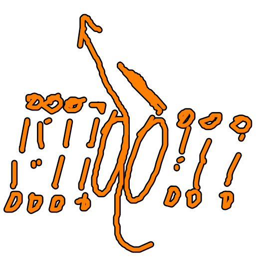


Achtung: Leitfähigkeit g_{Ion} ist nicht linear/hängt von V

$\Rightarrow g_{Ion} = \bar{g}$ "gating variable"

max. Leitfähigkeit

spannungsabhängig



gate open

$g(V) = \bar{g}\, p(V)$

Wahrscheinlichkeit für "offen"

, $\text{Offen: O} \underset{\alpha}{\overset{\beta}{\rightleftarrows}} \text{geschlossen C = closed}$

$\dot{p} = -\beta p + \alpha (1-p)$

angewendet auf K und Na:

$I_K = \bar{g}_K n^4 (V - E_K)$ mit $\dot{n} = \alpha_n(V)(1-n) - \beta_n(V)\, n$

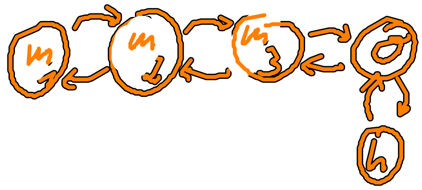
$$\alpha_n(V) = 0.01 \frac{10 - V}{e^{(10-V)/10} - 1}$$

$$\beta_n(V) = 0.125 e^{-V/80}.$$

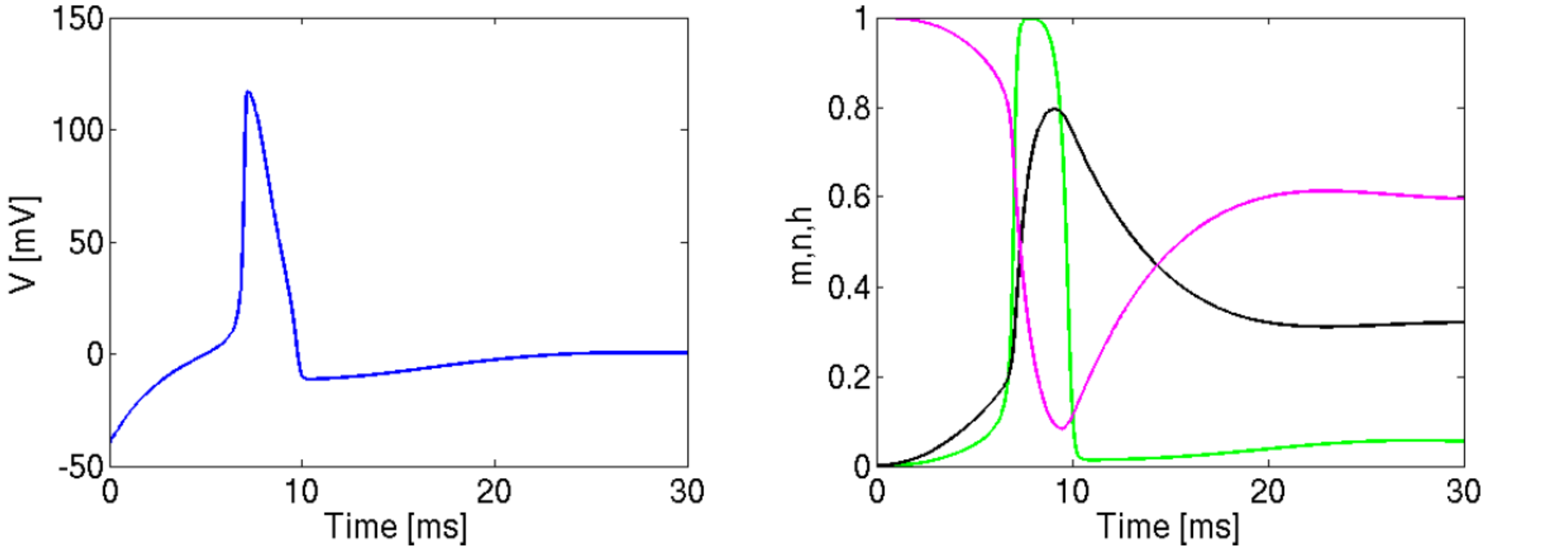
$$C_1 \underset{\alpha}{\overset{\beta}{\rightleftarrows}} C_2 \underset{\alpha}{\overset{\beta}{\rightleftarrows}} C_3 \underset{\alpha}{\overset{\beta}{\rightleftarrows}} C_4 \underset{\alpha}{\overset{\beta}{\rightleftarrows}} O$$

$$I_{Na} = \bar{g}_{Na}\, m^3 h \,(V - E_{Na}) \quad \text{mit} \quad \begin{aligned} \dot{m} &= \alpha_m(V)\,(1-m) - \beta_m(V)\, m \\ \dot{h} &= \alpha_h(V)\,(1-h) - \beta_h(V)\, h \end{aligned}$$

2 unabhängige Kanäle



$$\alpha_m(V) = 0.1\frac{25-V}{e^{(25-V)/10}-1}$$
$$\beta_m(V) = 4e^{-V/18}$$
$$\alpha_h(V) = 0.07e^{-V/20}$$
$$\beta_h(V) = \frac{1}{e^{(30-V)/10}+1}.$$



Left panel: Membrane voltage. Right panel: Activation potential, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).

Gleichung für Membranpotenzial:

$$C_m \dot{V} = -\bar{g}_K\, n(V)^4 (V - E_K) - \bar{g}_{Na}\, m(V)^3 h(V)(V - E_{Na}) - \bar{g}_{Leak}(V - E_{Leak})$$

Fortsetzung folgt ...